Siep
XIII CONFERENZA

STATO O MERCATO?
Intervento pubblico e architettura dei mercati
Pavia, Università, 5 - 6 ottobre 2001

LISA GRAZZINI

# AD VALOREM AND PER UNIT TAXATION IN AN OLIGOPOLY MODEL

pubblicazione internet realizzata con contributo della COMPAGNIA di San Paolo

Società italiana di economia pubblica

Dipartimento di economia pubblica e territoriale – Università di Pavia

# Ad Valorem and Per Unit Taxation in an Oligopoly Model

Lisa Grazzini*

April 2001

**Abstract**

This paper compares the welfare effects of ad valorem and per unit commodity taxation, in a model of oligopolistic interaction. Our main result is that, when the number of consumers is sufficiently high, per unit taxes welfare dominate ad valorem taxes.

Keywords: Imperfect competition, Strategic market game, Commodity Taxation.

JEL Classification: H22, L13, C72, D51.

*CORE, Université Catholique de Louvain, Belgium and Dipartimento di Scienze economiche, Università di Firenze, Italy. E-mail: grazzini@cce.unifi.it.

I am deeply grateful to Jean J. Gabszewicz, Maurice Marchand and Pierre Pestieau for their helpful comments and suggestions.

The author gratefully acknowledges a European Commission Trade and Mobility fellowship No ERBFMBICT971936.

# 1 Introduction

The main purpose of taxation is to collect resources to finance a given public budget requirement. When lump-sum taxes are not available, commodity taxes have to be chosen optimally in order to minimize the welfare loss due to the distortions following their introduction. One way to pursue this aim consists in comparing the welfare properties of different types of commodities taxes, and accordingly selecting the one minimizing the distortions introduced. In imperfectly competitive economies, this welfare comparison between different tax instruments becomes particularly urgent since the distortions caused by fiscal mechanisms are added to other distortions due to the strategic behaviour of agents.[1]

In the public finance literature, the welfare comparison between two particular types of tax systems, namely ad valorem and per unit taxes, is at the basis of a long tradition, and several contributions show the superiority of ad valorem over per unit taxation when firms behave strategically.[2] In this respect, also the present paper aims at comparing the welfare properties of ad valorem and per unit taxes when some agents - the firms - behave strategically while others - the consumers - behave competitively. However, we provide a counter-example which shows that per unit taxation may welfare dominate ad valorem taxation. In our set-up, a shift from an ad valorem tax to a per unit tax which raises an equal amount of tax revenue, leads to a greater aggregate welfare, when the number of consumers is sufficiently high with respect to the number of firms.

Our analysis uses a strategic market game formulation, firstly developed to describe the strategic behaviour of agents in the case of exchange economies (Shapley (1976), Shapley and Shubik (1977)). Following this line of research, we consider the oligopoly model proposed by Gabszewicz and Michel (1997) which allows to study strategic behaviour within a productive economy cast into a general equilibrium context. More specifically, this market game can be viewed as an extension of the Cournot oligopoly model, in which the market is composed of two goods and two groups of agents.[3] The first group of agents is made of consumers who share a competitive behaviour on the exchange market, and initially own only the first good. The second group is composed of agents who do not own initially any good, but own a linear technology transforming the first good into a second one. Furthermore, these agents behave noncompetitively when they sell the second good, and use quantities as strategies in order to manipulate the exchange rate between the two commodities. Finally, all agents would like to consume both goods. In this context, we analyse the welfare

[1] Notice that, with imperfect competition, taxation can also serve the purpose of correcting the distortions generated by the market mechanism, like in Guesnerie and Laffont (1978), Myles (1989) and Gabszewicz and Grazzini (1999).

[2] See Keen (1998) for a survey on this topic.

[3] In the case of an exchange economy, see Gabszewicz and Michel (1997) for a general definition of an oligopoly equilibrium which also covers a situation of multilateral oligopoly with an arbitrary number of goods.

effects of commodity taxes levied on the good produced by strategic agents. More precisely, we compare the different impact of ad valorem and per unit taxes, when a given public revenue has to be collected. The question we raise is whether the aggregate welfare given by the sum of the utility levels of consumers and oligopolists is greater under ad valorem or per unit taxation.

The welfare comparison between ad valorem and per unit taxes has been extensively studied in the public finance literature.[4] Under perfect competition, per unit and ad valorem taxes which levy equal revenue also result in equal price, and thus they are equivalent (Suits and Musgrave (1953) and Bishop (1968)). On the contrary, under imperfect competition, the superiority of ad valorem over per unit taxation is usually established. In the monopoly case, Suits and Musgrave (1953) show that, if the same yield is obtained from a per unit and an ad valorem tax, the final price will be higher (the output smaller) under the per unit tax. Furthermore, Skeath and Trandel (1994) show an even stronger result, namely that it is possible to shift from any given per unit tax to a suited ad valorem tax such that, at equilibrium, profit, consumer surplus, and tax revenue are each greater under ad valorem taxation than they would be under per unit taxation.

The dominance of ad valorem over per unit taxation has been extended to the oligopoly case only recently by Delipalla and Keen (1992) for the case of symmetric Cournot oligopoly, with both a fixed number of firms and entry, and Denicolò and Matteuzzi (2000) for the asymmetric case.[5] More precisely, Denicolò and Matteuzzi (2000) show that, with constant but asymmetric marginal costs, if the tax rates are sufficiently high, ad valorem taxation leads to greater tax revenue, consumer surplus and industry profits than per unit taxation. The superiority of ad valorem over per unit taxation is also obtained by Delipalla and Keen (1992), but within a general equilibrium model. They analyse the optimal combination of ad valorem and per unit taxes, and show that the optimal per unit tax is zero, so that any public revenue needed requires maximum reliance on ad valorem taxation.[6] In this respect, our contribution aims at providing a counter-example which shows that a per unit tax may welfare dominate an ad valorem tax. Our result is obtained within a particular example, and the different characterizations used to model imperfect competition should be emphasized. Firstly, even if also Delipalla and Keen (1992) use a general equilibrium framework, their analysis is cast into a model inspired

[4] Notwithstanding this issue is one of the oldest in the public finance literature, it continues to be nowaday for its policy implications. For example, see Delipalla and Keen (1992) and Keen (1998) for discussions on the importance of this topic in the European Union, especially with respect to the industries of tobacco and alcoholics.

[5] In the case of a price-setting oligopoly, Colangelo and Galmarini (1997) show that product differentiation may play a crucial role on the Pareto ranking of commodity taxes. For example, they show that a low degree of product differentiation may prevent ad valorem taxation from Pareto dominating per unit taxation.

[6] Myles (1996) studies how the simultaneous use of both ad valorem and per unit taxes can eliminate the welfare loss due to imperfect competition.

from Gabszewicz and Vial (1972), which constitutes a much more general set-up than the one considered here. This model involves a productive sector with firms behaving noncompetitively while consumers are price-takers. Secondly, in order to perform the welfare comparison between ad valorem and per unit taxes, they assume that a single representative consumer is the sole owner of all firms in the noncompetitive industry. Thus, they show that a conflict of interests arises since the representative agent prefers ad valorem taxes in his role of consumer, and per unit taxes in his role of firms' owner. On the contrary, in our model, the strategic agents are simultaneously producers and consumers of the taxed good and prefer per unit taxes, while the competitive agents, who are only consumers, are in favor of ad valorem taxes. Accordingly, the conflict of interests arises between these two groups of agents. Finally, their paper differs from ours with respect to the basis of comparison used to compare ad valorem and per unit taxes. More precisely, they consider a tax change that, while not fully revenue-neutral, leaves total tax payments at the initial equilibrium price unchanged. Differently, as a basis of comparison, we use ad valorem and per unit taxes which generate the same tax revenue, i.e. are revenue-neutral (Suits and Musgrave (1953)).

The next section provides the model of homogeneous oligopoly with ad valorem and per unit taxes, and section 3 provides a welfare comparison between the two forms of taxation. Section 4 contains a short conclusion.

## 2 The model

Consider a productive economy with two goods, 1 and 2, and including $n+m$ agents, falling into two types.[7] Agents $i$, $i = 1, ..., n$, -the *consumers*- behave competitively on the market and their initial endowment consists only of good 1. Agents $j$, $j = 1, ..., m$, -the *oligopolists*- do not own initially any good, but own each a linear technology allowing to produce good 2 using good 1 as input. More precisely, consider the following economy. All agents have the same utility function $U$ defined by

$$U(x^1, x^2) = x^1 x^2.$$

Initial endowments are defined by

$$a_i = (\frac{1}{n}, 0), \qquad i = 1, ..., n \tag{1}$$

and

$$a_j = (0, 0), \qquad j = 1, ..., m. \tag{2}$$

Furthermore, agents of type 2 own a linear technology, defined by

$$y_j = \frac{1}{\alpha_j} z_j, \qquad \alpha_j > 0 \tag{3}$$

[7] This model of homogeneous oligopoly has been proposed by Gabszewicz and Michel (1997).

where $y_j$ denotes the amount of good 2, which can be produced out of an amount $z_j$ of good 1. More specifically, notice that agents of type 2 have to take two distinct decisions. Firstly, they have to decide how much of good 2 to produce, which also determines via (3) the amount $z_j$ of good 1 to buy from agents of type 1. Secondly, they have to choose which share $q_j$ of the amount $y_j$ produced of good 2 to send to the market for trade (and the resulting amount $y_j - q_j$ to keep for private consumption). Clearly the equilibrium exchange rate between good 1 and good 2 depends on the amount $q_j$ of good 2 sent by each oligopolist $j$ to the market. This amount influences the total supply $\sum_{k=1}^{m} q_k$ of good 2, compared with the fixed total supply $\sum_{k=1}^{n} a_k^1$ of good 1. Consequently, each oligopolist $j$ can individually manipulate the exchange rate by choosing the share $q_j$. This gives rise to a game whose *players* are the oligopolists, with *strategies* for oligopolists $j$, $j = 1, ..., m$, defined by pairs $(q_j, y_j)$ with $q_j \leqslant y_j$.

Now consider that a commodity tax is levied on good 2. Two forms of commodity taxation will be considered, namely, ad valorem and per unit taxes. In the former type, the tax is proportional to the price of the good sold and thus it is a tax on the *value* of sales; in the latter, the tax is a fixed amount levied on each unit of the good sold and consequently, it is a tax on the *volume* of sales.[8]

Let $t$ denote an ad valorem tax, $0 < t < 1$, and $\tau$ a per unit tax, $0 < \tau < \frac{1}{\alpha_j}$, levied on good 2.[9] In the case of an ad valorem tax $t$, the producer price for good 2 obtains as $P^2 = p^2(1-t)$, where $p^2$ is the consumer price for good 2. Consequently, the total product of the tax is given by $R_t = tp^2 \sum_{k=1}^{m} q_k$. If, in contrast, a per unit tax $\tau$ is imposed, the producer price is defined by $P^2 = p^2 - \tau$, and accordingly the total tax product obtains as $R_\tau = \tau \sum_{k=1}^{m} q_k$.

Given a price vector $(p^1, p^2)$, a competitive agent $i$, $i = 1, ..., n$, solves the problem

$$\begin{array}{ll} \max\limits_{x^1, x^2} & x^1 x^2 \qquad s.t. \\ & x^1 + px^2 \leqslant \frac{1}{n}, \end{array}$$

giving rise to individual demand

$$x_i(p) = \left(\frac{1}{2n}, \frac{1}{2np}\right), \qquad i = 1, ..., n \tag{4}$$

where $p = \frac{p^2}{p^1}$, and total demand for good 2 equals to $\frac{1}{2p}$. Thus, the indirect

[8] Notice that in modeling an ad valorem tax, some authors prefer to define it as a percentage of consumer price (Suits and Musgrave (1953), Delipalla and Keen (1992)) while others prefer to specify it as a percentage of producer price (Skeath and Trandel (1994), Colangelo and Galmarini (1997)). In the following, we will use the former formulation but our results remain valid should we switch to the latter one.

[9] In this model, taxes are expressed in real terms, and since the maximum amount of good 2 which can be produced out of good 1 is $1/\alpha_j$ via the linear technology in (3), we assume that $\tau < 1/\alpha_j$.

utility function $S$ of consumers obtains as

$$S(p) = \left(\frac{1}{2n}\right)\left(\frac{1}{2np}\right). \tag{5}$$

Now we proceed to the definition of the *payoffs* of the game among the oligopolists. To this end, assume that producer $j$ has selected the strategy $(q_j, y_j)$, $j = 1, ..., m$. At a price vector $p$, the profit of oligopolist $j$ obtains as

$$p(1-t)q_j - z_j,$$

in the case of ad valorem taxation, and as

$$(p-\tau)q_j - z_j,$$

in the case of per unit taxation. With this profit, he can buy an amount of good 1 equal to $p(1-t)q_j - \alpha_j y_j$, in the case of an ad valorem tax and $(p-\tau)q_j - \alpha_j y_j$, in the case of a per unit tax, yielding resulting utility payoffs

$$(p(1-t)q_j - \alpha_j y_j)\,(y_j - q_j), \tag{6}$$

and

$$((p-\tau)q_j - \alpha_j y_j)\,(y_j - q_j), \tag{7}$$

respectively. Given these strategies $(q_j, y_j)$, $j = 1, ..., m$, the value of $p$ at which supply equals demand on market for good 2 is given by

$$\sum_{k=1}^{m} q_k = \frac{1}{2p};$$

or

$$p = \frac{1}{2\sum_{k=1}^{m} q_k}. \tag{8}$$

By substituting this equilibrium exchange rate in the utility payoffs (6) and (7), we finally obtain the payoffs of the game, namely

$$V(q_j, y_j) = \left(\frac{1-t}{2\sum_{k=1}^{m} q_k} q_j - \alpha_j y_j\right)(y_j - q_j); \qquad j = 1, ..., m, \tag{9}$$

in the case of ad valorem taxation, and

$$V(q_j, y_j) = \left(\left(\frac{1}{2\sum_{k=1}^{m} q_k} - \tau\right) q_j - \alpha_j y_j\right)(y_j - q_j); \qquad j = 1, ..., m, \tag{10}$$

in the case of per unit taxation. At an *oligopoly equilibrium*, $V$ must be maximal with respect to $q_j$ and $y_j$, given the strategies $(q_k, y_k)$ chosen by the oligopolists $k$, $k \neq j$, and this must be satisfied for all $j$, $j = 1, ..., m$. The optimality conditions with respect to $q_j$ and $y_j$ give

$$\frac{1}{y_j - q_j} = \frac{\frac{1-t}{2Q} - \tau - \frac{(1-t)q_j}{2Q^2}}{\left(\frac{1-t}{2Q} - \tau\right) q_j - \alpha_j y_j} = \frac{\alpha_j}{\left(\frac{1-t}{2Q} - \tau\right) q_j - \alpha_j y_j}, \tag{11}$$

with $Q = \sum_{k=1}^{m} q_k$, and where $0 < t < 1$ and $\tau = 0$, in the case of wholly ad valorem taxation, while $\tau > 0$ and $t = 0$, in the case of wholly per unit taxation. From the second equality of the above equation, we obtain that for all $j$, $j = 1, ..., m$, the equality

$$1 - \frac{q_j}{Q} = 2\frac{\alpha_j + \tau}{1 - t}Q, \qquad j = 1, ..., m. \tag{12}$$

must hold at equilibrium. Summing up equations (12), we get, at equilibrium, with $\overline{\alpha} \underset{def}{=} \frac{1}{m}\sum_{j=1}^{m} \alpha_j$,

$$Q_h^* = \frac{(m-1)(1-t)}{2m(\overline{\alpha} + \tau)}, \qquad h = t, \tau; \tag{13}$$

and

$$p_h^* = \frac{m(\overline{\alpha} + \tau)}{(m-1)(1-t)}, \qquad h = t, \tau; \tag{14}$$

where the subscript $h$, $h = t, \tau$, denotes hereafter a variable obtained under ad valorem or per unit taxation. Accordingly, total tax revenue is equal to

$$R_t^* = tp_t^* Q_t^* = \frac{1}{2}t, \tag{15}$$

under ad valorem taxation, and

$$R_\tau^* = \tau Q_\tau^* = \frac{(m-1)\tau}{2m(\overline{\alpha} + \tau)}, \tag{16}$$

under per unit taxation. Notice that in (15) total tax revenue under ad valorem taxation does not depend on the number $m$ of firms which potentially can be active at equilibrium while in (16) total tax revenue with a per unit tax depends on $m$. Furthermore, using (12) and (13), we obtain

$$q_{jh}^* = \frac{m\overline{\alpha} + \tau - (m-1)\alpha_j}{2m^2(\overline{\alpha} + \tau)^2}(m-1)(1-t), \qquad h = t, \tau; \qquad j = 1, ..., m; \tag{17}$$

and

$$y_{jh}^* = \frac{(m\overline{\alpha} + \tau)^2 - (m-1)^2\alpha_j^2}{4m^2\alpha_j(\overline{\alpha} + \tau)^2}(1-t), \qquad h = t, \tau; \qquad j = 1, ..., m. \tag{18}$$

Notice that the oligopoly equilibrium has $m$ "active" firms if, and only if, $\forall j$, $j = 1, ..., m$,

$$\alpha_j < \frac{m\overline{\alpha} + \tau}{m-1}. \tag{19}$$

First we notice that, if all firms are identical, i.e. $\alpha_j = \overline{\alpha}$, the $m$ firms are active at the oligopoly equilibrium, both under ad valorem and per unit taxation. On the other hand, when the average costs $\alpha_j$ are not equal, the "survival condition"

(19) does not necessarily hold for all firms $j$ at the oligopoly equilibrium with $m$ firms. If this is the case, then those firms for which average cost $\alpha_j$ exceeds $\frac{m\overline{\alpha}+\tau}{m-1}$ have to leave the market, and the resulting (endogenous) number of active firms at the oligopoly equilibrium will be lower than $m$.[10] Furthermore, we notice that, under ad valorem taxation, the survival condition (19) is the same as in the case without taxation: the number of active firms in the industry does not depend on the ad valorem tax. With respect to the no-tax framework, the effect of an ad valorem tax is simply that of reducing proportionally all the variables of the model (see (13), (14), (15) and (16)), thus leaving unchanged the number of active firms. Consequently, ad valorem taxation has the desirable property of neutrality with respect to the number of firms operating in the market, namely *it does not distort the number of active firms*. On the contrary, introducing a per unit tax potentially allows less efficient firms to enter into the market, i.e. firms $j$, $j = 1, ..., m$, for which $\frac{m\overline{\alpha}}{m-1} < \alpha_j < \frac{m\overline{\alpha}+\tau}{m-1}$.

In order to compare the effects of ad valorem and per unit taxation, we consider firstly the comparative statics of each tax instrument. From equation (14), a per unit tax $\tau$ affects prices as

$$\frac{\partial p_\tau^*}{\partial \tau} = \frac{m}{m-1} > 0, \tag{20}$$

and an ad valorem tax $t$, as

$$\frac{\partial p_t^*}{\partial t} = \frac{m\overline{\alpha}}{(1-t)^2(m-1)} > 0. \tag{21}$$

Similarly, from (13), a per unit tax $\tau$ affects the aggregate amount of good 2 sent to the market for trade by oligopolists $j$, $j = 1, ..., m$, as

$$\frac{\partial Q_\tau^*}{\partial \tau} = -\frac{m-1}{2m(\overline{\alpha}+\tau)^2} < 0, \tag{22}$$

and an ad valorem tax $t$, as

$$\frac{\partial Q_t^*}{\partial t} = -\frac{m-1}{2m\overline{\alpha}} < 0. \tag{23}$$

Notice that these results are specific to oligopolistic models and in line with previous literature. From (20), per unit taxes are over-shifted: consumer price rises more than the increase in tax, i.e. $\frac{\partial p_\tau^*}{\partial \tau} > 1$. In the case of a Cournot model with linear costs, Delipalla and Keen (1992) show that per unit taxation is over-shifted if, and only if, the elasticity of the slope of inverse demand is greater than one, i.e. $E = -p_{x^2x^2}\frac{x^2}{p_{x^2}} > 1$.[11] In our context, per unit taxes are over-shifted, and from (4), it is easy to check that $E = 2$. On the contrary, the result on the

[10] By successive eliminations of firms, the oligopoly equilibrium results in an endogenous number of active firms. For more details on this point, see Gabszewicz and Michel (1997).

[11] See also Seade (1985), Stern (1987) and Myles (1995) p. 361.

incidence of ad valorem taxation shows that consumer price may rise less than the increase in tax (under-shifting). From (21), over-shifting (under-shifting) occurs if, and only if $\overline{\alpha} > (<) \frac{(m-1)(1-t)^2}{m}$. Finally, in both cases of per unit and ad valorem taxation, an increase in the tax rate leads to a reduction in the total quantity of good 2, $Q_h^*$, $h = t, \tau$, which oligopolists are willing to exchange on the market, thus reinforcing the distortion already generated by their strategic behaviour.

# 3 A comparison between ad valorem and per unit taxes

In this section, we compare the welfare effects of ad valorem and per unit taxation. To perform this analysis, as a basis of comparison, we use ad valorem and per unit taxes that are *revenue-neutral*, at oligopoly equilibrium. More precisely, we consider a shift from an ad valorem tax $t$ to a per unit tax $\tau$ which raises an equal amount of tax revenue.[12] Specifically, from equating (15) and (16), the value of $\tau$ which is used as a basis of comparison obtains as

$$\tau = \frac{m\overline{\alpha}t}{m(1-t)-1}, \tag{24}$$

with $\tau > 0$, under the assumption that $m > \frac{1}{1-t}$. In (24), notice that $m$ is the number of potential active firms and $\overline{\alpha}$ is the average cost of this pool of firms.

In the following proposition, we start by comparing the effects of ad valorem and per unit taxes on the number of active oligopolists, and their resulting utility level at the oligopoly equilibrium.

**Proposition 1** *A revenue-neutral shift from an ad valorem tax $t$ to a per unit tax $\tau$, leads to*

*(i) an increase in the number of active firms $j$, $j = 1, ..., m$, with firms $j$, for which $\frac{m\overline{\alpha}}{m-1} < \alpha_j < \frac{m\overline{\alpha}(1-t)}{m(1-t)-1}$, entering into the market;*

*(ii) a reduction in the utility level of oligopolists $j$, $j = 1, ..., m$, for which $0 < \alpha_j < \frac{m\overline{\alpha}\sqrt{1-t}}{m\sqrt{1-t}+1}$;*

*(iii) an increase in the utility level of oligopolists $j$, $j = 1, ..., m$, for which $\frac{m\overline{\alpha}\sqrt{1-t}}{m\sqrt{1-t}+1} < \alpha_j \leqslant \frac{m\overline{\alpha}(1-t)}{m(1-t)-1}$.*

[12] See Suits and Musgrave (1953) for a discussion on this point. A "small" tax shift that leaves total tax payments unchanged at the initial equilibrium price, but which is not fully revenue-neutral, is instead used by Delipalla and Keen (1992).

**Proof.** By substituting (24) into (19), under per unit taxation, the oligopoly equilibrium has $m$ active firms if, and only if,

$$\alpha_j < \frac{m\overline{\alpha}(1-t)}{m(1-t)-1}.$$

Since, from (19), we know that, under ad valorem taxation, the corresponding condition is $\alpha_j < \frac{m\overline{\alpha}}{m-1}$, with $\frac{m\overline{\alpha}}{m-1} < \frac{m\overline{\alpha}(1-t)}{m(1-t)-1}$, the shift from an ad valorem tax $t$ to the per unit tax $\tau$ in (24) allows the entry of those firms $j$, $j = 1, ..., m$, for which $\frac{m\overline{\alpha}}{m-1} < \alpha_j < \frac{m\overline{\alpha}(1-t)}{m(1-t)-1}$.

Finally, under ad valorem taxation, by substituting (17) and (18) into (9), the utility level reached by oligopolists $j$, $j = 1, ..., m$, obtains as

$$V(q_{jt}^*, y_{jt}^*) = \left(\frac{m\overline{\alpha} - (m-1)\alpha_j}{2m\overline{\alpha}}\right)^4 \frac{(1-t)^2}{\alpha_j}. \tag{25}$$

Similarly, with the per unit tax $\tau$ in (24), by substituting (17) and (18) into (9), the utility level reached by oligopolists $j$, $j = 1, ..., m$, obtains as

$$V(q_{j\tau}^*, y_{j\tau}^*) = \left(\frac{m\overline{\alpha}(1-t) - (m(1-t)-1)\alpha_j}{2m\overline{\alpha}}\right)^4 \frac{1}{\alpha_j}. \tag{26}$$

Accordingly, simple calculations show that the function $f(\cdot, \alpha_j) = V(q_{jt}^*, y_{jt}^*) - V(q_{j\tau}^*, y_{j\tau}^*)$ defined on $\alpha_j \in \left]0, \frac{m\overline{\alpha}}{m-1}\right]$ is strictly positive when $\alpha_j \in \left]0, \frac{m\overline{\alpha}\sqrt{1-t}}{m\sqrt{1-t}+1}\right[$, and strictly negative when $\alpha_j \in \left]\frac{m\overline{\alpha}\sqrt{1-t}}{m\sqrt{1-t}+1}, \frac{m\overline{\alpha}}{m-1}\right]$, which completes the proof of part *(ii)* and *(iii)* of proposition 1. ■

Part *(i)* of proposition 1 parallels similar results obtained in the literature. For a symmetric oligopoly with entry, Delipalla and Keen (1992) also show that a shift away from ad valorem taxation towards per unit leads to an increase in the number of active firms (proposition 4 (b)). However, by considering the asymmetric case, part *(i)* of our proposition 1 specifies the characteristics of the new firms entering into the market, which are high cost and thus less efficient firms. In a partial equilibrium model, a similar result is obtained by Denicolò and Matteuzzi (2000), also for the case of an asymmetric oligopoly. Furthermore, the rest of proposition 1 shows that a conflict of interests arises between the more efficient firms, which prefer ad valorem taxation (part *(ii)* of proposition 1) and the less efficient firms, which favor per unit taxation (part *(iii)* of proposition 1). Per unit taxation is obviously preferred by firms $j$, $j = 1, ..., m$, for which $\frac{m\overline{\alpha}}{m-1} < \alpha_j \leqslant \frac{m\overline{\alpha}(1-t)}{m(1-t)-1}$, that were excluded from the market, under ad valorem taxation, and become active, under per unit taxation. For the remaining firms $j$, for which $0 < \alpha_j < \frac{m\overline{\alpha}}{m-1}$, the fact that the more efficient firms prefer an ad valorem tax and the less efficient favour per unit taxation depends on further observations. Firstly, notice that the revenue-neutral shift from ad valorem to per unit taxation increases the amount of tax paid on each unit of good 2 sold

by each firm $j$, $j = 1, ..., m$, since $\tau = \frac{m\overline{\alpha}t}{m(1-t)-1}$ is greater than $tp_t^* = \frac{m\overline{\alpha}t}{(m-1)(1-t)}$. Secondly, such a revenue-neutral shift affects in a different way the amount of good 2 which firms $j$, $j = 1, ..., m$, are willing to supply on the market for trade. Accordingly, also the price changes, and it is easy to check that the producer price with the tax $\tau$ in (24) is higher than the producer price under ad valorem taxation. Thus, the increase in the producer price under per unit taxation compensates the higher amount of tax that now has to be paid on each unit of good 2, in the case of the less efficient firms, while this does not occur for the more efficient ones.

Now we proceed to a comparison of the effects of ad valorem and per unit taxes from the view point of consumers.

**Proposition 2** *A revenue-neutral shift from an ad valorem tax $t$ to a per unit tax $\tau$, leads to*

*(i) an increase in the consumer price;*

*(ii) a reduction in the utility level of consumers $i$, $i = 1, ..., n$.*

**Proof.** By substituting (24) into (14), it is easily checked that, under per unit taxation, the price at equilibrium obtains as

$$p_\tau^* = \frac{m\overline{\alpha}}{m(1-t)-1}, \tag{27}$$

which is strictly greater than the price under ad valorem taxation obtained in (14). Thus, by substituting (27) into (5), the utility level of consumers $i$, $i = 1, ..., n$, under per unit taxation, obtains as

$$S(p_\tau^*) = \frac{m(1-t)-1}{4n^2 m\overline{\alpha}}. \tag{28}$$

Similarly, by substituting (14) into (5), the utility level of consumers $i$, $i = 1, ..., n$, under ad valorem taxation, is given by

$$S(p_t^*) = \frac{(m-1)(1-t)}{4n^2 m\overline{\alpha}}, \tag{29}$$

which is strictly greater than (28). ■

Proposition 2 also parallels similar results obtained by Delipalla and Keen (1992), proposition 3 (a). In our context, the revenue-neutral shift from ad valorem to per unit taxation leads to a decrease in the aggregate amount of good 2 supplied on the market by firms, and accordingly an increase in consumer price. As a consequence, consumers $i$, $i = 1, ..., n$, can now buy a lower quantity

of good 2 at a higher price: the revenue-neutral tax shift leads to a decrease in their utility level.

To sum up the results of propositions 1 and 2, we conclude that a conflict of interests arises between consumers $i$, $i = 1, ..., n$, and low cost firms $j$, who both are in favor of ad valorem taxation, and high cost firms $j$ which, on the contrary, are in favor of per unit taxation. As a consequence, to compare the welfare properties of ad valorem and per unit taxes, we have to consider their effects on the *aggregate welfare*, namely the sum of the utility levels of consumers $i$, $i = 1, ..., n$, and oligopolists $j$, $j = 1, ..., m$ at equilibrium. To simplify the analysis, the welfare comparison is made under the assumption that all firms are symmetric, namely that $\alpha_j = \overline{\alpha}$, $\forall j$, $j = 1, ..., m$. As it was noticed above, in this case, there are $m$ active firms both under ad valorem and per unit taxation.

**Proposition 3** *A revenue-neutral shift from an ad valorem tax $t$ to a per unit tax $\tau$ increases aggregate welfare whenever $n > \frac{4m^2}{2-t}$.*

**Proof.** By substituting $\alpha_j = \overline{\alpha}$, $\forall j$, $j = 1, ..., m$, into (25) and (26), the difference in the utility level for each oligopolist $j$, $j = 1, ..., m$, under ad valorem and per unit taxes obtains as

$$V(q_{jt}^*, y_{jt}^*) - V(q_{j\tau}^*, y_{j\tau}^*) = -\frac{(2-t)t}{16m^4\overline{\alpha}}. \tag{30}$$

Similarly, by subtracting (28) from (29), the difference in the utility level for each consumer $i$, $i = 1, ..., n$, is given by

$$S(p_t^*) - S(p_\tau^*) = \frac{t}{4n^2m\overline{\alpha}}. \tag{31}$$

Finally, from (30) and (31), the difference in aggregate welfare with ad valorem and per unit taxes, obtains as

$$n \cdot \frac{t}{4n^2m\overline{\alpha}} + m \cdot \left(-\frac{(2-t)t}{16m^4\overline{\alpha}}\right) = \frac{t}{16nm^3\overline{\alpha}}(4m^2 - n(2-t)),$$

which is strictly negative if $n > \frac{4m^2}{2-t}$. ■

Proposition 3, which is in sharp contrast with the previous literature cited in our introduction, shows that per unit taxation welfare dominates ad valorem taxation that raises an equal amount of tax revenue, whenever the number of consumers is sufficiently high. Furthermore, notice that this assumption is not very demanding since it is in adequacy with the assumption that consumers behave as price-takers.[13]

[13] In this set-up, if a lump-sum tax were available, it would take the form of a levy on the amount of good 2 produced by firms. In this case, it is possible to show that a lump-sum tax would be welfare superior to an ad valorem tax that raises the same amount of tax revenue, whenever the number of consumers is sufficiently high and the number of oligopolists sufficiently low. On the contrary, per unit taxation would welfare dominate lump-sum taxation, whenever the number of consumers is sufficiently high. See Smart (1999) for a welfare comparison between lump-sum and excise taxation.

# 4 Concluding remarks

This paper compares the different welfare properties of ad valorem and per unit taxation in a general equilibrium model of oligopolistic interaction. Our main result shows that, when the number of consumers is sufficiently high, per unit taxation welfare dominates ad valorem taxation that raises an equal amount of tax revenue.

This result is intended to provide a counter-example to previous results obtained in the literature. For example, the superiority of ad valorem taxation over per unit has been shown by Suits and Musgrave (1953) and Skeath and Trandel (1994) for the monopoly case, and by Denicolò and Matteuzzi (2000) and Delipalla and Keen (1992) for oligopoly. With respect to the latter, even if their paper differs from ours in several respects, as stated in our introduction, notice that, under their assumption of a single representative consumer, i.e. $n = 1$, we also obtain the dominance of ad valorem over per unit taxation. However, in our context, this event has not any economic meaning since consumers represent the competitive side of the market, and for this reason their number has to be sufficiently high compared with the number of oligopolists which describe the strategic side.

Finally, notice that our analysis has been cast into a particular oligopoly model, in which oligopolists are simultaneously producers and consumers. This stylized description is suitable to represent a world in which each firm would be owned by a single individual, but not that of enterprises owned by shareholders who do not have the same preferences (Gabszewicz and Michel (1997)). Nevertheless, with the latter formulation, several difficulties arise in modelling imperfect competition into a general equilibrium framework, namely the oligopoly equilibrium is not invariant with respect to the normalization rule used to normalize price, and the profit maximization criterion may not be optimal from the shareholders' viewpoint (Gabszewicz and Vial (1972)).

# References


[1] Bishop, R. L. (1968), The Effects of Specific and Ad Valorem Taxes, *Quarterly Journal of Economics*, 82, 198-218.

[2] Colangelo, G., Galmarini, U. (1997), On the Pareto Ranking of Commodity Taxes in a Price-setting Oligopoly with Product Differentiation, *Public Finance*, 52, 50-63.

[3] Delipalla, S., Keen, M. (1992), The Comparison between Ad Valorem and Specific Taxation under Imperfect Competition, *Journal of Public Economics*, 49, 351-367.

[4] Denicolò, V., Matteuzzi, M. (2000), Specific and Ad Valorem Taxation in Asymmetric Cournot Oligopolies, *International Tax and Public Finance*, 7, 335-342.

[5] Gabszewicz, J. J., Grazzini, L. (1999), Taxing Market Power, *Journal of Public Economic Theory*, vol. 1, n. 4, 475-497.

[6] Gabszewicz, J. J., Michel, P. (1997), Oligopoly Equilibrium, in Exchange Economies in *Trade, Technology and Economics. Essays in Honour of Richard G. Lipsey* by B. C. Eaton, and R. G. Harris, Eds., Cheltenham: Edward Elgar.

[7] Gabszewicz, J. J., Vial, J. P. (1972), Oligopoly "à la Cournot" in General Equilibrium Analysis, *Journal of Economic Theory*, Vol. 4 pp. 381-400.

[8] Guesnerie, R., Laffont, J. J. (1978), Taxing Price Makers, *Journal of Economic Theory*, Vol. 19 pp. 423-455.

[9] Hamilton, S. F. (1999), Tax Incidence under Oligopoly: a Comparison of Policy Approaches, *Journal of Public Economics*, 71, 233-245.

[10] Keen, M. (1998), The Balance between Specific and Ad Valorem Taxation, *Fiscal Studies*, 19, 1-37.

[11] Myles, G. (1989), Ramsey Tax Rule for Economies with Imperfect Competition, *Journal of Public Economics*, 38, 95-115.

[12] Myles, G. (1995), *Public Economics*, Cambridge, Cambridge University Press.

[13] Myles, G. (1996), Imperfect Competition and the Optimal Combination of Ad Valorem and Per Unit Taxation, *International Tax and Public Finance*, 3, 29-44.

[14] Reinhorn, L. J. (1999), Optimal Taxation with Cournot Oligopoly, mimeo.

[15] Shapley, L. (1976), Noncooperative General Exchange, in *Theory and Measurement of Economic Externalities* by S. Lin Ed., New York: Academic Press.

[16] Shapley, L., Shubik, M. (1977), Trade Using One Commodity as a Means of Payment, *Journal of Political Economy*, Vol. 85 pp. 937-968.

[17] Skeath, S. E., Trandel, G. A. (1994), A Pareto Comparison of Ad Valorem and Unit Taxes in Noncompetitive Environments, *Journal of Public Economics*, 53, 53-71.

[18] Smart, M. (1999), A Simple Proof of the Efficiency of the Poll Tax, *Journal of Public Economics*, 71, 459-465.

[19] Suits, D. B., Musgrave, R. A. (1953), Ad Valorem and Unit Taxes Compared, *Quarterly Journal of Economics*, 67, 598-604.