Società italiana di economia pubblica

XIX CONFERENZA

ECONOMIA DEL CAPITALE UMANO
Istituzioni, incentivi e valutazioni

Pavia, Aule storiche Università, 13 - 14 settembre 2007

POPULATION GROWTH IN A MODEL OF ECONOMIC GROWTH WITH HUMAN CAPITAL ACCUMULATION AND HORIZONTAL R&D

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pubblicazione internet realizzata con contributo della COMPAGNIA di San Paolo

Population Growth in a Model of Economic Growth with Human Capital Accumulation and Horizontal R&D

Abstract

This paper reconsiders the effects of population growth on per-capita income growth within a Romerian (1990)-type endogenous growth model with human capital accumulation. One important novelty of our contribution is that in the human capital accumulation equation we explicitly consider the possibility that agents' investment in skill acquisition might be positively, negatively or not influenced at all by technological progress. We find that both the growth rate and the level of real per-capita income are independent of population size. Moreover, population growth may affect or not real per-capita income growth depending on the size of the degree of altruism of agents towards future generations and on the nature of technical progress, for given agents' degree of altruism.

Key Words: *Horizontal Innovation; Technological Change; Population Growth; Human Capital; Multi-Sector Endogenous Growth, Scale Effects.*

JEL Classification: *J10; J24; O31; O33; O41.*

1. Introduction

It is today well-known that most of the world population growth is concentrated in poorer countries (United Nations, 2001) and that such trend will persist even in the long run. The last fact is especially evident if one looks at the evolution over one century and more of the share of the world population living in three different sub-samples of countries (*more developed*, *less-developed* and *least-developed*, according to the repartition adopted by the United Nations): in the period 1950-2050 the share of world population residing in the more-developed regions is expected to decrease (from 32% to 13%), whereas it is expected to increase in the less-developed and, in particular, in the least-developed regions (respectively, from 60% to 67% and from 8% to 20%).[1] Thus, a natural concern arising from these data pertains to the long-run effects of population change on the economic performance of a country (the growth rate of real per-capita income of its inhabitants).

Until now, the literature has proposed three broad approaches to the analysis of this deep-rooted issue (see Bloom *et al.*, 2003, pp. 1-20). According to the *Pessimistic View*, population growth unambiguously hinders economic growth through two different channels: (a) in a world where economic resources are fixed and technological progress is low or totally absent, the food production activity is overwhelmed by the pressures of a rapidly growing population. The available diet would then fall below the *subsistence level* and so would the productivity growth rate also do (Malthus, 1798); (b) when population growth is rapid, a large part of investment (typically in physical capital) is used to satisfy the needs of the growing population (*"investment-diversion effect"* - Kelley, 1988, p. 1699), rather than to increase the level of per-capita capital endowments. As a consequence, per-capita economic growth would be lower in the presence of a higher population growth rate. As per the *Optimistic View*, population growth fuels economic growth. This is the main message coming from Kuznets (1960, 1967) and Simon (1981), according to whom larger economies can more easily build on, exploit and disseminate the flow of knowledge they produce. In other words, population growth may have a positive effect on technological progress and innovation which, in turn, are the main engines of economic development. Many papers that have analyzed the statistical correlation between population change and economic growth have found that, once other factors (such as country size, openness to trade, educational attainment of the population, and the quality of existing institutions) are taken into account, there exists little evidence that population growth might either slow down or encourage economic growth. Accordingly, a third view on the relationship between population growth and economic growth is the so-called *Population Neutralism View*.

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[1] See United Nations, 2001 and Bloom *et al.*, 2003, Fig. 1.1, p. 13. For a broad picture of the major present and future global demographic changes and their possible effects on countries' macroeconomic performance see also Bloom and Canning (2004).

Our paper takes for reasonable all of these three views[2] and combines them within the same analytical framework. Thus, the two main questions of the present contribution are the following: in an economy where human capital is not fixed and represents an indispensable input to firms' research and development (R&D) activity producing endogenous technological change, under which conditions can we account for the existence of a positive/negative/no relation at all between population change and economic growth? In a situation in which aggregate income growth is explained by technological progress (horizontal R&D activity), but ultimately driven by human capital investment, is economic growth sustainable in the long run even in the absence of any population change?

We try to answer these two questions by developing a theoretical, dynamic, general-equilibrium growth model with human capital accumulation and R&D activity. In more detail, we consider a multi-sector economy where an undifferentiated consumption good is produced by using human capital, the existing stock of *ideas* and intermediate goods. These goods are available in different varieties and produced under monopolistic competition conditions. Purposive R&D activity by firms, which combines human capital and *ideas*, is the source of technical progress. Population grows at an exogenous rate and each individual in the population is endowed with a certain amount of skills that may grow over time through formal investment in human capital. We also assume that human capital is fully employed and used to produce consumption goods, intermediates, *ideas* and new human capital.

The most important novelty of our model consists in the fact that in the human capital accumulation equation we explicitly take into account the possibility that the investment in skill acquisition by agents might be positively, negatively or not influenced at all by technological progress (the invention of new varieties of intermediate goods). In the first case, it is postulated that a faster technological progress, by increasing the demand for skills, induces agents to accumulate more human capital (*skill-biased technical change hypothesis*). In the second case, instead, the model captures all those situations where technological progress exerts a sort of *"erosion effect"* on human capital investment (*"eroding" technical change hypothesis*). Finally, the last case corresponds to the specific circumstance in which individual incentives to invest in schooling are totally independent of the nature and direction of technical change (*neutral technical change hypothesis*).

The main result of the paper is that, along the balanced growth path (BGP, henceforth) equilibrium, population growth may affect (positively or negatively) or not real per-capita income growth depending on: 1) the sign of the relation between some of the *technological* and *preference* parameters of the model (in particular, an important role is played by the size of agents' *degree of altruism* towards future generations as compared to the parameter reflecting the impact of technological progress on human

[2] *"…In some countries population growth may on balance contribute to economic development; in many others, it will deter development; and in still others, the net impact will be negligible"* (Kelley, 1988).

capital investment); 2) the *nature of technical change* (whether it is *skill-biased*, *"eroding"* or *neutral*), for given agents' degree of altruism. While the first conclusion (the link between population growth and economic growth is a function of the relationship between *preference* and *technological* parameters) is not new (see, among others, Dalgaard and Kreiner, 2001 and Strulik, 2005), we believe that tying the effect of population growth on economic growth to the direction of technical change is of particular interest. Intuitively, an increase in the population growth rate implies two possible consequences as far as per-capita income growth is concerned: on the one hand, it is likely to lead to a fall of per-capita income growth (for the simple reason that, *ceteris paribus*, when population grows the given, available, aggregate income has to be divided among a larger number of people). At the same time, however, the increase in the population growth rate also leads to a major number of innovations (technological progress).[3] If, as an example, such technical change is *skilled-biased* in nature (meaning that it spurs the demand and, thus, the consequent supply of human capital), then per-capita income growth (driven by skill investment in the model) rises. Accordingly, the final effect on economic growth of an increase in population growth can well be positive or negative or else exactly equal to zero. On the other hand, if technical change is of the *"eroding"* type (meaning that it lowers further human capital investment by agents and, thus, economic growth), the effect of population growth on per-capita income growth is unambiguously negative. Finally, we show that in the case of *neutral technical change* (technological progress does not influence at all agents' incentive to invest in schooling), population growth is more likely to bear an adverse effect on per-capita income growth (such effect can be, at most equal to zero).

However, even with respect to the above-mentioned works by Dalgaard and Kreiner (2001) and Strulik (2005), our paper presents some major differences. Unlike Dalgaard and Kreiner (2001), who use a one-sector growth model with human and technological capital accumulation (patents and education are generated by the same production function), we build a two-sector growth model reflecting the fact that the production of human capital is relatively intensive in human capital. Instead, unlike Strulik (2005), who uses a two-R&D-sector growth model (innovation expands the variety *and* quality of intermediate goods), we show that two further results of our paper (namely that economic growth is no longer semiendogenous -*i.e.*, driven solely by exogenous population growth- but fully endogenous -*i.e.*, ultimately driven by private incentives to invest in human capital, and that in the long run economic growth is sustainable even in the absence of population growth) can be obtained using a much simpler and more tractable model with only one R&D dimension (the horizontal one). Furthermore, while Strulik considers a purely Lucas (1988)-type per-capita human capital accumulation equation where, besides population growth, an *exogenous depreciation* rate of skills operates, in our model the presence of the

[3] *"More people means more Isaac Newtons and therefore more ideas"* (Jones, 2001, p.10).

growth rate of *ideas* in the law of motion of per-capita human capital acts as an endogenous mechanism of *depreciation/appreciation* of (embodied) knowledge.

The remainder of the paper is structured as follows. In Section 2 we set the basic model, whose BGP properties are analyzed in Section 3. In Section 4 we discuss the main results concerning the long-run relationship between population growth, the direction of technical change and economic growth. Finally, Section 5 concludes.

2. The Basic Model

The economy is composed of households and firms. Households receive wages and interest income, purchase consumption goods and choose how much to save and how much to invest in human capital. Firms produce goods (consumption and intermediate goods) and perform R&D activity. Population (*L*) coincides with the available number of workers (there exists full employment) and grows at an exogenous and constant rate, g_L.[4] Following Barro and Sala-i-Martin (2004, Chap.5, p.240) the total stock of human capital existing in the economy at time t (H_t) is given by the number of workers at t (L_t) times the average level of human capital of each worker (h_t). Thus, H ($\equiv L \cdot h$) grows not only because population rises, but also because the average quality of each worker may increase over time. Consumption goods are produced competitively, with prices being taken as given and each input compensated according to its own marginal product. In the intermediate goods sector, monopolistic firms produce horizontally differentiated products entering the production function of consumption goods as an input. Purposive R&D activity is the source of technological progress. In more detail, technical progress takes place by inventing new varieties of differentiated capital goods (*blueprints* or *ideas*) within a competitive R&D sector. In order to produce new *ideas*, human capital and (eventually) the existing stock of ideas are combined together through a homogeneous production function. When a new blueprint is discovered, an intermediate goods producer acquires the perpetual patent over it and, hence, s/he can practice monopoly pricing forever. It is worth mentioning here that human capital is employed in each economic sector. In other words, we assume that this factor input is used to produce consumption goods and intermediate inputs, to discover new ideas and to generate new human capital.

[4] The population growth rate is the difference between the fertility rate and the mortality rate. In this paper we abstract from *endogenous fertility* considerations (whereby rational agents choose their fertility by weighing the costs and benefits of rearing children) and neglect *migrations*.

2.1 *Production*

Two kinds of *material* goods are produced in the economy: consumption goods and intermediate inputs. Consumption goods act as *numeraire* goods (their price is normalized to one) and are produced competitively using human capital (H_Y), specialized inputs (indexed by *i* and employed in quantity x_i), and a fixed factor (*F*). In particular, we postulate that the production function of these goods is given by:

$$Y_t = \left(\frac{H_{Yt}}{n_t}\right)^{\alpha}\left[\int_0^{n_t} x_{it}^{\upsilon} di\right] F^{1-\alpha-\upsilon}, \qquad 0<\alpha\,,\ \upsilon\,,\ (\alpha+\upsilon)<1\,.$$

In the above equation Y_t is the output of the (homeogeneous) consumption goods at time *t*, n_t denotes the total number of intermediate input varieties used in production at the same date, α and $(1-\alpha-\upsilon)$ are the shares of output going to H_Y and *F*, respectively. In a symmetric equilibrium where each intermediate input is produced and employed in the same amount ($x_i = x$, $\forall i$), υ represents the share of intermediates in aggregate output.

This production function is borrowed by Dalgaard and Kreiner (2001). It exhibits constant returns to scale both to the *rival inputs* (H_{Yt}, x_{it} and *F*) and to the reproducible ones (H_{Yt} and n_t). As is well-known, the latter property represents a sufficient condition for endogenous growth.[5] Without any loss of generality, we can normalize *F* to one and obtain:

$$Y_t = \left(\frac{H_{Yt}}{n_t}\right)^{\alpha}\left[\int_0^{n_t} x_{it}^{\upsilon} di\right]. \tag{1}$$

In equilibrium each *rival* input (x_{it} and H_{Yt}) receives its marginal productivity. Hence:

$$\frac{\partial Y_t}{\partial x_{it}} = \upsilon\left(\frac{H_{Yt}}{n_t}\right)^{\alpha} x_{it}^{\upsilon-1} = p_{it}\,, \qquad \forall i \in [0, n_t], \qquad n_t \in [0,\infty) \tag{2}$$

$$\frac{\partial Y_t}{\partial H_{Yt}} = \alpha\frac{Y_t}{H_{Yt}} = w_{Yt} \tag{3}$$

In equations (2) and (3), p_{it} and w_{Yt} are respectively the price of the *i-th* intermediate input and the wage per unit of human capital employed in the consumption goods sector. Notice that, in the symmetric equilibrium ($x_i = x$, $\forall i$), w_{Yt} would read as:

$$w_{Yt} = \alpha\left(\frac{n_t}{H_{Yt}}\right)^{1-\alpha} x_t^{\upsilon}\,. \tag{3'}$$

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[5] According to Dalgaard and Kreiner (2001, p.193), the term $(H_{Yt}/n_t)^{\alpha}$ can be interpreted as capturing the fact that *"…production tends to become more human capital intensive through time as production complexity increases (Howitt, 1999)"*.

As for the intermediate inputs, we assume that they are produced by monopolistically-competitive firms. The generic firm *i* manufactures only a single variety (variety *i*) of intermediate inputs by employing the following *one-to-one* technology in human capital:

$$x_{it} = h_{it}, \qquad \forall i \in [0, n_t].$$

After purchasing the *i-th* idea from the R&D sector, intermediate firm *i* maximizes (with respect to x_{it}) the instantaneous profit under the inverse demand constraint (equation 2). From the first order conditions, it is possible to obtain the wage rate accruing to one unit of human capital employed in the intermediates production (w_{it}):

$$w_{it} = \upsilon^2 \left(\frac{H_{Yt}}{n_t} \right)^{\alpha} x_{it}^{\upsilon - 1}. \tag{4}$$

Because both technology and demand are the same for all intermediates we can focus on a symmetric equilibrium in which $x_{it} = x_t$, $\forall i \in [0, n_t]$. Accordingly, each local intermediate monopolist will face the same wage rate ($w_{it} = w_t$, $\forall i$). Equations (2) and (4) together yield the usual constant markup ($1/\upsilon$)-based pricing rule:

$$p_{it} = \frac{1}{\upsilon} w_{it} = \frac{1}{\upsilon} w_t = p_t, \qquad \forall i \in [0, n_t]. \tag{5}$$

Defining by $H_{it} \equiv \int_0^{n_t} h_{it} di$ the total demand for intermediate human capital and using the hypothesis of symmetry across intermediate firms, we obtain:

$$x_{it} = \frac{H_{it}}{n_t} = x_t, \qquad \forall i \in [0, n_t]. \tag{6}$$

Given x_t, the instantaneous profit accruing to the generic intermediate firm *i* is:

$$\pi_{it} = \upsilon(1 - \upsilon) \left(\frac{H_{Yt}}{n_t} \right)^{\alpha} \left(\frac{H_{it}}{n_t} \right)^{\upsilon} = \pi_t, \qquad \forall i \in [0, n_t]. \tag{7}$$

Notice that, as long as H_{Yt}/n_t and H_{it}/n_t are constant (as it will be along the BGP), x_t, w_t, p_t and π_t will be all constant.

2.2 *R&D activity*

This sector produces *ideas* (designs and/or blueprints for new varieties of intermediates) and is populated by a large number of small firms. The representative firm employs the following deterministic research technology (see Jones, 1995 and Arnold, 1998):

$$\dot{n}_t = \gamma H_{nt}^{\psi} n_t^{\chi}, \qquad \gamma > 0, \qquad \psi \in (0;1), \qquad \chi \in [0;1), \tag{8}$$

where n_t denotes the number of intermediate-inputs varieties existing at time *t*, H_n is the total amount of human capital employed in R&D activity and γ is a positive productivity parameter. The production function of new ideas used in (8) is Cobb-Douglas and states that research human capital (H_n) is indispensable for the invention of new varieties of inputs. Once obtained a new invention, its holder is granted an infinitely-lived patent.

Because this sector is competitive there is free entry into R&D. This implies:

$$\frac{1}{\gamma}\frac{H_{nt}^{1-\psi}}{n_t^{\chi}} w_{nt} = V_{nt} \tag{9}$$

$$V_{nt} = \int_t^{\infty} \exp\left[-\int_t^{\tau} r(\omega)d\omega\right]\pi_\tau d\tau, \qquad \tau > t. \tag{10}$$

In equations (9) and (10) w_n is the wage rate accruing to one unit of research human capital, $\exp\left[-\int_t^{\tau} r(\omega)d\omega\right]$ is a present value factor which converts a unit of profit at time τ into an equivalent unit of profit at time *t; r* is the real rate of return on the consumers' asset holdings (to be defined in a moment); π is the profit accruing to the generic producer of intermediates and V_n is the market value of one unit of research output (the *i-th* idea allowing to produce the *i-th* variety of intermediates). Equation (9) is the zero-profit condition, stating that firms will keep entering the R&D sector till when all profit opportunities are exhausted. In equation (10) V_n is the discounted value of the profit flow a local intermediate monopolist can potentially earn from *t* to infinity (V_n coincides with the market value of the *i-th* intermediate firm). When *r* is constant (as it will be along the BGP), V_n will be also constant.

2.3 *Consumers*

We consider a closed economy (there is no international trade in goods and/or services and no migrations across countries), where the total number of households is constant and normalized to unity. The size of the household (*L*), however, changes through time at the (constant and exogenous) rate of population growth (g_L).[6] There is no physical capital in this economy and the household uses the income it does not consume to accumulate more assets (taking the form of ownership claims on firms). Thus:

$$\dot{A}_t = r_t A_t + (\varepsilon_t H_t) w_t - C_t \, ,$$

where *A* denotes total assets, *r* is the real rate of return on household's asset holdings and ε is the employed fraction of the available household's stock of human capital, *H*. According to this equation, the household's investment in assets (the left hand side) equals the household's savings (the right hand side). In turn, savings are equal to the difference between total income (the sum of *interest income* - *rA* - and *human capital income*, εHw) and aggregate consumption (*C*). Since human capital accrues in equilibrium the same reward across sectors, we denoted the wage rate going to one unit of human capital simply by *w*, without any sector-specific subscript. Given the above expression, the assets law of motion in per-capita terms is:

$$\dot{a}_t = (r_t - g_L) a_t + (\varepsilon_t w_t) h_t - c_t \, , \qquad (11)$$

with $a_t (\equiv A_t / L_t)$, $h_t (\equiv H_t / L_t)$ and $c_t (\equiv C_t / L_t)$ denoting respectively per-capita asset holdings, per-capita human capital and per-capita consumption. The household uses the remaining fraction $(1 - \varepsilon_t)$ of the available stock of human capital (H_t) to produce new human capital. The law of motion of human capital at the economy-wide level is the following:

$$\dot{H}_t = \sigma(1 - \varepsilon_t) H_t - (\varphi g_n) H_t \, , \qquad (1 + \varphi) > 0 \, . \qquad (12)$$

σ and φ are technological parameters. While the first one (σ) represents the productivity of human capital in the production of new human capital and is positive, the second one (φ) reflects the impact of technological progress (given by the growth rate of the number of varieties of intermediate goods, g_n) on human capital investment. For given $\sigma > 0$ and $0 < \varepsilon < 1$, the restriction $\varphi > -1$ prevents the growth rate of the model's aggregate variables from either exploding ($\varphi = -1$) or being negative ($\varphi < -1$) along a BGP equilibrium where human and technological capital grow at the same rate (in a moment we shall give a more formal definition of BGP equilibrium). Equation (12) is borrowed from Sequeira and Reis (2006, p.8), with a major difference. Unlike that paper (in which φ>0), we consider the more general case in which a faster technical progress (higher g_n) may, *ceteris paribus*, increase ($-1 < \varphi < 0$), decrease ($\varphi > 0$), or else exert no impact ($\varphi = 0$) on the rate of human capital investment. Indeed, apart from the

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[6] As already mentioned, g_L is the difference between the population's birth and death rates, both taken as exogenous in this model. The reason why we take g_L as exogenous resides in the fact that in this paper we are interested in analyzing the conditions under which population growth can exert a positive, negative or no effect at all on the real per-capita growth rate in a *one-R&D-sector* model with human capital accumulation.

already analyzed case of $\varphi = 0$,[7] there exists wide literature (both empirical and theoretical) in favor of the hypothesis of a negative or positive effect of technological change on skill acquisition. On the one hand, it is recognized that *"…the time required for learning the new technology…increases with the rate of technological change"* (Galor and Moav, 2002, p.1148). This means that the rise of the technological complexity of the production process (in our model the introduction of a new technique/variety of intermediate inputs) brings about a cost in terms of more rapid obsolescence of the available human capital (in the words of Sequeira and Reis, 2006, p.8, technological progress *"…acts as an endogenous depreciation mechanism"* in the human capital accumulation equation). Recent empirical support to the hypothesis that technical progress may have a negative effect on human capital comes, among others, from Tamura (2006) and Kumar (2003). Following Galor and Moav (2002), we label the case where $\varphi > 0$ in equation 12 as the *Eroding Technological Change (ETC*, henceforth*) hypothesis*. On the other hand, instead, it is also equally reasonable to think of a positive effect of technological change on human capital investment ($-1 < \varphi < 0$ in equation 12). In this regard, it is a very well documented fact that, starting in the 1980s, we observe in most of OECD countries a sensible decline in the demand for unskilled labor (with a consequent upsurge in the unskilled unemployment rate) and a rise in the wage inequality between skilled and unskilled (at least in the US and the UK), together with a simultaneous large increase in the supply of skilled labor, namely in the number of college or university graduates (see, among other studies, OECD 1993, 1996, 2000; Katz and Murphy, 1992, and Greiner *et al*., 2004). One of the most often advocated reasons behind these concomitant changes is that technological change occurred from that time forth might have been of the *skilled-biased* type, whereby new technologies and human capital are complements. If this were the case, then technological progress would act as an endogenous *"appreciation"* mechanism in the human capital investment equation, in the sense that it would spur the demand for -and (in the absence of *"technological"* constraints to the production of human capital) the subsequent supply of- skills.[8] Many empirical tests (for example, Autor *et al.*, 1998; Desjonqueres *et al.*, 1999; Caselli, 1999) tend to favor the hypothesis that technical progress may have a positive effect on human capital. Following this line of research, we label the case where $-1 < \varphi < 0$ in equation 12 as the *Skilled-Biased Technological Change (SBTC*, henceforth*) hypothesis*. Although we believe that in real life the previous two are the most relevant cases, in this paper we shall also consider as a special case the one where $\varphi = 0$ (technological change has no effect on human capital investment). We label this hypothesis as the *Neutral Technological Change (NTC*, henceforth*) hypothesis*. Later on, we shall see that the effect

[7] See, in particular, Arnold (1998, p.85, equation 1) and Funke and Strulik (2000, p. 494, equation 5).
[8] Galor and Moav (2000) have pointed to the role of skilled-biased technical change in explaining the evolution over time not only of technology and wage inequality, but also of educational attainments.

that population growth has on real per-capita income growth is crucially related to the sign of the impact of technological progress on skill acquisition, *i.e.* to the direction of technical change.

Given $\dot{H}_t$, the law of motion of human capital in per-capita terms is given by:

$$\dot{h}_t = \frac{\dot{H}_t}{L_t} - g_L h_t = \sigma(1-\varepsilon_t)h_t - (\varphi g_n + g_L)h_t . \tag{12'}$$

This equation reproduces the same *congestion effect* in human capital accumulation described by Dalgaard and Kreiner (2001, pp. 190-91): the smaller the first term of (12'), $\dot{H}_t / L_t$ (*i.e.*, the ratio of the input in the human capital sector -which in our model consists solely of human capital- to the entire population), the less the average level of quality (human capital, *h*) of each individual expands. The second term of (12'), $g_L h_t$, represents the cost (in terms of human capital) of upgrading the level of quality of the newborns (who are uneducated) to the average level of quality of the existing population. With a *constant intertemporal elasticity of substitution* (CIES) instantaneous utility function, the objective of the household is to maximize under constraints its own intertemporal utility deriving from per capita consumption:

$$\underset{\{c_t,\varepsilon_t,a_t,h_t\}_{t=0}^{\infty}}{Max} U \equiv \int_0^\infty \left(\frac{c_t^{1-\lambda} - 1}{1-\lambda} \right) e^{-(\rho - m g_L)t} dt , \qquad \rho > 0 ; \qquad (\rho - m g_L) > 0 ; \quad m \in [0;1]; \qquad \lambda \ge 1 \tag{13}$$

s.t.:

$$\dot{a}_t = (r_t - g_L)a_t + (\varepsilon_t w_t)h_t - c_t , \qquad \varepsilon_t \in [0;1]; \quad g_L \gtreqless 0 \tag{11}$$

$$\dot{h}_t = \sigma(1-\varepsilon_t)h_t - (\varphi g_n + g_L)h_t , \qquad \sigma > \rho ; \qquad (1+\varphi) > 0 \tag{12'}$$

$$\lim_{t\to\infty} \mu_{at} a_t = 0 ; \qquad \lim_{t\to\infty} \mu_{ht} h_t = 0 \tag{14}$$

a_0 and h_0 given.

The household decides on the amount of per-capita consumption and on the share of human capital to be devoted to production activities (respectively c_t and ε_t). Equation (13) is the household's intertemporal utility function, equation (11) is the per-capita budget constraint and equation (12') represents the per-capita human capital supply function. We denoted by ρ the pure rate of time preference of each individual, by *w* the wage going to one unit of human capital (taken as given by the household) and by $1/\lambda$ the constant intertemporal elasticity of substitution. The hypothesis $\lambda \ge 1$ is in line with the evidence according to which the intertemporal elasticity of substitution in consumption is lower than (or, at most, not significantly different from) one.[9] The assumption $\rho > mg_L$ ensures that *U* is bounded and that the two transversality conditions (14) are checked (see the *Appendix* for algebraic details), while the constraint $\sigma > \rho$ requires that the productivity of education is sufficiently large compared with the individual discount rate. This condition is standard in models with human and technological capital accumulation (see, among others, Arnold, 1998, p.85 and Strulik, 2005, p.135). The reason why in our paper it is also useful will be clear soon. Instead, we do not do any specific assumption on g_L, which in this model may be positive, negative or even equal to zero. As to *m*, this is a preference parameter controlling for the degree of *altruism* towards future generations. The limiting case of *m*=0 defines the minimal degree of altruism, or the case of *perfect egoism* (the household maximizes only the utility of per capita consumption of its actual members and does not care at all about its future generations), whereas the opposite limiting case of *m*=1 defines the situation of *perfect altruism* (the household maximizes the utility of per capita consumption of all of its members, both actual and future, taking explicitly into account the fact that its own size may grow over time). Clearly, $0 < m < 1$ describes an intermediate degree of altruism (see Strulik, 2005, p.135). Finally, as ε is a fraction, it must belong to the closed set $[0;1]$ - for an interior solution to exist ε should be strictly between zero and one.

3. Equilibrium and BGP Analysis

All markets always clear. Since human capital is fully employed between production and education activities and equally productive in the production of consumption goods, intermediate inputs and ideas, in equilibrium the following equalities must hold:

$$H_{Yt} + H_{it} + H_{nt} = \varepsilon_t H_t \tag{15}$$

$$w_{Yt} = w_{it} \,. \tag{16}$$

$$w_{it} = w_{nt} \,. \tag{16'}$$

Moreover, total household's asset holdings (*A*) must equal the aggregate firm value (nV_n):

$$A_t = n_t V_{nt} \,, \tag{17}$$

where V_{nt} is given by equation (10) and satisfies the usual no-arbitrage condition:

$$\dot{V}_{nt} = r_t V_{nt} - \pi_{it} \,.$$

In the model, the *i-th* idea allows the *i-th* intermediate firm to produce the *i-th* variety of intermediates. This explains why in equation (17) total assets (*A*) equal the number of profit-making intermediate firms

[9] See Growiec (2006, pp. 17-19) for a short but comprehensive survey on empirical estimates of the intertemporal elasticity of substitution.

(n) times the market value (V_n) of each of them (equal, in turn, to the market value of the corresponding idea). Finally, the no-arbitrage equation suggests that the return on the value of the *i-th* intermediate firm $(r_t V_{nt})$ must equal in equilibrium the sum between the instantaneous monopoly profit of the *i-th* input producer (π_{it}) and the capital gain/loss matured on V_n during the time interval *dt* ($\dot{V}_{nt}$).

We can now move to a formal definition and characterization of the BGP of the model. In what follows we denote by g_B the growth rate of generic variable *B*.

Definition: Balanced Growth Path (BGP)

We define a BGP as a state where: (i) *All variables depending on time grow at constant (possibly positive) rate;* (ii) *Both the existing number of varieties* (n_t) *and the available stock of human capital* (H_t) *grow at the same rate,* $g_n = g_H$ *(implying that the aggregate human to technological capital ratio,* H_t / n_t*, remains invariant over time);* (iii) *The sectoral shares of human capital* (H_{jt} / H_t*, j=Y, i, n) are constant.*

This definition implies:

PROPOSITION 1

For a BGP equilibrium to exist the R&D technology has to display constant returns to scale to H_n *and n together (i.e.,* $\psi = 1 - \chi$*).*

Moreover, along the BGP the employed fraction of the household's total stock of human capital is constant ($\varepsilon_t = \varepsilon$, $\forall t$).

Proof:

To prove the first part of the proposition, take equation (8) and use the definition of BGP stated above. The second part of the proposition, instead, derives immediately from the fact that the growth rate of all the time-dependant variables is constant along the BGP (see 12).■

According to Arnold (1998, p. 85): *"…It can be shown that if the R&D technology is homogeneous it must either have the Cobb Douglas form…or else reveal constant returns to scale"*. Thus, the first part of Proposition 1 says that the production of new ideas must occur under constant returns to scale to human and technological capital (H_n and *n*, respectively) in order for a BGP equilibrium to exist in our model.

Using Proposition 1, it is possible to show that the following results must hold along the BGP equilibrium (mathematical derivation of such results is in the *Appendix*):

$$g_n = g_H = \frac{\left[\sigma - \rho - (1 - m - \lambda) g_L\right]}{(\lambda + \varphi)} \tag{18}$$

$$g_c = g_a = g_h = g_y = g_n - g_L = \left(\frac{\sigma - \rho}{\lambda + \varphi}\right) - \left(\frac{1 + \varphi - m}{\lambda + \varphi}\right) g_L \tag{19}$$

$$r = \frac{\lambda\sigma + \varphi\left[\rho + (1 - m - \lambda) g_L\right]}{(\lambda + \varphi)} \tag{20}$$

$$\frac{H_{nt}}{n_t} = \left(\frac{g_n}{\gamma}\right)^{\frac{1}{1-\chi}}; \qquad \frac{H_{it}}{n_t} = \frac{r\upsilon}{\gamma(1-\upsilon)}\left(\frac{g_n}{\gamma}\right)^{\frac{\chi}{1-\chi}}; \qquad \frac{H_{Yt}}{n_t} = \frac{\alpha r}{\gamma\upsilon(1-\upsilon)}\left(\frac{g_n}{\gamma}\right)^{\frac{\chi}{1-\chi}} \tag{21}$$

$$\frac{H_t}{n_t} = \frac{1}{\varepsilon_t}\frac{H_{nt}}{n_t}\left[1 + \frac{r}{\gamma(1-\upsilon)}\left(\upsilon + \frac{\alpha}{\upsilon}\right)\left(\frac{H_{nt}}{n_t}\right)^{\chi - 1}\right] \tag{22}$$

$$\varepsilon = 1 - \frac{(1 + \varphi)\left[\sigma - \rho - (1 - m - \lambda) g_L\right]}{\sigma(\lambda + \varphi)}. \tag{23}$$

Equation (18) gives the BGP equilibrium growth rate of the economy's number of intermediate inputs varieties (*n*) and total stock of human capital (*H*). According to equation (19), per capita consumption (*c*), asset holdings (*a*), human capital (*h*) and income ($y \equiv Y / L$) grow in the long run at the same constant rate. Together, equations (18) and (19) say that along the BGP equilibrium the growth rate of real per-capita income (g_y) is explained by technological progress (g_n) but is ultimately driven by human capital investment (g_H). Equations (20), (21) and (22) provide, respectively, the equilibrium value for the real interest rate (*r*), the ratios H_{jt} / n_t (*j*=*n*,*i*,*Y*) and H_t / n_t. Finally, equation (23) represents the fraction of human capital employed in equilibrium in non-education activities (production of intermediate inputs and consumption goods and invention of new ideas).

Given the results (18) through (23), it is possible to show that - with $\lambda \geq 1$, $(1 + \varphi) > 0$ and $(\rho - mg_L) > 0$ - the condition $\sigma > (\rho - mg_L)$ is sufficient to guarantee simultaneously that the household's problem has an interior solution ($0 < \varepsilon < 1$) and that the real interest rate (*r*), the growth rate of aggregate variables (g_n and g_H) and the ratios in equations (21) and (22) are all positive. Moreover, when $\sigma > (\rho - mg_L) + (1 + \varphi) g_L$ the growth rate of variables in per-capita terms is positive as well (see the *Appendix* for further details). In what follows we postulate that these two constraints on σ [namely,

$\sigma > (\rho - mg_L)$ and $\sigma > (\rho - mg_L) + (1+\varphi)g_L$] hold together.[10] We can now state the first two central results of this paper:

PROPOSITION 2

In an economy with human capital accumulation, horizontal innovation-based R&D activity, and in which the impact of technological progress (the growth rate of intermediate-goods variety) on skill investment may be positive, negative or even equal to zero ($\varphi > -1$), both the growth rate (g_y) and the level (y) of real per-capita income are independent of population size along the BGP equilibrium.

Proof:

Equation (19) reveals that per-capita income growth (g_y) depends on population growth (g_L), but not on population size (*L*). Furthermore, after simple algebraic manipulations and employing the assumption of symmetry across intermediate inputs ($x_{it} = x_t$ for each *i*), the level of per-capita income along the BGP equilibrium can be recast as:

$$y_t \equiv \frac{Y_t}{L_t} = \frac{\tau_Y^{\alpha} \tau_i^{\upsilon}}{(H_t / n_t)^{1-\alpha-\upsilon}} h_0 e^{g_y t},$$

where h_0 is the initial (*i.e.*, at time t=0) per-capita skill level and τ_Y and τ_i are respectively the shares of the available stock of human capital devoted to consumption goods and intermediate inputs production - *i.e.*, $\tau_Y \equiv \frac{H_{Yt}}{H_t} = \frac{H_{Yt}}{n_t}\frac{n_t}{H_t}$ and $\tau_i \equiv \frac{H_{it}}{H_t} = \frac{H_{it}}{n_t}\frac{n_t}{H_t}$. Simple inspection of the above equation suggests that *y* is independent of *L*, for given h_0 (per-capita income is proportional to per-capita human capital).■

PROPOSITION 3

In an economy with human capital accumulation, horizontal innovation-based R&D activity, and in which the impact of technological progress (the growth rate of intermediate-goods variety) on skill investment may be positive, negative or even equal to zero ($\varphi > -1$), real per-capita income growth depends positively on a constant (a positive combination of preference - ρ and λ - and technological - σ and φ - parameters). The effect of population growth on economic growth, instead, rests with the size of the degree of altruism (m) present in the economy. However, given our assumptions on parameter values, economic growth is surely positive even in the presence of a population growth rate equal to zero.

Proof:

In order to prove the first part of the Proposition, see (19) and our assumptions on the parameter values (12' and 13). Equation 19 also suggests that the sign of $\partial g_y / \partial g_L$ depends on the sign of ($m - 1 - \varphi$). Finally, and according to the same equation, $g_y = \frac{\sigma - \rho}{\lambda + \varphi} > 0$ when $g_L = 0$.■

[10] We are analysing the most general possible case ($g_L \gtreqless 0$). If $g_L \geq 0$, then the inequality $\sigma > (\rho - mg_L) + (1+\varphi)g_L \geq (\rho - mg_L)$ would certainly hold, since $(1+\varphi)$ is positive. As $\lambda \geq 1$, technically $\sigma > \rho$ ensures positive growth of per-capita variables in the BGP equilibrium, even when $g_L = 0$.

The next section studies more deeply the effect of population growth on economic growth.

4. Population growth, human capital accumulation, the direction of technological progress and economic growth: a discussion

In order to analyze the fundamental interactions among population growth, human capital investment, technological change and real per-capita economic growth, we first rewrite equation (19) as:

$$g_y = \underbrace{\frac{\sigma(1-\varepsilon)}{(1+\varphi)}}_{\equiv g_n = g_H} - g_L \tag{19'}$$

Thus, the impact of population growth on per-capita income growth can be decomposed into two separate effects:

- The *direct* and *negative population effect* (the term $-g_L$ in equation 19'), according to which a larger population leads directly to a fall of per-capita income growth.

This happens for the simple and intuitive reason that, *ceteris paribus* (*i.e.*, for given L_0, Y_0 and $g_Y = g_n = g_H$), when population grows (g_L increases) the available aggregate income has to be divided among a greater number of people, with each of them now accruing a lower level of per-capita income;

- The *indirect* and *positive accumulation effect* (the term $\sigma(1-\varepsilon)/(1+\varphi)$ in equation 19').

To see how this effect works, we re-write g_n along the BGP equilibrium (when $\psi = 1-\chi$) as follows:

$$g_n = \gamma\left(\frac{H_{nt}}{n_t}\right)^{1-\chi} = \gamma\left(\tau_n \frac{H_t}{n_t}\right)^{1-\chi} = \gamma\left(\tau_n \frac{L_t h_t}{n_t}\right)^{1-\chi},$$

where we denoted by τ_n the constant BGP equilibrium share of human capital devoted to R&D activity. We see from the equation above that (for given L_0) an increase of g_L leads to a larger population size (L_t rises). This, in turn, implies (for given average quality level of each individual, h_t) an increase of the available stock of human capital ($H_t \equiv L_t h_t$) and, thus, for given n_t and τ_n, a higher g_n (which, in equilibrium, is equal to the human capital investment rate, g_H). In brief, and *ceteris paribus,*[11] the higher the growth rate of population, the faster technological progress and skill investment ($g_n = g_H$) and, through this channel, the larger the per-capita income growth rate.

[11] Namely, for given L_0, h_t, n_t and τ_n.

Indeed, the idea that population growth might exert a positive impact on economic development through factor accumulation (and, more specifically, the accumulation of knowledge capital) is not new and dates back to Kuznets (1960, 1967) and Simon (1981). In the words of Bloom *et al.* (2003, p.15): *"…Larger societies – with the capacity to take advantage of economies of scale – are better positioned to develop, exploit, and disseminate the increased flow of knowledge they receive (Kuznets, 1960, 1967)"*. Our paper formalizes this view through a model in which human capital is an indispensable input within the invention process.

With this in mind, we are now able to summarize the effects that population growth has on economic growth.

THEOREM 1

- *When* $m > (1+\varphi)$*, the positive accumulation effect prevails over the negative population effect and* $\frac{\partial g_y}{\partial g_L} > 0$ *(population growth always exerts a positive effect on economic growth).*
- *When* $m < (1+\varphi)$*, the negative population effect outweighs the positive accumulation effect and* $\frac{\partial g_y}{\partial g_L} < 0$ *(population growth always exerts a negative effect on economic growth).*
- *When* $m = (1+\varphi)$*, the positive accumulation effect and the negative population effect cancel each other and* $\frac{\partial g_y}{\partial g_L} = 0$ *(population growth never affects economic growth).*

Proof:

The theorem follows immediately from equation (19), the discussion above and the fact that $(\lambda+\varphi) > 0$. ■

The intuition behind the theorem is as follows. When the degree of *altruism* (*m*) is sufficiently high (low), implying that the future size of the family is (not) sufficiently taken into account, households are more (less) patient - the term ($\rho - mg_L$) in equation 13 is, *ceteris paribus*, lower (higher) - and, hence, save more (less). This implies a higher (lower) aggregate investment in R&D activity and human capital accumulation. Accordingly, when the size of the dynastic family rises, this increase exerts ultimately a positive (negative) effect on g_y.

Theorem 2 relates the effect of population growth on economic growth to the nature of technological progress. In stating the theorem we explicitly consider the constraints $m \in (0;1)$ and $(1+\varphi) > 0$. As special cases, we shall separately study what happens when $m = 0$, $m = 1$ and $\varphi = 0$.

THEOREM 2

- *Under the SBTC hypothesis* ($-1 < \varphi < 0$), *population growth may exert either a positive, or a negative, or else no effect on real per-capita income growth:* $\frac{\partial g_y}{\partial g_L} \gtreqless 0$.
- *Under the ETC hypothesis* ($\varphi > 0$), *instead, population growth exerts an unambiguously negative effect on real per-capita income growth:* $\frac{\partial g_y}{\partial g_L} < 0$.

Proof:

According to Theorem 1, $\frac{\partial g_y}{\partial g_L} > 0$ when $m > (1+\varphi)$. The three inequalities: $m > (1+\varphi)$, $m \in (0;1)$ and $(1+\varphi) > 0$ are simultaneously checked when the following is true:

$$0 < (1+\varphi) < m < 1,$$

which, in turn, is compatible with the *SBTC hypothesis* ($-1 < \varphi < 0$).

Instead, $\frac{\partial g_y}{\partial g_L} < 0$ when $m < (1+\varphi)$. The three inequalities: $m < (1+\varphi)$, $m \in (0;1)$ and $(1+\varphi) > 0$ are simultaneously checked either when:

$$0 < m < 1 < (1+\varphi),$$

or when:

$$0 < m < (1+\varphi) < 1.$$

The first of the two relations written above is compatible with the *ETC hypothesis* ($\varphi > 0$), whereas the second one is compatible with the *SBTC hypothesis* ($-1 < \varphi < 0$).

Finally, $\frac{\partial g_y}{\partial g_L} = 0$ when $m = (1+\varphi)$. The three inequalities: $m = (1+\varphi)$, $m \in (0;1)$ and $(1+\varphi) > 0$ are trivially checked when:

$$0 < m = (1+\varphi) < 1,$$

which is clearly compatible with the *SBTC hypothesis* ($-1 < \varphi < 0$).■

Ceteris paribus, and in particular for given h_t, an increase in the population size (g_L) leads in our model to a faster technological progress, g_n (the intuition being that in the presence of larger population, and at least for the industrialized countries, the probability of having a greater number of scientists and engineers is higher and so is also the capacity of generating new discoveries). For given ε and with $\varphi > 0$, the rise of g_n unambiguously reduces economic growth (g_y):

$$g_y = \sigma(1-\varepsilon) - \varphi g_n - g_L .$$

Hence, under the *ETC hypothesis* ($\varphi > 0$) population growth always exerts a negative effect (both direct and indirect, through technological progress) on real per-capita income growth. Instead, under the *SBTC hypothesis* ($-1 < \varphi < 0$), and following an increase in population, the term $-\varphi g_n$ becomes increasingly positive in the expression above. Thus, the whole effect of population growth on real per-capita income growth may be positive, or negative, or else exactly equal to zero.

The next Proposition analyses the relationship between population growth and economic growth in three special cases: $m = 0$, $m = 1$ and $\varphi = 0$.

PROPOSITION 4

In an economy with human capital accumulation, horizontal innovation-based R&D activity, and in which the impact of technological progress (the growth rate of intermediate-goods variety) on skill investment may be positive, negative or even equal to zero ($\varphi > -1$), the effect of population growth on real per-capita income growth is:

- *Unambiguously negative if the weight assigned to per-capita consumption of future generations (m) is zero - i.e.,* $\frac{\partial g_y}{\partial g_L} < 0$, $\forall \varphi > -1$.
- *Ambiguous, and depending on the direction of technological change, if the weight assigned to per-capita consumption of future generations (m) is one - i.e.,* $\frac{\partial g_y}{\partial g_L} > 0$ *under the SBTC hypothesis* ($-1 < \varphi < 0$)*;* $\frac{\partial g_y}{\partial g_L} = 0$ *under the NTC hypothesis* ($\varphi = 0$)*, and* $\frac{\partial g_y}{\partial g_L} < 0$ *under the ETC hypothesis* ($\varphi > 0$)*.*

When technological change is neutral ($\varphi = 0$), the impact of population growth on real per-capita income growth is unambiguously negative, $\forall \; m \in [0;1)$.

Proof: See equation (19) and recall that $\lambda \geq 1$ and that $(\lambda + \varphi)$ is always positive. ■

When $m = 0$ the inter-temporal utility function in (13) is of the Millian type. This corresponds to the case in which each individual in the economy is totally selfish, since s/he does not take at all into account the utility of his/her descendants. In this situation it seems intuitive that any increase in the population size would affect negatively, and without ambiguity, individual welfare (per-capita income growth). This is stated in the first part of Proposition 4. The second part of the same proposition, instead, focuses on the opposite case. When $m = 1$ the inter-temporal utility function in (13) is of the Benthamite type. This corresponds to the case of *perfect altruism*, in the sense that now the household maximizes utility of all of its members (both present and future). In this situation the effect of having larger dynastic families is not clear-cut and crucially depends on how technological progress (induced by a bigger population) is able to influence the individual incentives to invest in human capital and, ultimately, the income growth rate of each member of the dynasty. In the special case in which incentives to invest in human capital are totally independent of technological progress ($\varphi = 0$), then the last part of Proposition 4 suggests that population growth is very likely to lower per-capita income growth.

Table 1 summarizes Theorem 2 and Proposition 4 in analyzing the relationship between the *degree of altruism* towards future generations (*m*), the *nature of technological progress* (φ), and the *sign* of the effect of population growth on per-capita income growth $\left(\frac{\partial g_y}{\partial g_L}\right)$:

	$\frac{\partial g_y}{\partial g_L} > 0$	$\frac{\partial g_y}{\partial g_L} < 0$	$\frac{\partial g_y}{\partial g_L} = 0$
SBTC $-1 < \varphi < 0$	$m \in (0;1]$	$m \in [0;1)$	$m \in (0;1)$
ETC $\varphi > 0$	---	$\forall m \in [0;1]$	---
NTC $\varphi = 0$	---	$m \in [0;1)$	$m = 1$

Table 1: *Degree of altruism (m), the direction of technological change (φ) and the effects of population growth on real, per-capita income growth* $(\partial g_y / \partial g_L)$

In words the effect of population growth on income growth is a priori indefinite under the *SBTC hypothesis* and with $m \in (0;1)$. However, under the same hypothesis on the direction of technical change, population growth unambiguously discourages economic growth in the presence of the lowest level of altruism in the population (*m*=0), whereas it unambiguously raises economic growth in the presence of the highest possible degree of altruism (*m*=1).

Under the *ETC hypothesis*, instead the effect of population growth is always negative, irrespective of the degree of altruism towards future generations and, hence, of the form of the inter-temporal utility function.

Finally, if technological change is neutral with respect to individual incentives to acquire skills (*NTC hypothesis*), then the effect played by population growth on economic growth is negative for nearly all of the possible degrees of altruism (population growth and income growth being, at most, uncorrelated when m=1).

In sum, Table 1 suggests that under the assumption most of growth theories (both exogenous and endogenous) do about the inter-temporal households' utility function (namely, m=1) the effect of population change on economic growth is ultimately tied to the nature of technical progress. In the opposing case of m=0, instead, such an effect is unambiguously negative (irrespective of the direction of technological change), whereas it is still ambiguous for a continuum of intermediate values of m (in such case the effect is definitely negative in the presence of *eroding* and *neutral technical change* and can be positive, negative or equal to zero when *technical change* is *skilled-biased*). This amounts to saying that, except for the case of m=0, the direction of technical change is of paramount importance in assessing the long-run effects of population growth on per-capita income growth.

5. Conclusions

Since the early 18th century world population has considerably increased to over 6 billion people and is expected to reach 9 billion people by 2050. More importantly, *"…Past and projected additions to world population have been, and will increasingly be, distributed unevenly across the world. The disparities reflect the existence of considerable heterogeneity in birth, death, and migration processes, both over time and across national populations …"* (Bloom and Canning, 2004, p.3). In this regard, it is well known that most of the explosive population growth is mainly concentrated in developing countries, with the developed regions of the world succeeding in maintaining roughly constant the number of their inhabitants especially thanks to the migration dynamics from abroad.

By taking the population growth rate as exogenous and constant, in this paper we built an endogenous *Romerian (1990)-type* growth model with human accumulation, the objective being to analyze the long-run effects of population change on real per-capita income growth. One peculiarity of our contribution is that technological progress (the growth of the number of *ideas*) is explicitly recognized as (potentially) able to influence agents' decision to invest in skill acquisition, the engine of economic growth in our model. Thus, and depending on whether technical change is postulated to affect positively, negatively or not affect at all human capital accumulation, we may have respectively *"skilled-biased"*, *"eroding"* or else *"neutral" technical change*. Hence, and unlike the standard approach (in which population growth adds to the *exogenous depreciation* rate of per-capita skills), the presence of the growth rate of ideas in the law of motion of human capital acts in our model as an *endogenous* mechanism of *depreciation* or *appreciation* of (embodied) knowledge. This is important because allows us tying the effects of population growth on per-capita income growth to the nature and the direction of technical change. Still, we find that, for given type of technical progress, such effects may also significantly depend on the size of the agents' degree of altruism towards future generations (the intuition is that this preference parameter influences households propension to save and, thus, to invest in human and R&D capital). Furthermore, we also find that, even in a framework where economic growth is explained by *horizontal R&D* (even though it is ultimately driven by human capital accumulation), population growth is neither necessary nor conducive to long-run growth in per-capita real income.

Clearly, more work (both theoretical and empirical) is needed to resolve the ambiguities concerning the effect of population change on economic growth and to come to more definitive conclusions on this topic. As an example, from a purely theoretical point of view it would be interesting to study how the results of this paper might change in the presence of endogenous population growth (*i.e.*, endogenous *fertility* and *migration* decisions). Moreover, it is well recognized that population growth is only one (yet very partial) facet of global demographic change and that the shifting age structure of the world population is another aspect that certainly needs to be considered in the analysis. We leave these and other paths of study to future research.

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Appendix

In these notes we derive the set of results (18) through (23) in the main text. Along the BGP equilibrium ε is constant (Proposition 1 in the main text) and endogenously determined through the solution to the household's constrained maximization problem (equations 13, 11, 12' and 14 in the text). Letting μ_{at} and μ_{ht} denote respectively the shadow price of per-capita financial wealth and human capital, the first order conditions of this problem read as:

(i) $$\frac{e^{-(\rho - m g_L)t}}{c_t^{\lambda}} = \mu_{at},$$

(ii) $$\mu_{at} = \mu_{ht}\frac{\sigma}{w_t},$$

(iii) $$\mu_{at}(r_t - g_L) = -\dot{\mu}_{at}$$

(iv) $$\mu_{at}\varepsilon_t w_t + \mu_{ht}\left[\sigma(1-\varepsilon_t) - (\varphi g_n + g_L)\right] = -\dot{\mu}_{ht},$$

a_0 and h_0 given.

Equations (i) and (iii) give the usual Ramsey-Keynes rule governing the optimal path of per-capita consumption. Equation (ii) states that in equilibrium human capital must be equally productive in education and manufacturing activities, whereas equation (iv) provides the evolution of the shadow price of human capital over time. Combining (ii) and (iv) and using (iii) yields respectively:

(v) $$\frac{\dot{\mu}_{ht}}{\mu_{ht}} = -(\sigma - \varphi g_n - g_L),$$

(vi) $$\frac{\dot{\mu}_{at}}{\mu_{at}} = -(r_t - g_L).$$

Equation (ii) implies:

(vii) $$\frac{\dot{\mu}_{at}}{\mu_{at}} = \frac{\dot{\mu}_{ht}}{\mu_{ht}} - \frac{\dot{w}_t}{w_t},$$ or:

(viii) $$r_t = (\sigma - \varphi g_n) + \frac{\dot{w}_t}{w_t}.$$

In the BGP equilibrium (long-run equilibrium) the human capital wage (w) is the same across sectors and grows at constant rate, together with all other time-dependant variables. Accordingly, the real return on asset holdings (r) is constant. With r, H_{Yt}/n_t, H_{it}/n_t and H_{nt}/n_t constant, and using 7 in the main text, equation 10 becomes:

(ix) $$V_{nt}=\underbrace{\int_t^{\infty}\upsilon(1-\upsilon)\left(\frac{H_{Y\tau}}{n_\tau}\right)^{\alpha}\left(\frac{H_{i\tau}}{n_\tau}\right)^{\upsilon}}_{\equiv\pi_\tau}e^{-r(\tau-t)}d\tau=\frac{\upsilon(1-\upsilon)}{r}\left(\frac{H_{Yt}}{n_t}\right)^{\alpha}\left(\frac{H_{it}}{n_t}\right)^{\upsilon}, \qquad \tau>t, \qquad \alpha,\upsilon\in(0;1).$$

Thus, the value of an innovation, *i.e.* the present value of the monopoly profits that innovation make possible, is constant in the long run ($\dot{V}_{nt}/V_{nt}=0$). Given V_{nt}, and making use of equation 9 in the main text, w_{nt} (the wage rate accruing to research human capital) is equal to:

(x) $$w_{nt}=\gamma V_{nt}\left(\frac{n_t}{H_{nt}}\right)^{\chi}=\gamma\frac{\upsilon(1-\upsilon)}{r}\left(\frac{H_{Yt}}{n_t}\right)^{\alpha}\left(\frac{H_{it}}{n_t}\right)^{\upsilon}\left(\frac{n_t}{H_{nt}}\right)^{\chi}, \qquad \psi=1-\chi, \qquad \chi\in[0;1).$$

Using (4) and (6) in the text:

(xi) $$w_{it}=\upsilon^2\left(\frac{H_{Yt}}{n_t}\right)^{\alpha}\left(\frac{H_{it}}{n_t}\right)^{\upsilon-1}.$$

Equating w_{nt} and w_{it} (see equation 16' in the main text), one can determine the equilibrium constant ratio H_{it}/n_t:

(xii) $$\frac{H_{it}}{n_t}=\frac{r\upsilon}{\gamma(1-\upsilon)}\left(\frac{H_{nt}}{n_t}\right)^{\chi}.$$

In a symmetric equilibrium where $x_{it}=x_t$, $\forall i$ we know that (see 3' in the text):

(xiii) $$w_{Yt}=\alpha\left(\frac{n_t}{H_{Yt}}\right)^{1-\alpha}\left(\frac{H_{it}}{n_t}\right)^{\upsilon}$$

Equating w_{Yt} and w_{it} (see equation 16), yields:

(xiv) $$\frac{H_{Yt}}{n_t}=\frac{\alpha}{\upsilon^2}\frac{H_{it}}{n_t}=\frac{\alpha r}{\gamma\upsilon(1-\upsilon)}\left(\frac{H_{nt}}{n_t}\right)^{\chi}.$$

Equations (x), (xi) and (xiii) suggest that wages are constant along the BGP:

(xv) $$g_{w_n}=g_{w_i}=g_{w_Y}\equiv g_w\equiv\frac{\dot{w}_t}{w_t}=0.$$

Combining equations (i) and (vi) in these notes we are able to obtain the usual Euler equation, giving the optimal path of per-capita consumption:

(xvi) $$\frac{\dot{c}_t}{c_t}\equiv g_c=\frac{1}{\lambda}\left[r-\rho-(1-m)g_L\right].$$

From equation (17) in the text and using (ix) in these notes, we obtain:

(xvii) $$g_a=g_n+g_{V_n}-g_L=g_n-g_L, \qquad a_t\equiv A_t/L_t \text{ and } g_{V_n}=0.$$

Merging equations 11 (in the main text) and (vi) yields:

(xviii) $\frac{\dot{\mu}_{at}}{\mu_{at}} = -g_a + \varepsilon w \frac{h_t}{a_t} - \frac{c_t}{a_t}$.

Instead, from the combination of 12' (in the main text) and (v), we get:

(xix) $\frac{\dot{\mu}_{ht}}{\mu_{ht}} = -g_h - \sigma\varepsilon$, $h_t \equiv H_t / L_t$, $g_h \equiv g_H - g_L = g_n - g_L = g_a$.

Since $g_w \equiv \frac{\dot{w}_t}{w_t} = 0$, (vii) implies $\frac{\dot{\mu}_{at}}{\mu_{at}} = \frac{\dot{\mu}_{ht}}{\mu_{ht}}$, that is:

(xx) $\frac{c_t}{a_t} = \varepsilon w \frac{h_t}{a_t} + \sigma\varepsilon$, where $g_h = g_a$ has been used.

With w and ε constant and $g_h = g_a$, the last equation leads to :

(xxi) $g_c = g_a = g_h = g_n - g_L$.

Equating (xxi) and (xvi) one obtains:

(xxii) $r = \lambda g_n + \rho + (1 - m - \lambda) g_L$.

Next, by equalizing (xxii) to (viii) and using $\frac{\dot{w}_t}{w_t} = 0$, we get:

(xxiii) $g_n = \frac{[\sigma - \rho - (1 - m - \lambda) g_L]}{(\lambda + \varphi)}$.

Along the BGP the available stock of human capital (H_t) and the existing number of varieties (n_t) grow at the same rate (*i.e.* $g_n = g_H$). Using this fact and equation (12) in the main text we conclude:

(xxiv) $g_H = g_n = \frac{\sigma(1 - \varepsilon)}{(1 + \varphi)}$.[12]

Finally, the equalization of (xxiii) and (xxiv) allows us obtaining the optimal fraction of human capital employed in production activities (ε):

(xxv) $\varepsilon = 1 - \frac{(1 + \varphi)[\sigma - \rho - (1 - m - \lambda) g_L]}{\sigma(\lambda + \varphi)}$.

Given (xxiii), it is now possible to calculate (see equations xxi and xxii above):

(xxi') $g_c = g_a = g_h = g_n - g_L = \frac{(\sigma - \rho) - (1 + \varphi - m) g_L}{(\lambda + \varphi)}$;

[12] Notice that, with $\sigma > 0$ and $\varepsilon \in (0;1)$, the restriction $\varphi > -1$ prevents $g_H = g_n$ from being either explosive ($\varphi = -1$) or negative ($\varphi < -1$).

(xxii') $r = \dfrac{\lambda\sigma + \varphi[\rho + (1 - m - \lambda)g_L]}{(\lambda + \varphi)}$.

With $\lambda \geq 1$, $(1+\varphi) > 0$ and $(\rho - mg_L) > 0$, the condition $\sigma > (\rho - mg_L)$ is sufficient to guarantee simultaneously that: the household's problem has an interior solution ($0 < \varepsilon < 1$), the common growth rate of aggregate variables ($g_n = g_H$) is positive, and that the real interest rate (*r*) is also greater than zero. Finally, when $\sigma > (\rho - mg_L) + (1+\varphi)g_L$ the growth rate of variables in per-capita terms is positive as well ($g_c = g_a = g_h > 0$). Note that we are not making any a-priori assumption on the population growth rate ($g_L \gtreqless 0$ in our model),[13] and that the assumption $\sigma > \rho$ ensures that g_c is positive even when $g_L = 0$.

Under the assumption of symmetry across intermediate inputs ($x_{it} = x_t$ for each *i*), and employing equations (1) and (6) in the main text, we can write per-capita output as:

$$y_t \equiv \frac{Y_t}{L_t} = \left(\frac{H_{Yt}}{n_t}\right)^{\alpha}\left(\frac{H_{it}}{n_t}\right)^{\upsilon}\frac{n_t}{L_t},$$

where $\dfrac{H_{Yt}}{n_t}$ and $\dfrac{H_{it}}{n_t}$ are given constants (to be determined in a moment). Taking logs of both sides of this expression and totally differentiating with respect to time, we obtain:

(xxi'') $g_c = g_a = g_h = g_y = g_n - g_L = \dfrac{(\sigma - \rho) - (1 + \varphi - m)g_L}{(\lambda + \varphi)}$;

To find out the equilibrium values of $\dfrac{H_{Yt}}{n_t}$ and $\dfrac{H_{it}}{n_t}$, consider the production function of ideas (with $\psi = 1 - \chi$) in the main text and (xxiii) above and get:

(xxvi) $\dfrac{H_{nt}}{n_t} = \left\{\dfrac{[\sigma - \rho - (1 - m - \lambda)g_L]}{\gamma(\lambda + \varphi)}\right\}^{\frac{1}{1-\chi}}$, $\chi \in [0;1)$.

With γ and $(\lambda + \varphi)$ both always positive and $\lambda \geq 1$, the condition $\sigma > (\rho - mg_L)$ is also sufficient to guarantee that the ratio H_{nt} / n_t is positive.

From (xii) and (xiv) we can easily compute the BGP equilibrium ratios:

(xxvii) $\dfrac{H_{it}}{n_t} = \dfrac{r\upsilon}{\gamma(1-\upsilon)}\left(\dfrac{H_{nt}}{n_t}\right)^{\chi}$ and $\dfrac{H_{Yt}}{n_t} = \dfrac{\alpha r}{\gamma\upsilon(1-\upsilon)}\left(\dfrac{H_{nt}}{n_t}\right)^{\chi}$,

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[13] In the much simpler case where $g_L \geq 0$, the inequality $\sigma > (\rho - mg_L) + (1+\varphi)g_L \geq (\rho - mg_L)$ would certainly hold since $(1+\varphi)$ is positive. The same is also true for the inequality $\sigma > \rho \geq (\rho - mg_L)$ that would be certainly satisfied for each $m \in [0;1]$ and each $g_{L\geq 0}$.

where r and H_{nt}/n_t are given respectively by equations (xxii') and (xxvi). Plugging H_{nt}/n_t, H_{it}/n_t and H_{Yt}/n_t into equation (15) in the main text leads to:

(xxviii) $$\frac{H_t}{n_t}=\frac{1}{\varepsilon_t}\frac{H_{nt}}{n_t}\left[1+\frac{r}{\gamma(1-\upsilon)}\left(\upsilon+\frac{\alpha}{\upsilon}\right)\left(\frac{H_{nt}}{n_t}\right)^{\chi-1}\right].$$

Equations (xxvi), (xxvii) and (xxviii) allow obtaining the BGP equilibrium shares of the available stock of human capital devoted to consumption goods and intermediate inputs production (τ_Y and τ_i) and to research activity (τ_n):

(xxix) $$\tau_Y\equiv\frac{H_{Yt}}{H_t}=\frac{H_{Yt}}{n_t}\frac{n_t}{H_t};\qquad \tau_i\equiv\frac{H_{it}}{H_t}=\frac{H_{it}}{n_t}\frac{n_t}{H_t};\qquad \tau_n\equiv\frac{H_{nt}}{H_t}=\frac{H_{nt}}{n_t}\frac{n_t}{H_t}.$$

In the end of this appendix we want also to verify that the two transversality conditions ($\lim_{t\to\infty}\mu_{at}a_t=0$ and $\lim_{t\to\infty}\mu_{ht}h_t=0$) are checked. Combining (v), (vi), (viii), (xv) and (xxi) these two conditions can be rewritten as:

(xxx) $$\lim_{t\to\infty}\mu_{at}a_t=\mu_{a0}a_0\lim_{t\to\infty}e^{-(r-g_n)t}=0 \quad\text{and}\quad \lim_{t\to\infty}\mu_{ht}h_t=\mu_{h0}h_0\lim_{t\to\infty}e^{-(r-g_n)t}=0,$$

where a_0, h_0, μ_{a0} and μ_{h0} are the (given) initial values (at t=0) of the two state-variables (*a* and *h*) and their respective shadow prices (μ_a and μ_h). Equation (xxx) suggests that the transversality conditions are trivially satisfied when:

$$r>g_n.$$

Using (xxii') and (xxiii), and with $\lambda\geq 1$ and $(1+\varphi)>0$, the requirement $r>g_n$ is met when $(\rho-mg_L)>0$. This is the same constraint that assures the intertemporal utility function to be bounded.