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EXTERNALITIES, REGULATION AND TAXATION IN ELECTRICITY GENERATION

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Externalities, Regulation and Taxation in Electricity Generation*

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Abstract

In this paper we explore the interactions between environmental externalities and intertemporal exercise of market power in electricity generation industries where imperfect competition occurs among firms adopting thermal and hydraulic processes. We identify the implications in terms of intertemporal output profile, prices, environmental quality. We then investigate how taxation, price cap and contracts for water differences ($CWDs$) should be designed to achieve efficient outcomes. We assess the flexibility value of adjusting policy instruments according to per-period conditions. We establish that, even when each activity within the industry is subject to a specific mechanism, proper design still requires that a global view of the concerned sector be maintained.

Keywords: Electricity generation, Externalities, Price cap, Taxation, Water quotas, Contracts for differences

J.E.L. Classification Numbers: L13, L51, L9; D62; H23

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1 Introduction

In this paper we explore the interactions between environmental externalities and intertemporal exercise of market power in electricity generation industries where imperfect competition occurs among firms adopting different technological processes. After assessing the implications in terms of output profile, prices and environmental quality, we take a policy-oriented approach and figure out how policy instruments that are widely used in real-world electricity sectors as well as alternative ones should be properly designed for efficient outcomes to be achieved. We (more generally) shed light on policy relationships in oligopolies that are characterized by relevant dynamic aspects and constraints on fundamental inputs.

Our study hinges on the real-world evidence that electricity is often generated in oligopolies where thermal stations compete with hydraulic stations that use water stored in dams. A good example can be found in New Zealand, where a thermal generator competes with ECNZ, the firm that manages the two major reservoirs storage systems (Scott and Read [27]). Thermal and hydraulic technology display different features, which are at the core of our analysis.

The thermal technology is based on the usage of hydrocarbon stocks, which do not constitute a scarce resource in the short run. The thermal process is thus a static one. Production decisions are made independently at each period of time, hence market power can only be statically exerted. Nevertheless, hydrocarbons release polluting emissions, so that an *environmental externality* is associated with the thermal activity.

The hydraulic technology we focus on relies upon water kept in dams. The latter is exhausted and renewed over time according to the hydrological pattern of the concerned region. Resource paucity makes huge economic rents available in each time period, which opens the door to static exercise of market power (Bushnell [11]; Ambec and Doucet [2] etc.). However, rents can be further increased by strategically (and costlessly) transferring water across periods that exhibit different market conditions. Dynamic market power of this sort can be exerted in closed-loop Cournot frameworks, where the hydraulic generator acquires a first-move advantage *vis-à-vis* the thermal competitors (Crampes and Moreaux [13]). *Intertemporally strategic behaviour* is thus associated with the hydraulic activity.

So far the interplay between environmental externalities and intertemporal exercise of market power has not been investigated. It is thus unclear which implications the joint work of these two "evils" yields. Building on Crampes and Moreaux [13], in the present work, we focus on a two-period duopoly game to explore how the externality that is induced by the thermal operator releasing polluting emissions interacts with the market power that is exerted by the hydraulic generator moving water over time. To begin with, the analysis allows us to emphasize how the allocation that is implemented at the market equilibrium diverges from the one that would arise in a first-best setting where environmental quality were taken into account. Furthermore, it helps us identify conditions under which strategic water misallocation between periods alleviates/exacerbates environmental problems through the interactions with the thermal activity.

Moving next to policy issues, we initially explore a scenario where industry-wide price cap regulation is in place together with pollution taxation. Price cap regulation tends to survive even in liberalized electricity sectors, especially at the early stages of the liberalization process. In England and Wales, the economic Regulator (OFFER) has capped the prices of the electricity pool purchases from 1994 to 1996 (Acutt and Elliott [1]). In Ontario, under the Market Power Mitigation Agreement, a price cap has been introduced on the electricity sold to the wholesale pool (Gillen and Wen [17]). On the other hand, the carbon tax is being increasingly favoured as an instrument for controlling polluting emissions, hinging on the argument that it affects a "bad" rather than a "good". Despite wide reliance, mismanagement of these policies has sometimes worsened/compromised performance (as in the 2000-1 California crisis), which points to the importance of improving upon mechanism design. To address this issue, we establish how both price cap and tax should be set to best fit industries of the kind of our interest.

We begin by establishing that there is a flexibility value to adjusting policy instruments on a per-period basis according to specific market conditions, *i.e.* to enlarging the space of feasible instruments to the cardinality of the periods set. This is readily in support of those rules that span different discipline on activities in different moments in time, such as peak-time consumption curtailing.

We then explore flexible (per-period) taxation. We assess that the latter intervenes on thermal distortions trading-off countervailing effects so as to strike proper *net* incentives. Indeed, the tax is meant to correct for the environmental damage the activity induces and yet it accounts for the welfare cost that is associated with the ensuing output reduction. This reflects the imperfectly competitive structure of the industry where the polluting agent operates and is in line with Barnett [4]'s finding about taxation of monopolies generating externalities. Reasonably enough, we establish that polluting activities can be more vigorously charged when cleaner processes can be called upon to replace them. This clearly points to the importance that (neat) products be available as substitutes for dirty ones, especially in highly captive markets. We as well conclude that taxation should not be too severe when the price elasticity of market demand is small, which is typically the case in electricity sectors. Therefore, *ceteris paribus*, taxation is milder at peak time, when demand is relatively less price elastic, and tighter at off peak time. The intuition behind this can be easily seen. When demand is rigid, quantity reduction stemming from tax increase triggers important price raise. Taxation is thus softened to contain this effect.

We subsequently move to investigate flexible price regulation. We point that the latter can be performed by constructing an intertemporal constraint that forces per-period market prices to obey a global cap. We show that, as long as this instrument can be adopted jointly with the flexible tax, the Regulator can focus on the *hydraulic* process only, unless the participation of the thermal operator is to be ensured. An interesting insight thus emerges. Although the intertemporal cap is targeted to a Cournot framework for industry-wide implementation, it proves similar to "traditional" price regulation of multiproduct monopoly, each period being approached as a different product. This circumstance directly follows from the interaction between cap and tax. More precisely, proper design of the cap requires that the authority be able to identify how much of the scarce resource should be used in each period, given the available stock, so as to reach the most efficient outcome that is feasible in the concerned setting. When the competitor's financial viability is an issue, it is as well necessary to determine the (constrained) efficient amount of thermal electricity. In any event, it is essentially a matter of assessing (constrained) optimal quantities.

Having this result in mind, the most natural alternative to price cap one could think of consists in (directly) implementing per-period water quotas through *contracts for water differences* ($CWDs$). The latter are similar to the so-called contracts for (price) differences frequently used in wholesale electricity markets to reduce generators' incentives to raise price.

At later stage, we study a scenario where $CWDs$ are adopted jointly with pollution taxation. Economic regulation is here targeted to the sole operator that can exert intertemporal market power. According to $CWDs$, the hydraulic generator goes subject to a penalty if the amount of resource that is used in either period diverges from the target water quota, which is fixed by the Regulator. We pin down the appropriate payment for efficient water allocation to be decentralized.

As a more general contribution, this analysis highlights that, even in environments where each activity goes under a specific instrument and single policies do not follow as a compromise over industry averages, it is still necessary to maintain a global view of the concerned sector. This is in sharp contrast with the finding previously recalled that, instead, industry-wide price regulation (generally) collapses onto a price cap calibrated on the hydraulic generator only. From this standpoint, regulating hydraulic production appears more requiring than regulating market prices, even in an industry where quantities are the relevant choice variables and water quotas appear to be a natural control instrument.

As long as various policies are adopted by different Regulators, coordination puzzles eventually arise, which in turn points to the relevance of promoting cooperation and policy harmonization. This problem alleviates whenever a unique policy is adopted, which is the kind of situation we lastly investigate focusing on industry-wide taxation. We find that, under some conditions, electricity taxation is a suitable instrument for discouraging power overusage and improving environmental quality. It proves also a valid tool for extracting hydraulic rents. This is in line with the arguments in Gillen and Wen [17], who are favorable to charging royalty fees on hydraulic electricity.

The remainder of the paper is organized as follows. In Section 2, we revise the related literature. In Section 3, we describe the model and present the first-best scenario. In Section 4, we first recall open and closed-loop Cournot competition, as from Crampes and Moreaux [13], and then identify the interaction between environmental externality and intertemporal water (mis)allocation. In Section 5 to 7, we focus on policy issues. Section 8 concludes.

2 Related Literature

Three main domains of economic literature contribute to the theoretical background of our work, namely the papers that focus on competition in electricity generation when hydropower is produced, the studies that concentrate on price cap regulation in the presence of intertemporal concerns and those that investigate environmental taxation.

To the first category of works belongs the aforementioned article by Crampes an Moreaux [13]. The latter build a model of competition between a thermal station and a hydraulic station to show that the (intrinsically static) thermal output turns is in fact determined by intertemporal considerations, stemming from the scarcity of the water resource and the possibility of transferring the latter over time at zero cost. They highlight the dynamic market power the hydraulic generator can exert, which constitutes the core starting point of the present paper. Within the same domain of literature, Scott and Read [27] rely upon a market simulation model embodying Cournot sub-games to optimize hydro-reservoir operation in a deregulated electricity market where both hydraulic and thermal capacity are present. In a dynamic setting, Ambec and Doucet [2] study hydropower generation in the presence of both water (resource) and reservoir (capacity) constraints and compare monopoly and competitive performance. In turn, Bushnell [11] explores Cournot competition among firms possessing both hydroelectric and thermal plants. As based on a mixed linear complementarity model, he concludes that water allocation in electricity generation is a great instrument for strategic behaviour and, as such, it should be considered while designing regulatory and environmental policies. Garcia, Reitzes and Stacchetti [16] analyze the dynamic strategic behaviour of two hydraulic producers facing a perfectly inelastic demand and competing in a Bertrand-Nash fashion. In particular, they investigate the impact of price caps on the producers' conduct by looking at price caps that are kept constant and thus are not intertemporally modulated. By contrast, adhering to Crampes and Moreaux [13], we assume that the scarce water is managed by a unique firm and that competition with the thermal producer takes place *à la* Cournot. Furthermore, we adopt a normative approach in that, within this setting, we construct the optimal intertemporal price cap, which attaches different weights to the prices in different periods. Like this each period is treated as if it were a specific market, albeit the weights are determined to embody the intertemporal link that is created by the available resource. Garcia, Reitzes and Stacchetti [16] find that, in the environment they consider, the reliability of the system can be compromised by the imposition of a price cap. Indeed, whenever the latter is set sufficiently low, hydraulic generation can totally replace thermal generation and shortage can follow. They do not suggest any policy adjustment to handle with these implications though, sticking on a purely positive perspective. Yet their result confirms the necessity of adequately designing the regulatory constraint, so that relevant intertemporal aspects are taken into account. One last work that corroborates the importance of properly considering intertemporal issues in electricity price cap regulation is the one by Johnsen [23]. The latter investigates hydropower generation and storage under monopoly in the presence of uncertainty about future water inflows and possibility of exchange with neighboring regions. As based on a stylized numerical model, Johnsen concludes that introducing a price cap only in the second period of generation unequivocally increases expected welfare, whereas the impact of imposing the same price cap in both periods depends upon monopoly generation in the first period.

Moving next to the second domain of literature, price cap regulation in different intertemporal perspectives is investigated by Hagerman [22], Braeutigam and Panzar [10], Foreman [15], Dobbs [14] and Roques and Savva [26]. Most of our interest, Foreman [15] is concerned with the possibility that the weighting scheme employed in price cap plans be subject to welfare-reducing manipulation over time. Focusing on price cap regulation of a multiproduct monopolist, he shows that, whenever the weight that is assigned to some service is given by the share of total revenue for the basket generated by that service in the previous period, the firm can intertemporally manipulate the weight to get higher profits. He then proposes an alternative weighting scheme based on relative quantities, which is shown to be less prone to intertemporal manipulation. Our environment displays a few major differences with respect to Foreman's. As aforementioned, he concentrates on a monopolistic market where the supplier produces several goods and there is no other link between periods than the one induced by the weights in the price cap. By contrast, we look at a duopolistic industry where firms produce a homogeneous good using two different technological processes, one of which creates a relationship between the two periods. Indeed, allocation of some water to either period determines how much water is left disposable in the other period and both such amounts depend on the total quantity of available resource. Therefore, while in Foreman incentives to intertemporal strategic behaviour are provided by the peculiar structure of the regulatory scheme, in our environment they stem from the characteristics of one of the technologies that are involved in the market game, relative to those of the competing process. Similarly to us, Roques and Savva [26] focus on price cap regulation of an oligopoly *à la* Cournot. Analysis and objectives are otherwise different though. Indeed, Roques and Savva study price cap effects on investment in new capacity under stochastic demand, while investment is of no concern to us and we rule out any demand uncertainty.

Finally, the bunch of papers about environmental pollution and taxation is a rather rich one and we content ourselves with mentioning the studies that more closely relate to the present work. Building on Buchanan (1969) and criticizing Baumol and Oates (1975), Barnett [4] highlights the complexities that need to be dealt with when taxing externalities that are generated by monopolistic firms. By contrast, we focus on a duopolistic framework where environmental taxation of a polluting firm coexists with price cap regulation. From this viewpoint, our approach is reminiscent of Baron [5], who analyzes a model in which two public authorities, namely a regulatory and an environmental agency, respectively regulate the price and the polluting emissions of a firm. Baron still looks at a monopolistic sector though. Departing from monopoly and focusing on the electricity generation industry, Acutt and Elliott [1] explore the interaction between environmental policies, which are aimed at reducing pollution, and a price cap that is aimed at reducing market power. As compared to the latter study, our work further specifies the analysis by devoting attention to the case where hydraulic and thermal stations compete in power generation and dynamics and resource scarcity are primary aspects. This allows us to first investigate the interaction between environmental externality and intertemporal market power, which is unaddressed in the existing literature, and then explore the interplay between different policy instruments in settings where intertemporal aspects and constraints on fundamental inputs are to be accounted for. At later stage, we also investigate taxation of energy, whether generated by the polluting process or by the clean one. This latter approach is reminiscent of Levin [24], who examines the conditions under which taxation is effective at reducing environmental externalities within Cournot oligopolies. Yet he sticks on a static framework where all producers release pollutants and strategic control of a scarce resource is not an issue.

3 The Model

Hinging on Crampes and Moreaux [13], we consider a discrete intertemporal model and suppose that electricity is generated by two firms using different technological processes, namely a hydraulic process (firm H) and a thermal process (firm T). Firms compete *à la* Cournot and programme generation over a time span of two periods ($t = 1, 2$) at zero intertemporal discount.

The thermal output during period t is denoted q_t^T. The associated variable cost of production is given by $c\left(q_t^T\right)$, the function $c(\cdot)$ being increasing and convex in its argument. A fixed cost F^T is also incurred. Moreover, the installed thermal capacity is supposed to be so large that it is never saturated. In each period, the thermal generation process releases polluting emissions $e\left(q_t^T\right)$, which are larger the larger the thermal production $\left(\partial e/\partial q_t^T > 0,\ \forall t\right)$. Emissions create environmental damage $D\left(e\left(q_t^T\right)\right)$, with $D(\cdot)$ a smooth function increasing in the level of emissions $(\partial D/\partial e > 0)$.

As for the hydraulic technology, the exogenous stock of water, which is denoted S, can be utilized between the beginning of the first period and the end of the second period. The intertemporal water constraint writes as

$$q_1^H + q_2^H \leq S,$$

where q_t^H, $t = 1, 2$, expresses the hydroelectric output during period t. Whenever the available resource is scarce, the constraint is binding and holds as an equality. This is the relevant situation in our setting, which captures the fact that hydraulic operators are generally prevented from free disposal by legal requirements[1]. Related to this is the assumption that the initial stock of water is commonly known in the industry, which catches the circumstance that information on reservoir filling is made available by the public authorities to the various market participants[2]. Furthermore, hydro-power generation is supposed to incur a fixed cost F^H but no variable cost, in order to capture the circumstance that cost does not change with water taking. The possibility that the installed turbines be saturated is ruled out.

Electricity is a standardized commodity, hence firms offer perfectly substitute products. The total utility that is obtained from the consumption of $Q_t = \left(q_t^T + q_t^H\right)$ units of power during period t is denoted $u_t\left(Q_t\right)$, the function $u_t\left(\cdot\right)$ being increasing and strictly concave. Electricity consumption is unaffected by environmental externalities.

Lastly, in each period, the demand for power is supposed to be perfectly known, hinging on the observation that the main part of the yearly variability of demand can be predicted with reasonable accuracy.

3.1 The First-Best Scenario

We begin by exploring the first-best scenario. The social welfare function is given by

$$\begin{aligned} W\left(q_1^H, q_1^T, q_2^H, q_2^T\right) &= u_1\left(Q_1\right) + u_2\left(Q_2\right) - c\left(q_1^T\right) - c\left(q_2^T\right) \\ &\quad - F^H - F^T - D\left(e\left(q_1^T\right)\right) - D\left(e\left(q_2^T\right)\right). \end{aligned} \tag{1}$$

Welfare is assumed to be the unweighed sum of consumer surplus and firms' profits, net of the environmental externality that is induced by the thermal process.

Looking exclusively at interior solutions[3], the first-best profile of output is pinned down by maximizing the social welfare function subject to the intertemporal water constraint. More

[1] These requirements are innocuous with regard to our study in that we are essentially interested in intertemporal misallocation of some given amount of water. Letting the firm decide whether to use or not the whole stock would eventually exacerbate, though not change the nature of the market power problem we investigate.

[2] For instance, as from December 2002, following to a tightening in the Norwegian supply situation, the Norwegian Water Resources and Energy Directorate (NVE) decided to provide more detailed information about reservoir filling in the country as compared to the past. In particular, information about aggregate reservoir levels for four different regions started being published instead of information about aggregate reservoir levels for Norway as a whole (Grønli and Costa [20]). This was highly recommended (also) at the aim of improving upon the existing monitoring practices on security of supply (Grønli and Costa [21]).

[3] Unless differently specified, the analysis will focus on interior solutions all along the article.

precisely, the first-best allocation satisfies the set of conditions

$$\begin{aligned} p_1 &= p_2 \\ &= \mu \\ &= \frac{\partial c}{\partial q_1^T} + \frac{\partial D}{\partial e}\frac{\partial e}{\partial q_1^T} \\ &= \frac{\partial c}{\partial q_2^T} + \frac{\partial D}{\partial e}\frac{\partial e}{\partial q_2^T}, \end{aligned} \tag{2}$$

where μ is the Lagrange multiplier associated with the resource constraint. The conditions listed in (2) say that, at the social optimum, electricity should be equally priced over time. Furthermore, in each period, the energy price should equal the marginal virtual cost of water (μ) as well as the marginal social cost of thermal output.

4 Cournot Competition

The first-best environment previously described does not materialize in a power generation industry that is structured as a duopoly where each firm controls one technological process.

In what follows, we revisit the analysis presented in Crampes and Moreaux [13] to recall how the sector performs as long as firms compete in the absence of any corrective intervention. To highlight the ensuing kind of intertemporal market power and illustrate the advantage it yields to the hydraulic generator, we first present Cournot competition in an open-loop context and then compare with a closed-loop situation.

4.1 The Open-Loop Game

When duopolists engage in an open-loop game, firm T takes firm H's decisions as given and chooses output q_t^T, $t = 1, 2$, so as to maximize the profit function

$$\pi^T = q_1^T p_1\left(q_1^H + q_1^T\right) + q_2^T p_2\left(q_2^H + q_2^T\right) - c\left(q_1^T\right) - c\left(q_2^T\right) - F^T.$$

This yields

$$p_t + q_t^T \frac{\partial p_t}{\partial Q_t} = \frac{\partial c}{\partial q_t^T},\ t = 1, 2. \tag{3}$$

suggesting that firm T sets quantities so that, in each period, marginal revenues (the left-hand side of (3)) are equal to marginal cost (the right-hand side). Hence, condition (3) identifies the intertemporal profile of thermal output, for any given level of hydraulic production.

In turn, firm H selects quantity q_t^H, $t = 1, 2$, so as to maximize the profit function

$$\pi^H = q_1^H p_1\left(q_1^H + q_1^T\right) + q_2^H p_2\left(q_2^H + q_2^T\right) - F^H$$

subject to the water constraint. From the first-order condition with respect to the hydraulic per-period output, one obtains

$$p_1 + q_1^H \frac{\partial p_1}{\partial Q_1} = p_2 + q_2^H \frac{\partial p_2}{\partial Q_2}. \tag{4}$$

Condition (4) characterizes the intertemporal profile of hydraulic power, for any given thermal output. It states that, as long as an open-loop game is played, firm H allocates the available stock of water so that, given the competitor's output profile, the marginal revenues from hydraulic generation equal over time.

A discrepancy between equilibrium prices, which does not arise in the first-best scenario, results in the open-loop game. It stems from the fact that either firm is endowed with market power and exerts the latter in each period (*i.e.*, statically), as jointly expressed by (3) and (4).

4.2 The Closed-Loop Game

Suppose next that generators play a closed-loop Cournot game. As Crampes and Moreaux [13] highlight, the peculiarity of a closed-loop game is that market agents base current decisions on the history of past actions. In our setting, this means that duopolists take firm H's action in period 1 to be the pertinent variable for choosing quantities in period 2, provided that the size of the stock of water is commonly known. Hence, as long as the stock is scarce and entirely used, at period 2 we have $q_2^H = S - q_1^H$ and $q_2^T = Q_2^T\left(S - q_1^H\right)$ for firm H and firm T respectively.

In the framework described above, condition (3) is still the profit-maximizing rule for firm T, which exerts market power in each period as it does in the open-loop setting.

On the other hand, firm H's profit function specifies as

$$\pi^H = q_1^H p_1\left(q_1^H + q_1^T\right) + \left(S - q_1^H\right) p_2\left(S - q_1^H + Q_2^T\left(S - q_1^H\right)\right) - F^H,$$

because the hydraulic producer anticipates how its competitor is going to react in period 2, after observing the amount of water that has been used at $t = 1$. Therefore, the first-order condition with respect to q_1^H is given by

$$\begin{aligned} p_1 + q_1^H \frac{\partial p_1}{\partial Q_1} &= p_2 + q_2^H \frac{\partial p_2}{\partial Q_2}\left(1 + \frac{dQ_2^T}{dq_2^H}\right) \\ &= p_2 + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}, \end{aligned} \tag{5}$$

which establishes that firm H's marginal revenues are again equal in period 1 and 2. Notice that (5) is obtained by considering that it is $\left(-dq_2^H/dq_1^H\right) = 1$. This means that, under the water constraint, any additional unit of hydraulic power that is generated in the first period requires that one unit be given up in the second period and vice versa.

The difference between open-loop and closed-loop game emerges through the term

$$q_2^H \frac{\partial p_2}{\partial Q_2}\frac{dQ_2^T}{dq_2^H}$$

as soon as (5) is contrasted with (4). The term above expresses the variation occurring in firm H's period-2 marginal revenues as a result of water transfer over time. It captures the dynamic market power firm H exerts at period 1, when it chooses how to allocate water over time, anticipating firm T's reaction in period 2. That is, in the closed-loop game, firm H exerts market power not only statically but also dynamically because in period 1 it acts as a "first-mover" with respect to the decision its competitor will make in period 2. This involves that the two agents play as Cournot competitors in each period and as Stackelberg competitors over time. Thus in the closed-loop game one has

$$p_1 + q_1^H \frac{\partial p_1}{\partial Q_1} > p_2 + q_2^H \frac{\partial p_2}{\partial Q_2},$$

meaning that, for any given output profile, the marginal revenues from hydraulic production in period 1 exceeds the marginal revenues that would entail in period 2 if an open-loop game were played. Although thermal quantities are still pinned down by (3), when firm H behaves according to (5), rather than to (4), the intertemporal issue exacerbates and the equilibrium allocation no longer coincides with the open-loop one.

Remark that the phenomenon aforementioned appears inasmuch as firm H has an interest in transferring water from one period to the other, as compared with the equilibrium allocation of the open-loop game. This is indeed the case whenever the firm does not face the same demand in both periods, *i.e.* it has different incentives to produce at peak and off-peak time[4].

4.2.1 Water (Mis)Allocation and Environmental Externalities

We have previously identified how the hydraulic operator exerts intertemporal market power by allocating water between periods. Both the intertemporal output profile associated with the thermal generation process and the overall thermal production are thereby affected. The relevance of this aspect is twofold. Firstly, the per-period electricity price that consumers face at equilibrium stems from the per-period total quantity of generated power, which depends upon the strategic interaction between rival productions (a direct effect on output volume). Secondly, by affecting thermal production, the intertemporal exercise of market power turns out to interplay with the environmental diseconomy (an indirect effect on social welfare).

To explore how firm H's actions influence thermal output, we check how firm T's quantities vary as q_1^H is changed. From condition (3), we have

$$\frac{dq_1^T}{dq_1^H} = \frac{\frac{\partial p_1}{\partial Q_1} + q_1^T \frac{\partial^2 p_1}{\partial Q_1^2}}{-\left(2\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial \left(q_1^T\right)^2} + q_1^T \frac{\partial^2 p_1}{\partial Q_1^2}\right)} \tag{6}$$

[4] In Scandinavian countries, peak demand appears at the second period of generation. Indeed, reservoir levels sharply increase during spring and early summer due to snow melting, though fall rains contribute to reservoir filling as well. On the other hand, as outflows follow the consumption pattern, they are more important in winter, when lower temperatures require intense heating (Grønli and Costa [20]). The situation is reasonably similar in Italy, where reservoirs are mainly located in mountain areas and winter has historically represented the peak period. However, the traditional Italian timing has progressively evolved during the last years and 2006 has been the first year of peak shifting with summer displaying peak consumption, due to systematic resort to air conditioning (Terna [29]).

together with

$$\frac{dQ_2^T}{dq_2^H}\frac{dq_2^H}{dq_1^H} = -\frac{dQ_2^T}{dq_2^H} = \frac{-\left(\frac{\partial p_2}{\partial Q_2} + q_2^T\frac{\partial^2 p_2}{\partial Q_2^2}\right)}{-\left(2\frac{\partial p_2}{\partial Q_2} - \frac{\partial^2 c}{\partial\left(q_2^T\right)^2} + q_2^T\frac{\partial^2 p_2}{\partial Q_2^2}\right)}, \tag{7}$$

the denominator being positive in both expressions from the second-order condition for a maximum of firm T's profit function. (6) and (7) respectively reveal how q_1^T and q_2^T vary as q_1^H is increased. Provided that quantities are strategic substitutes, as usual in a Cournot duopoly, q_t^T is diminished as q_t^H is raised, at any t[5]. Recall however that, in the specific environment under scrutiny, q_2^H is reduced as q_1^H is increased, as long as the resource constraint binds. This involves that q_2^T raises as so does the hydraulic output q_1^H $\left(\left(dQ_2^T/dq_2^H\right)\left(dq_2^H/dq_1^H\right) > 0\right)$.

The overall effect of a change in q_1^H on total thermal production $Q^T = \sum_t q_t^T$ is given by the sum of (6) and (7), namely by (see Appendix for details)

$$\begin{aligned} \frac{dQ^T}{dq_1^H} &= \frac{dq_1^T}{dq_1^H} + \frac{dQ_2^T}{dq_2^H}\frac{dq_2^H}{dq_1^H} \qquad (8)\\ &= \frac{-q_1^T\frac{\partial^2 p_1}{\partial Q_1^2}\left(\frac{\partial p_2}{\partial Q_2} - \frac{\partial^2 c}{\partial\left(q_2^T\right)^2}\right) + q_2^T\frac{\partial^2 p_2}{\partial Q_2^2}\left(\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial\left(q_1^T\right)^2}\right) + \frac{\partial p_1}{\partial Q_1}\frac{\partial^2 c}{\partial\left(q_2^T\right)^2} - \frac{\partial p_2}{\partial Q_2}\frac{\partial^2 c}{\partial\left(q_1^T\right)^2}}{\sum_t\left(2\frac{\partial p_t}{\partial Q_t} + q_t^T\frac{\partial^2 p_t}{\partial Q_t^2} - \frac{\partial^2 c}{\partial\left(q_t^T\right)^2}\right)}. \end{aligned}$$

As it is immediately evident, no general rule can be identified. Indeed, whether total thermal quantity increases in q_1^H $\left(dQ^T/dq_1^H > 0\right)$ depends upon the specific functional form demands and costs take.

Suppose, for instance, that the demand function is linear in both periods. Then, with $c(\cdot)$ strictly convex, having $dQ^T/dq_1^H > 0$ requires that

$$\frac{\partial^2 c/\partial\left(q_2^T\right)^2}{\partial^2 c/\partial\left(q_1^T\right)^2} < \frac{-\partial p_2/\partial Q_2}{-\partial p_1/\partial Q_1} = \frac{-\partial^2 u_2/\partial Q_2^2}{-\partial^2 u_1/\partial Q_1^2}. \tag{9}$$

According to (9), in the closed-loop game with linear per-period demand, total thermal production increases as more water is used in period 1 (and reduces as water is transferred to period 2) as long as the relative curvature of the cost function at the period-2 and period-1 equilibrium is smaller than that of consumer utility functions.

If, instead, costs are linear and $\partial^2 p_1/\partial Q_1^2 > 0$, for Q^T to be positively related to q_1^H one needs to have

$$\left(\frac{\partial^2 p_2/\partial Q_2^2}{\partial^2 p_1/\partial Q_1^2}\right)\left(\frac{-\partial p_2/\partial Q_2}{-\partial p_1/\partial Q_1}\right) < \frac{q_1^T}{q_2^T}, \tag{10}$$

which is instead a condition on the relative slope and curvature of the demand functions and the equilibrium per-period thermal quantities[6]. In particular, if the demand function takes a linear form in period 2 $\left(\partial^2 p_2/\partial Q_2^2 = 0\right)$, then the inequality above simply calls for $q_1^T/q_2^T > 0$,

[5] One can also check that it is $dq_1^H/dq_1^T < 0$. This follows as long as, at all possible outputs, the marginal revenues of either operator, at any given quantity the latter produces, is a decreasing function of total industry output (compare Levin [24], who recalls Hahn, 1962, p.331).

[6] The inequality in (10) would be reversed with $\partial^2 p_1/\partial Q_1^2 < 0$.

which holds true as long as firm T does produce electricity at the period-1 equilibrium.

Lastly, under the specific circumstance that both demand and cost functions are linear in quantities, total thermal output Q^T does not vary as q_t^H changes, although the intertemporal profile of thermal output does change. To see that $dQ^T/dq_1^H = 0$, it is enough to check that one has $dq_1^T/dq_1^H = -1/2$ together with $\left(dQ_2^T/dq_2^H\right)\left(dq_2^H/dq_1^H\right) = 1/2$. In words, as an additional unit of hydraulic power is generated in period 1, q_1^T is decreased by half unit and q_2^T increased by precisely the same amount.

As long as firms pursue maximum profits, their policy choices do not reflect any environmental concern. Nevertheless, as previously mentioned, by affecting the intertemporal profile and the total amount of thermal output, the market power that is dynamically exerted by firm H interacts with the environmental externality that is induced by the thermal technological process.

Recall that, as q_t^T increases (resp., reduces), emissions increase (resp., reduce) in period t. Furthermore, as already assessed, firm H's strategic behaviour does have an impact on both per-period and overall thermal production. In the end, the way it interferes with the environmental externality, through the strategic interaction with the thermal activity, depends upon the shape of the damage function and the relative emission levels at the equilibrium of either period.

Total damage is unaffected only if demand and cost functions are linear, emissions are directly proportional to the produced quantities and the marginal damage is constant with respect to the emission level. Suppose that the first two conditions holds but not so does the third one as the marginal damage increases with emissions. Then, despite total thermal output is unaffected by water intertemporal transfer, the environmental damage does not need to stay alike as the scarce resource is redistributed between periods.

On one side, the circumstance that the hydraulic generator strategically allocates scarce water between periods is an expression of the fact that it is able to exert market power not only statically (in every single period) but also dynamically (over time). This raises obvious efficiency concerns.

On the other side, depending on the specific market and technological characteristics, strategic intertemporal transfer of water can either alleviate or exacerbate the environmental problem by affecting the amount of power firm T generates. In case a pollution reduction is triggered, the exercise of dynamic market power helps contain the environmental externality. On the opposite, when an increase in thermal emissions is induced, the efficiency loss that comes along with the intertemporal strategic behaviour is coupled with the welfare loss that is caused by the environmental externality. This suggests that socially desirable institutional setting is one where the interplay between these two evils is internalized in the best interest of the collectivity.

5 Pollution Taxation and Price Cap Regulation

In this Section, we explore an institutional framework in which policy interventions are in place, namely environmental and economic regulation. We restrict attention to some of the instruments that are most widely used (or advocated) in real-world electricity generation industries, *i.e.* environmental taxation and price cap regulation. With reference to those instruments, we first investigate the performance they yield and then take a normative approach in order to access how they should be designed.

In the standard formulation, price cap writes as

$$p_t \leq P, \ t = 1, 2,$$

meaning that, in each period, the energy charge cannot exceed a threshold P that is fixed by the economic Regulator. In turn, constant environmental taxation typically consists in imposing a tax liability on the amount of released emissions, according to a rate here denoted τ.

The Price Cap is Binding in Either Period. When the cap is binding in either period, firm H's profits specify as

$$\pi^H = PS - F^H.$$

The same price is attached to each unit of hydraulic power, whatever the period in which water is used. For this reason, firm H has no incentive to strategically allocate water over time. Thus any $q_t^H \in [0, S]$ is a period$-t$ candidate output. It follows that, under uniform price cap, firm H can be expected to make efficient choices, given the rival production. Also remark that a binding cap that is fixed over time induces the hydraulic operator to exhaust the whole stock of water even if the firm is not legally compelled to do so. Indeed, using less resource than available in nature would reduce the firm's net benefits[7].

Let us next turn to the thermal operator. Under binding price cap, firm T chooses output so as to equal the relevant marginal cost, including the marginal tax liability, to the highest attainable price

$$P = \frac{\partial c}{\partial q_t^T} + \tau \frac{\partial e}{\partial q_t^T}.$$

Given the cap, the tax rate that yields the maximum level of social welfare is found to be

$$\tau = \sum_t \frac{\partial D}{\partial e} \frac{\partial e}{\partial q_t^T} \frac{dq_t^T}{d\tau} \Bigg/ \sum_t \frac{\partial e}{\partial q_t^T} \frac{dq_t^T}{d\tau}. \tag{11}$$

The expression above says that the tax rate is obtained as an *average* of the marginal damage of pollution over the two time intervals.

As long as cost and emission functions are the same over time, firm T has an incentive to generate the same quantity of energy in either period[8]. Importantly, while this is consistent with the socially optimal rule in (2), it does not necessarily involve implementation of the first-best output profile, even if P can be set at the efficient level. First best does arise whenever the damage function is linear in polluting emissions. In this case, the optimal tax

[7]If the cap binds in one period only, then the hydraulic generator has an incentive to use all the water in that period, which yields maximum attainable profits. To see this formally, suppose first that the cap binds in period 1. Then firm H chooses q_1^H such that $P - p_2 = q_2^H \left(\partial p_2 / \partial Q_2\right) \left(dQ_2 / dq_2^H\right)$, which requires $q_2^H = 0$ and so $q_1^H = S$. Indeed, with $q_2^H > 0$, it would be $P < p_2$, a contradiction. Suppose next that the cap binds in period 2. Then firm H selects q_1^H so that $P - p_1 = q_1^H \left(\partial p_1 / \partial Q_1\right)$, which calls for $q_1^H = 0$ and so $q_2^H = S$. Indeed, with $q_1^H > 0$, it would be $P < p_1$, again a contradiction.

[8]Consider instead the case where the cap binds in period t but not in period $z \neq t$. One can show that, as long as the emission function is linear in quantity, in period t firm T produces more and so environmental quality is worse than in period z.

rate equals the marginal damage, which is constant, and thus output is not distorted.

The finding above suggests that distortions substantially follow from the rigidity of the tax instrument. To restore efficiency the environmental Regulator should (be allowed to) adopt more *flexible* policies, *i.e.* the space of instruments at Her disposal should be enlarged. To see this, suppose that the tax can be modulated on a per-period basis. Then the optimal rate is pinned down as $\tau_t = \partial D\left(e\left(q_t^T\right)\right)/\partial e$, $t = 1, 2$. That is, in each time period, the rate equals the marginal environmental damage evaluated at the most efficient thermal output that is achievable under price cap regulation. First best thus arises, provided that P can be set optimally. This can be hardly done though, without driving (at least) one of the firms out of the industry.

When operators' financial viability prevents first-best implementation, intertemporal price uniformity is unlikely efficient. This points to the opportunity of exploring flexibility in price regulation as well. In the following Section, we illustrate the potential benefits from coupling optimal per-period taxation with optimal intertemporal price capping.

5.1 Intertemporal Flexibility in Price Regulation

Suppose that the economic Regulator can modulate the price cap so as to ensure per-period flexibility. One way to do so is to impose the constraint

$$\sum_t \alpha_t p_t \leq P.$$

Within the latter, a weight $\alpha_t > 0$ is exogenously attached to each price p_t, so that the weighed sum of the energy prices is not allowed to exceed a threshold P[9]. Remark that this is consistent with the first-best rule as the possibility that prices equal over time is not ruled out. On the other hand, by allowing prices to diverge, the cap is suitable to match concerns that are likely to arise in second-best environments. In the latter, such aspects as generators' budgetary requirements (*i.e.*, the opportunity that firms' participation be secured) are accounted for.

Let γ_τ^T and γ^H the Lagrange multiplier associated with the price constraint for the thermal and the hydraulic operator respectively. Under the maintained hypothesis that the environmental Regulator can apply a different tax rate τ_t in each period, the first-order condition for a maximum of the profit function with respect to output writes

$$p_t - \frac{\partial c}{\partial q_t^T} = \tau_t \frac{\partial e}{\partial q_t^T} - \left(q_t^T - \gamma_\tau^T \alpha_t\right) \frac{\partial p_t}{\partial Q_t}, \; t = 1, 2, \tag{12}$$

and

$$p_1 - p_2 = \left(q_2^H - \gamma^H \alpha_2\right) \frac{\partial p_2}{\partial Q_2} \frac{dQ_2}{dq_2^H} - \left(q_1^H - \gamma^H \alpha_1\right) \frac{\partial p_1}{\partial Q_1} \tag{13}$$

for firm T and firm H respectively. Condition (12) and (13) fully characterize the benchmark allocation to be decentralized through the regulatory mechanism, given the tax policy.

In the framework under scrutiny, the optimal tax rate is given by

$$\tau_t = \frac{\partial D}{\partial e} + \left(q_t^T - \gamma_\tau^T \alpha_t\right) \frac{\partial p_t / \partial Q_t}{\partial e / \partial q_t^T}, \; t = 1, 2. \tag{14}$$

[9] With such weights, a unit increase in p_t tightens the constraint by an amount equal to α_t, $t = 1, 2$.

The expression in (14) is not an explicit one because τ_t is present in the right-hand side as well. Nevertheless, we can interpret (14) in a clear way, identifying three different terms that compose the welfare-maximizing tax rate.

First of all, the rate includes the marginal damage of polluting emissions in period t $(\partial D/\partial e)$. All else equal, it is larger the larger the external cost that is induced by the last unit of emissions released by the thermal process.

Secondly, the rate incorporates the welfare loss associated with the reduction in the firm's output that follows from introducing emission taxation $\left(q_t^T\left(\partial p_t/\partial Q_t\right)/\left(\partial e/\partial q_t^T\right)\right)$. This reflects the imperfectly competitive structure of the market in which the polluting agent operates (compare Barnett [4]). *Ceteris paribus*, τ_t gets lower as the market power effect becomes more important, in which case energy generation is to be encouraged to force price down.

Thirdly, the rate embodies the portion of the welfare cost aforementioned that is recovered as the intertemporal price cap limits market power $\left(\gamma_\tau^T\alpha_t\left(\partial p_t/\partial Q_t\right)/\left(\partial e/\partial q_t^T\right)\right)$. All else equal, τ_t is higher the more severe the discipline that is imposed by the economic Regulator. The reason is that, as energy price decreases, thermal output increases and environmental problems exacerbate. Hence, it becomes necessary to tighten environmental discipline.

To learn more about the relationship between the policy instruments at play, we more deeply explore how per-period taxation varies as per-period weighing is changed. For this purpose, we totally differentiate (14) with respect to α_t, which yields

$$\begin{aligned}\frac{d\tau_t}{d\alpha_t} &= \frac{dq_t^T}{d\alpha_t}\left\{\frac{\partial^2 D}{\partial e^2}\frac{\partial e}{\partial q_t^T}+\frac{\partial p_t/\partial Q_t}{\partial e/\partial q_t^T}\right. \\ &\quad \left. +\left(q_t^T-\gamma_\tau^T\right)\left[\frac{\partial^2 p_t/\partial Q_t^2}{\partial e/\partial q_t^T}-\frac{\left(\partial p_t/\partial Q_t\right)\left(\partial^2 e/\partial\left(q_t^T\right)^2\right)}{\left(\partial e/\partial q_t^T\right)^2}\right]\right\} \\ &\quad -\gamma_\tau^T\frac{\partial p_t/\partial Q_t}{\partial e/\partial q_t^T}+\left(q_t^T-\gamma_\tau^T\right)\frac{dq_t^H}{d\alpha_t}\frac{\partial^2 p_t/\partial Q_t^2}{\partial e/\partial q_t^T},\end{aligned}\tag{15}$$

where one also has

$$\frac{dq_t^T}{d\alpha_t}=\frac{\frac{d\tau_t}{d\alpha_t}\frac{\partial e}{\partial q_t^T}-\left(\frac{dq_t^H}{d\alpha_t}-\gamma_\tau^T\right)\frac{\partial p_t}{\partial Q_t}-\left(q_t^T-\gamma_\tau^T\alpha_t\right)\frac{\partial^2 p_t}{\partial Q_t^2}\frac{dq_t^H}{d\alpha_t}}{2\frac{\partial p_t}{\partial Q_t}-\frac{\partial^2 c}{\partial\left(q_t^T\right)^2}-\tau_t\frac{\partial^2 e}{\partial\left(q_t^T\right)^2}-\left(q_t^T-\gamma_\tau^T\alpha_t\right)\frac{\partial^2 p_t}{\partial Q_t^2}}$$

from the first-order condition for a maximum of π_τ^T with respect to q_t^T under intertemporal price regulation. The sign of $d\tau_t/d\alpha_t$ is hardly assessed for the general expression in (15). To get a clue, we thus focus attention on the case where demand, damage and emission functions take a linear form $\left(\partial^2 p_t/\partial Q_t^2=\partial^2 D/\partial e^2=\partial^2 e/\partial\left(q_t^T\right)^2=0\right)$. Within this context, (15) collapses onto

$$\frac{d\tau_t}{d\alpha_t}=\left(\frac{-\partial p_t/\partial Q_t}{\partial e/\partial q_t^T}\right)\left[\gamma_\tau^T+\frac{dq_t^H}{d\alpha_t}\Big/\left(1+\frac{\partial^2 c/\partial\left(q_t^T\right)^2}{-\partial p_t/\partial Q_t}\right)\right].$$

Suppose that hydraulic output raises with the price weight $\left(dq_t^H/d\alpha_t>0\right)$. Then the expression above says that the tax rate unambiguously increases with α_t[10]. The reason is that, with α_t large, firm T is induced to expand production, hence to pollute more, which in turn calls for more severe taxation. All else equal, the positive impact of α_t on τ_t is bigger the more stringent the price constraint for the thermal operator $\left(\gamma_\tau^T\right)$. It is also larger the more reactive the rival production to an increment in α_t. Intuitively, polluting activities can be more vigorously discouraged when cleaner processes can be called upon to replace them. This clearly points to the importance that (neat) products be available as substitutes for dirty ones, especially in highly captive markets.

Lastly introduce the following definitions for $i = H, T$ and $t = 1, 2$:

$$
\begin{aligned}
s_t^i &\equiv \frac{q_t^i}{Q_t} = \text{ Firm } i\text{'s market share in period } t \\
\eta_t &\equiv \frac{-\partial Q_t}{\partial p_t}\frac{p_t}{Q_t} = \text{ (Absolute value of) Price elasticity of market demand in period } t \\
r_t^T &\equiv \frac{p_t q_t^T}{e\left(q_t^T\right)} = \text{ Thermal revenues per emission unit in period } t \\
\epsilon_t^T &\equiv \frac{\partial e}{\partial q_t^T}\frac{q_t^T}{e\left(q_t^T\right)} = \text{ Elasticity of emissions to thermal quantity in period } t
\end{aligned}
$$

Using the definitions listed above, (14) rewrites

$$\tau_t = \frac{\partial D}{\partial e} - \frac{s_t^T}{\eta_t^T}\frac{r_t^T}{\epsilon_t^T} - \gamma_\tau^T \alpha_t \left(\frac{-\partial p_t/\partial Q_t}{\partial e/\partial q_t^T}\right),\ t = 1, 2,$$

suggesting that, all else equal, τ_t decreases with thermal market share and thermal revenues per emission unit. Instead, it gets larger the higher the elasticity of emissions to thermal quantity and the higher the price elasticity of market demand. Particularly this last finding deserves a few more words.

Firstly, it reveals that the environmental Regulator has to refer to the price elasticity of the whole market demand to properly calibrate the tax liability. It follows that She needs to form a global view of the concerned sector, albeit the tax is solely targeted to the polluting agent[11].

Secondly, the finding above says that taxation should not be too severe when the price elasticity of demand is small, which is typically the case in electricity markets (see, for instance, Bernstein and Griffin [6]). More specifically, it involves that, *ceteris paribus*, taxation is milder at peak time, when demand is relatively less price elastic, and tighter at off peak time. This result, which might appear counter-intuitive at a first glance, is explained on

[10] The positive sign of $d\tau_t/d\alpha_t$ follows as it is $(-\partial p_t/\partial Q_t)/\left(\partial e/\partial q_t^T\right) > 0$ together with $1/\left(1 + \frac{\partial^2 c/\partial\left(q_t^T\right)^2}{-\partial p_t/\partial Q_t}\right) > 0$, which is true with a convex cost function. With $\partial^2 e/\partial\left(q_t^T\right)^2 \neq 0$, things become more complex. One has $d\tau_t/d\alpha_t$ unequivocally larger than zero whenever it is $\frac{\frac{dq_t^H}{d\alpha_t}\left(\frac{\partial p_t/\partial Q_t}{\partial e/\partial q_t^T}\right)\left[\frac{\partial e}{\partial q_t^T} - \left(q_t^T - \gamma_\tau^T\right)\frac{\partial^2 e}{\partial\left(q_t^T\right)^2}\right]}{\frac{\partial p_t}{\partial Q_t} - \frac{\partial^2 c}{\partial\left(q_t^T\right)^2} - \tau_t\frac{\partial^2 e}{\partial\left(q_t^T\right)^2} + \left(\frac{\partial p_t/\partial Q_t}{\partial e/\partial q_t^T}\right)\left(q_t^T - \gamma_\tau^T\right)\frac{\partial^2 e}{\partial\left(q_t^T\right)^2}} > 0$, which might not be the case with a convex emission function.

[11] We use the pronoun She for the environmental Regulator and the pronoun He for the economic Regulator.

the basis of the market power argument aforementioned. When demand is rigid, quantity reduction stemming from tax increase triggers important price raise. To contain this effect taxation is softened.

No Budget Constraint is Binding. We begin the analysis of the intertemporal price cap by considering the simple case where the budget constraint slacks for either generator. The efficient period-1 hydraulic output meets the condition

$$p_1 - p_2 = \left(p_2 - \frac{\partial c}{\partial q_2^T} - \frac{\partial D}{\partial e} \frac{\partial e}{\partial q_2^T} \right) \frac{dQ_2^T}{dq_2^H}. \tag{16}$$

Observe that (16) has been obtained by anticipating firm T's reaction in period 2, just as firm H does while making production decisions in period 1. Under optimal taxation, firm T efficiently chooses output q_t^T, given the hydraulic production, and (16) reduces to

$$p_1 = p_2.$$

Hence the economic Regulator needs to identify values of γ^H, α_1 and α_2 that satisfy the condition

$$\left(q_2^H - \gamma^H \alpha_2\right) \frac{\partial p_2}{\partial Q_2} \frac{dQ_2}{dq_2^H} = \left(q_1^H - \gamma^H \alpha_1\right) \frac{\partial p_1}{\partial Q_1}.$$

One option is given by

$$\gamma^H \alpha_t = q_t^H, \ t = 1, 2. \tag{17}$$

This suggests that the economic Regulator can normalize γ^H to 1 and then adjust α_t to the optimal level of the hydraulic quantity in the scenario under scrutiny. This requires that the Regulator be able to determine the portion of water to be efficiently allocated to each period.

Let us try and understand the result above. As previously mentioned, the emission tax accounts for environmental externalities as well as for exercise of market power under price cap regulation. That is, the tax reflects *both* sources of thermal distortions and internalizes the impact of the price constraint on the latter. It follows that sole relevant distortions, on which the cap is to be explicitly calibrated, are those related to the hydraulic activity. It thus suffices to regulate prices as if firm H were a *multiproduct monopolist*, each period being approached as a different market. This explains why per-period prices can be sensibly weighed with the optimal per-period hydraulic quantities.

Contrasting this finding with the results about taxation, an interesting difference emerges between the two regulatory processes under scrutiny. Taxation of the sole polluting activity calls for the environmental authority to keep a comprehensive view of the overall industry. This holds true whether or not taxation is coupled with fare regulation[12]. On the opposite, to regulate the price of the whole sector, the economic agency can exclusively focus on the hydraulic process, so as to directly solve the intertemporal issue associated with water allocation. However, as we shall see in a while, this is only valid as long as tax and cap are simultaneously in place and firm T's budget constraint slacks.

[12] Check Section 6.

Price Regulation without Taxation when No Budget Constraint Binds. Further inspection of (14) reveals that taxation persists as long as the environmental benefit is larger than the net welfare cost associated with thermal output reduction. When the reverse is true, the thermal activity is, instead, untaxed. The simple scenario where no constraint binds is a suitable one to explore what happens in the absence of taxation.

When the intertemporal price cap is the sole available instrument, it is to be structured so as to meet the following set of conditions

$$\frac{\partial D}{\partial e}\frac{\partial e}{\partial q_t^T} = \left(\gamma^T \alpha_t - q_t^T\right)\frac{\partial p_t}{\partial Q_t}, \qquad t = 1, 2^{13} \tag{18a}$$

$$\begin{aligned}\left(p_2 - \frac{\partial c}{\partial q_2^T} - \frac{\partial D}{\partial e}\frac{\partial e}{\partial q_2^T}\right)\frac{dQ_2^T}{dq_2^H} &= \left(\gamma^H \alpha_1 - q_1^H\right)\frac{\partial p_1}{\partial Q_1} \\ &\quad - \left(\gamma^H \alpha_2 - q_2^H\right)\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H},\end{aligned} \tag{18b}$$

where γ^T is the Lagrange multiplier associated with the cap when firm T is not taxed. The most immediate way to satisfy these conditions is to normalize γ^T to 1 and fix price weights

$$\begin{aligned}\alpha_t &= q_t^T + \frac{\partial D}{\partial e}\frac{de}{dp_t} \\ &< q_t^T, \qquad\qquad t = 1, 2.\end{aligned} \tag{19}$$

According to (19), absent taxation, per-period weights are no longer modulated on the hydraulic activity. They are rather targeted to the thermal activity. In particular, α_t is given by the optimal period$-t$ thermal quantity as *diminished* by the external cost (evaluated at the optimal quantity) associated with the output increase that is triggered as the price falls under the pressure of the cap. In other words, the weight is downward distorted from the optimal thermal quantity so that the external effect is internalized. Instead, α_t would exactly equal the efficient value of q_t^T if the economic Regulator would not be concerned with environmental quality and solely care about control of market power[14].

Once weights are set as in (19), it remains to adjust the overall cap P so that the shadow cost of the price constraint for firm H $\left(\gamma^H\right)$ satisfies (18b), with γ^T normalized to 1. Inspection of condition (18b) reveals how this adjustment is to be performed. In particular, it is to reflect the importance of the advantage from a reduction in intertemporal (hydraulic) market power relatively to the benefit from a reduction in static (thermal) market power.

[13] Condition (18a) precisely reflects that $\tau_t = 0$.

[14] One should however be aware that, in such a case, firm T would incur a loss with α_t set at q_t^T. Indeed, economic regulation would force the firm to price at marginal cost, so that fixed costs would remain uncovered. But then the thermal operator's budget constraint would bind.

Firm T's Budget Constraint is Binding. Suppose next that firm T's budget constraint is binding. Letting λ_τ^T the associated shadow cost, (16) rewrites

$$\begin{aligned} p_1 - p_2 &= \left(p_2 - \frac{\partial c}{\partial q_2^T} - \frac{\partial D}{\partial e}\frac{\partial e}{\partial q_2^T}\right)\frac{dQ_2^T}{dq_2^H} \\ &\quad + \lambda_\tau^T\left(p_2 - \frac{\partial c}{\partial q_2^T} - \tau_t \frac{\partial e}{\partial q_2^T}\right)\frac{dQ_2^T}{dq_2^H} \\ &\quad + \lambda_\tau^T\left(q_2^T \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} - q_1^T \frac{\partial p_1}{\partial Q_1}\right). \end{aligned} \tag{20}$$

Using (14) and then relying upon (13), one finds that the economic Regulator should choose γ^H and α_t, $t = 1, 2$, so as to satisfy the condition

$$\lambda_\tau^T\left(q_2^T \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} - q_1^T \frac{\partial p_1}{\partial Q_1}\right) = \left(q_2^H - \gamma^H\alpha_2\right)\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} - \left(q_1^H - \gamma^H\alpha_1\right)\frac{\partial p_1}{\partial Q_1}.$$

This quickly obtains by setting

$$\begin{aligned} \gamma^H\alpha_t &= q_t^H - \tilde{q}_t^T \\ &< q_t^H, \qquad t = 1, 2, \end{aligned} \tag{21}$$

where we have defined $\tilde{q}_t^T \equiv \lambda_\tau^T q_t^T$. For instance, normalizing γ^H to 1, at period t, the appropriate price weight equals the (constrained) efficient hydraulic quantity as *diminished* by an adjusted measure of the (constrained) efficient thermal output, adjustment rate being the shadow cost of firm T's budget constraint.

The result above compares interestingly with the one in (17). Recall that, as long as the economic Regulator is not concerned with firms' financial viability and taxation is in place, price weights can be calibrated on the efficient levels of hydraulic output only. The possibility of focusing on firm H, rather than looking at the whole industry, is lost as soon as the regulatory body has to ensure firm T's participation in the regulated sector. In this case, proper weight is a combination of the two (constrained) efficient quantities in each period. That is, α_t is downward distorted from the (constrained) optimal period$-t$ quota of water so much as to reflect firm T's budgetary requirements and production in the (constrained) efficient situation.

Firm H's Budget Constraint is Binding. We terminate by considering the case where the budget constraint binds for the hydraulic generator. Then, using (14), (16) becomes

$$p_1 - p_2 = \frac{\lambda^H}{1+\lambda^H}\left(q_2^H \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} - q_1^H \frac{\partial p_1}{\partial Q_1}\right),$$

where λ^H is the Lagrange multiplier associated with firm H's constraint. Therefore, the economic Regulator needs to select values for γ^H and α_t, $t = 1, 2$, so as to meet the condition

$$\frac{\lambda^H}{1+\lambda^H}\left(q_2^H \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} - q_1^H \frac{\partial p_1}{\partial Q_1}\right) = \left(q_2^H - \gamma^H\alpha_2\right)\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} - \left(q_1^H - \gamma^H\alpha_1\right)\frac{\partial p_1}{\partial Q_1}.$$

This obtains by setting

$$\alpha_t = q_t^H, \quad t = 1, 2, \tag{22a}$$

together with

$$\gamma^H = \frac{1}{1+\lambda^H}. \tag{22b}$$

(22a) suggests that, because no budgetary concern arises for firm T, the economic Regulator can again calibrate the price weight on the (constrained) efficient hydraulic output in each period. That is, He can still concentrate on water intertemporal allocation only. However, as financial viability is to be guaranteed to firm H, the cap cannot be made tighter than the operator's budget constraint allows for, when evaluated at the (constrained) efficient quantities. P is to be fixed so as to reflect (22b). Clearly, in the environment under scrutiny, the joint performance of cap and tax does not replicate the outcome that arises when both budget constraints slack, although analogous structure of the cap applies.

5.1.1 Brief Summary and Remarks

In the presence of per-period taxation, the intertemporal price cap is a good instrument to provide (direct) incentives to firm H. This is the case whether firm H's budget constraint is binding or not. However, the target allocation to be decentralized changes according to whether firm $H's$ financial viability is an issue or not.

When the price cap is coupled with the emission tax, it contributes to limit firm T's exercise of market power, but explicitly refers to firm T only when the latter's participation is to be ensured. Flexible taxation intervenes on thermal distortions trading-off countervailing effects, so as to strike proper *net* incentives. Indeed, the tax is meant to correct for the environmental damage the activity induces. Yet it needs to account for the welfare cost that is associated with the reduction in firm T's output, net of the benefit induced by price regulation.

Absent taxation, instead, it is easier to explicitly set the cap so as to provide (direct) incentives to the thermal operator. As long as the economic Regulator is concerned with environmental problems, this calls for a similar compromise to the one otherwise reflected in the tax rate. That is, the Regulator gives up some discipline on market power so as to exert control on polluting emissions.

Lastly remark that, though modulated on a per-period basis, a (binding) cap of the form $p_t \leq P_t$, $t = 1, 2$, would perform differently from the intertemporal price constraint so far examined. It would lead the hydraulic operator to use the whole stock of resource during the period in which the ceiling is higher and to give up production in the other period. As compelled to such a cap, the agent no longer benefits from manipulating its quantity choice to anticipate the competitor's decisions in period 2. Hence, the incentives of the hydraulic producer to exert intertemporal market power are destroyed by the very fact that the cap is set to create asymmetry over time. Yet, this instrument is suitable to achieve efficiency under quite specific conditions only. More precisely, the sole efficient allocation it can decentralize is a bang-bang one, *i.e.* an allocation such that water is entirely used in the period with higher allowable fare[15].

[15] A corner allocation of the kind $q_t^H = S$, $q_z^H = 0$, $q_t^T = 0$ and $q_z^T > 0$ meets the social optimality criterion under the condition that $\mu < \left(\partial c\,(0) / \partial q_t^T\right) + \left(\partial D / \partial e\right) \left(\partial e\,(0) / \partial q_t^T\right)$. This says that the entire stock of water

6 Pollution Taxation and Contracts for Water Differences

In progress. Preliminary and incomplete

Our investigation about flexible price regulation has lead to the prediction that, as long as the per-period emission tax is imposed on the polluting activity, the economic Regulator can (essentially) focus on the issue of water (mis)allocation between periods displaying different market conditions, even if the price cap applies industry-wide. This basically requires that the Regulator be able to identify how much of the scarce resource should be used in each period, given the available stock, so as to reach the most efficient outcome that is feasible in the concerned setting.

Having this result in mind, a very natural alternative to price cap one could think of is the implementation of per-period water quotas through *contracts for water differences* $(CWDs)$. We hereafter illustrate how these instruments should work.

To begin with, the economic Regulator, who now solely disciplines the hydraulic generator, identifies the resource quota $\varphi_t^H \in [0, S]$ He would like firm H to consume in period t. Reasonably enough, φ_t^H equals the period$-t$ socially efficient amount of hydraulic electricity. Once φ_t^H is fixed, the water quota to be used in period $z \neq t$ is implicitly determined as $\varphi_z^H = S- \varphi_t^H$, which is the period$-z$ socially efficient amount of hydraulic power. To make sure that the hydraulic operator has an incentive to precisely choose $q_t^H = \varphi_t^H$ and so $q_z^H = \varphi_z^H$, the Regulator obliges the firm to contract over the quantity wedge $\Delta^H \equiv \left|q_t^H - \varphi_t^H\right| = \left|q_z^H - \varphi_z^H\right|$, which is non-zero whenever the regulatory target is mismatched.

Technically speaking, when $CFDs$ are relied upon, firm H's (net) profits write

$$\pi_\varphi^H = \sum_t q_t^H p_t - F^H - \nu \Delta^H,$$

where Δ^H measures the amount of water that is inefficiently allocated through intertemporal transfer. Having $q_t^H > \varphi_t^H$ (respectively, $q_t^H < \varphi_t^H$) means that the hydraulic operator uses too much (respectively, too little) resource in period t, as compared to the regulatory objective[16]. It is thus compelled to purchase a right for excess (respectively, insufficient) production in period t at the (regulated) unit price ν. Clearly, this obligation disappears if the operator perfectly adheres to the regulatory target $\left(\Delta^H = 0\right)$.

$CWDs$ are close to the so-called *contracts for (price) differences* frequently adopted in wholesale electricity markets as regulatory instruments for reducing generators' incentives to exert market power. This is the case, for instance, in the England and Wales power pool. In those contracts, which are put in place between generators and retailers, a countervailing effect is created by the following clause. If the wholesale price index in any time period proves higher than the regulated strike price, then the generator is obliged to refund the difference between strike and actual price for that period to the retailer. Something similar occurs in the $CWDs$ previously presented. That is, if the amount of resource that is used in period t diverges from the socially efficient one, then the hydraulic generator is compelled to repay for the associated social loss. One can imagine that the payment is made by the producer to

should be exhausted at period t and thermal production solely occur at period $z \neq t$, as long as the (dual) marginal cost of water is smaller than the social marginal cost of thermal power with $q_t^T = 0$.

[16] When one has $q_t^H > \varphi_t^H$, it is clearly also $q_z^H < \varphi_z^H$ and vice versa.

the Regulator and then transferred to the collectivity.

Observe that in a contract for (price) difference the incentive stems from the price wedge that is attached to the quantity to be contracted for. Hence, the main difficulty for the Regulator is to define the average cost level, at which the strike price is to be fixed. By contrast, in the $CWDs$ here analyzed, the incentive for the hydraulic producer hinges on the wedge between actual production and quota, given the unit penalty. Thus a major problem for the Regulator is to identify the liability $\nu > 0$ that solicits efficient performance by destroying the operator's benefits from resource misusage[17].

To see how ν is to be chosen, one should first recall that, under the scrutinized regime, firm T is not subject to economic regulation and is only compelled to pay the pollution tax. Thus the optimal per-period tax rate equals

$$\tau_t = \frac{\partial D}{\partial e} - \frac{s_t^T}{\eta_t}\frac{r_t^T}{\epsilon_t^T},\ t = 1, 2,$$

i.e. it is now given by the marginal environmental damage of emissions net of the (full) welfare loss that stems from reducing firm T's output. Therefore, as long as optimal per-period taxation is in place and firm H's budget constraint slacks, the best the economic Regulator can do is to set (see Appendix for details)

$$\nu = \left| \Gamma_2 p_2 \frac{s_2^H}{\eta_2} - p_1 \frac{s_1^H}{\eta_1} \right|, \tag{23}$$

where

$$\begin{aligned} \Gamma_2 &\equiv \frac{Q_2\eta_2}{Q_2\eta_2 + q_2^T\theta_2^T} \\ \theta_2^T &\equiv \frac{dQ_2^T}{dp_2}\frac{p_2}{q_2^T} = \text{Price elasticity of thermal supply at } t = 2. \end{aligned}$$

This allows to wash out the marginal revenues firm H obtains, in excess of unit price, by transferring water over time.

Let us inspect the finding above. The term $p_t s_t^H/\eta_t$ expresses firm H's revenue share $\left(p_t s_t^H\right)$ as deflated by the price elasticity of total market demand (η_t) in period $t = 1, 2$. On the other hand, $\Gamma_2 < 1$ is the ratio between price elasticity of market demand and price elasticity of market demand plus thermal supply in period 2, each such elasticity being weighed by the relevant period-2 quantity (*i.e.,* the quantity demanded on the market and the quantity supplied by firm T respectively). Γ_2 thus reflects the necessity of accounting for both the overall market conditions and the competitor's supply conditions in period 2. This necessity stems from the circumstance that, as already explained, intertemporal market

[17] When agreed upon for hedging purposes, contracts for difference also work the other way around, *i.e.* if the wholesale price index is *lower* than the strike price, the retailer refunds the difference between strike and actual price to the generator. In this case, contracts are said to be two-way. They are said to be one-way otherwise (Green [19]). In our $CWDs$ the two-way option is ruled out by taking the water wedge Δ^H in absolute value. The reason is to be found in that, by the very nature of the stock constrained intertemporal allocation problem, excess of water in period t corresponds to an equivalent shortage in period $z \neq t$ and vice versa.

power is exerted anticipating period-2 thermal actions. It as well drives the prediction that the unit liability ν should be set equal to the (absolute value of the) difference between a *portion* of the deflated revenue share in period 2 and the *whole* deflated revenue share in period 1[18].

Observe that, with $\lambda^H > 0$, the obligation rate is to be decreased as much as it is necessary to let the firm break even. This calls for $\nu^\lambda = \nu / \left(1 + \lambda^H\right)$, which means that, *ceteris paribus*, the rate is smaller the tighter the budget constraint of the regulated agent. More precisely, the hydraulic agent can be required to contribute only a portion $\left[1/\left(1+\lambda^H\right)\right] < 1$ of the welfare cost associated with each unit of inefficiently allocated water[19].

It is noteworthy that, in the environment under scrutiny, each activity goes under a specific instrument and single policies do not follow as a compromise over industry averages. In particular, environmental regulation only concerns the thermal generator, whereas economic regulation is only targeted to the operator that controls the scarce resource and can exert intertemporal market power. Yet the results of our analysis reveal that either Regulator should maintain a global perspective over the concerned sector. This conclusion, which has already been drawn with regard to the environmental Regulator, here extends to the economic Regulator as well. We have just highlighted that, for properly setting the unit penalty on water misallocation, the latter should be able to estimate such figures as output and demand elasticity at the industry level. He should also be able to assess the efficient level of output and supply elasticity for the unregulated firm. This deduction is to be contrasted with the one previously made about industry-wide price regulation. Recall that, as long as flexible taxation is in place and control is exerted on market prices through an intertemporal cap, the economic Regulator can basically concentrate on the hydraulic activity, which He can address similarly to multiproduct monopoly. Despite the cap is imposed on both firms, the Regulator needs to identify the (constrained) optimal thermal output only when the polluter's participation in the regulated sector is to be ensured. Interestingly enough, this does not suffice when $CWDs$ are introduced to solicit efficient behaviour by firm H, independently of the competitor's financial conditions. From this standpoint, regulating hydraulic production appears more requiring, hence less at the authority's hand, than regulating market prices, even in an industry where quantities are the relevant choice variables and water quotas seem to be a very natural control instrument.

7 Energy Taxation without Economic Regulation

In progress. Preliminary and incomplete

As a final step, we hereafter explore a scenario in which the whole generated power is taxed but there is no economic regulation. More precisely, whether electricity is produced by the thermal or the hydraulic plant, each output unit yields a tax liability equal to τ^E to the generating firm. We thus focus on an environment where the same policy instrument is

[18] In particular, it should be $\nu = \left(\Gamma_2 p_2 s_2^H / \eta_2^H - p_1 s_1^H / \eta_1^H\right)$ for $q_1^H > \varphi_1^H$ (or, equivalently, $q_2^H < \varphi_2^H$) and $\nu = \left(p_1 s_1^H / \eta_1^H - \Gamma_2 p_2 s_2^H / \eta_2^H\right)$ for $q_1^H < \varphi_1^H$ (or, equivalently, $q_2^H > \varphi_2^H$). In the former case, absent the obligation to pay, one has $p_1 < p_2$, which explains why firm H has an incentive to use too little water in period 1. In the latter case, the converse is true.

[19] Remark that, while the *expression* for ν in ν^λ is the same as in (23), the *value* it takes is not.

targeted to the two asymmetric operators.

The tax we look at can be seen as a royalty on electricity production. A tax of similar kind is currently applied in Ontario, where the Government is entitled to levy charges on electricity sales (Gillen and Wen [17]). The specificity of the Ontario regime is that the tax is, in fact, a water charge on hydraulic producers because electricity is (almost) entirely generated by hydraulic plants. We here consider the case where this type of charge is implemented industry-wide. Within this framework, we investigate how energy production would react to the introduction of the latter in order to understand what could be achieved by resorting to this policy instrument.

Under energy taxation, firm H's net profits write as

$$\begin{aligned}\pi^H_{\tau^E} &= \sum_t \left(p_t - \tau^E\right) q^H_t - F^H \\ &= \sum_t p_t q^H_t - \tau^E S - F^H,\end{aligned}$$

so that the first-order condition for a maximum of $\pi^H_{\tau^E}$ with respect to q^H_1 is still given by (5). This says that, when energy is taxed, the decision of the hydraulic generator about water intertemporal allocation is not marginally distorted, as compared to the case where (hydraulic) electricity is untaxed. This is so because, as long as the stock of resource is entirely used on a yearly basis, taxation has no marginal impact on hydraulic production. Since the energy tax induces no contraction in hydraulic power, it works as a lump-sum tax *vis-à-vis* firm H[20]. Nevertheless, taxation does allow to extract (some of) the rent firm H would otherwise obtain, profits being now diminished by the total tax liability $\left(\tau^E S\right)$. This illustrates the argument Gillen and Wen [17] put forward to support the adoption of royalty fees on hydro-power as a rent appropriation device[21].

In turn, in the presence of a tax on energy, firm T's net profits are equal to

$$\pi^T_{\tau^E} = \sum_t p_t q^T_t - \sum_t c\left(q^T_t\right) - \tau^E Q^T - F^T,$$

so that the first-order condition for a maximum of $\pi^T_{\tau^E}$ with respect to q^T_t, $t = 1, 2$, writes

$$p_t + q^T_t \frac{\partial p_t}{\partial Q_t} = \frac{\partial c}{\partial q^T_t} + \tau^E. \tag{24}$$

Differentiating both sides of (24) with respect to τ^E and rearranging terms we obtain (see Appendix for details)

$$\frac{dq^T_t}{d\tau_E} = \frac{\left(\frac{\partial p_t}{\partial Q_t} + q^T_t \frac{\partial^2 p_t}{\partial Q^2_t}\right)\frac{dQ_t}{d\tau_E} - 1}{\frac{\partial^2 c}{\partial\left(q^T_t\right)^2} - \frac{\partial p_t}{\partial Q_t}}, \tag{25a}$$

[20] Instead, a tax rate that were to differ between periods $\left(\tau^E_t\right)$ would marginally affect hydraulic choices. Indeed, it would work as a (constant) period$-t$ marginal cost, hence for firm H the profit-maximizing rule would become $p_1 + q^H_1\left(\partial p_1/\partial Q_1\right) - \tau^E_1 = p_2 + q^H_2\left(\partial p_2/\partial Q_2\right)\left(dQ_2/dq^H_2\right) - \tau^E_2$.

[21] Analyzing the situation in Ontario, Gillen and Wen [17] emphasize that another reason why royalties from hydraulic production are beneficial is that they remove the disincentive to invest in new generation. Albeit investment issues are not part of our investigation, we do acknowledge the relevance of this aspect.

which expresses how per-period thermal output is affected by the introduction of energy taxation, depending upon the variation that occurs in total output in period t $(dQ_t/d\tau_E)$. On the other hand, differentiating both sides of (5) with respect to τ^E returns

$$\frac{dq_1^H}{d\tau_E} = \frac{-\left(\frac{\partial p_1}{\partial Q_1} + q_1^H \frac{\partial^2 p_1}{\partial Q_1^2}\right)\frac{dQ_1}{d\tau_E} + \left(\frac{\partial p_2}{\partial Q_2} + q_2^H \frac{\partial^2 p_2}{\partial Q_2^2}\frac{dQ_2}{dq_2^H}\right)\frac{dQ_2}{d\tau_E} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{dq_2^H d\tau_E}}{\frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{d\left(q_2^H\right)^2}}, \tag{25b}$$

which strikes the relationship between the change in period-1 hydraulic quantity and the variations that occur in total output in both periods $(dQ_1/d\tau_E$ and $dQ_2/d\tau_E)$. Summing (25b) and (25a) for $t = 1$, we can thus assess the impact that having a positive tax rate triggers in total industry quantity in period 1, namely

$$\begin{aligned}
\frac{dQ_1}{d\tau_E} &= \frac{dq_1^H}{d\tau_E} + \frac{dq_1^T}{d\tau_E} \qquad (26)\\
&= \frac{-\left(\frac{\partial p_1}{\partial Q_1} + q_1^H \frac{\partial^2 p_1}{\partial Q_1^2}\right)\frac{dQ_1}{d\tau_E} + \left(\frac{\partial p_2}{\partial Q_2} + q_2^H \frac{\partial^2 p_2}{\partial Q_2^2}\frac{dQ_2}{dq_2^H}\right)\frac{dQ_2}{d\tau_E} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{dq_2^H d\tau_E}}{\frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{d\left(q_2^H\right)^2}}\\
&\quad + \frac{1 - \left(\frac{\partial p_1}{\partial Q_1} + q_1^T \frac{\partial^2 p_1}{\partial Q_1^2}\right)\frac{dQ_1}{d\tau_E}}{\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial\left(q_1^T\right)^2}}.
\end{aligned}$$

This expression is a rather complex one. In order to further investigate, we need to focus on a framework that can be more easily handled. For this purpose, we make the following assumptions:

$$\begin{aligned}
\frac{\partial^2 p_t}{\partial Q_t^2} &= \frac{\partial^2 c}{\partial\left(q_t^T\right)^2} = 0, \quad t = 1, 2\\
\frac{d^2 Q_2^T}{d\left(q_2^H\right)^2} &= \frac{d^2 Q_2^T}{dq_2^H d\tau_E} = 0.
\end{aligned}$$

The first assumption means that both inverse demand and thermal cost function are linear in quantity in either period. The second assumption requires that firm T's reaction function be linear in the hydraulic quantity in period 2 and that its slope be constant in the tax rate. Under these restrictions, (26) reduces to

$$\frac{dQ_1}{d\tau_E} = \frac{\frac{\partial p_1}{\partial Q_1}\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{d\tau_E} + \frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}}{\frac{\partial p_1}{\partial Q_1}\left(3\frac{\partial p_1}{\partial Q_1} + 2\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}\right)}.$$

In turn, (25a) collapses onto

$$\frac{dq_t^T}{d\tau_E} = -\frac{dQ_t}{d\tau_E} + \frac{1}{\partial p_t/\partial Q_t}, \; t = 1, 2,$$

whereas (25b) becomes

$$\frac{dq_1^H}{d\tau_E} = \frac{-\frac{\partial p_1}{\partial Q_1}\frac{dQ_1}{d\tau_E} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{d\tau_E}}{\frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}},$$

so that we as well obtain

$$\frac{dq_2^H}{d\tau_E} = \frac{-\frac{\partial p_1}{\partial Q_1}\frac{dQ_1}{d\tau_E} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{d\tau_E}}{-\left(\frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}\right)}.$$

Investigating the expressions above allows to ultimately reach the results that are summarized in the following two Lemmas concerning the thermal activity (the Proof is relegated to the Appendix):

Lemma 1 *Under the assumptions previously made, electricity taxation triggers a reduction in industry output both on a per-period and on a yearly basis. It thus works as an instrument to curtail power overusage.*

Lemma 2 *Under the assumptions previously made, electricity taxation induces a decrease in thermal output in either period. Hence, it can be adopted as a tool for improving environmental quality both on a per-period and on a yearly basis.*

Furthermore, we can state the following Lemma with regard to the hydraulic activity:

Lemma 3 *Electricity taxation has no impact on the profit-maximizing water allocation rule and works as a lump-sum charge (or a royalty fee) vis-à-vis the hydraulic generator. Yet it affects water intertemporal distribution through the strategic interaction with the thermal activity, depending upon the relative market conditions in the two periods and the impact of the charge on per-period industry output.*

8 Conclusions

Preliminary and incomplete

How do environmental externalities and intertemporal exercise of market power interact in electricity generation industries where Cournot competitors adopt thermal and hydraulic processes respectively? In this paper, we have addressed this issue assessing the implications in terms of output profile, prices, environmental quality. We have then taken a policy-oriented approach and figured out how policy instruments that are widely used in real-world electricity sectors, but also alternative ones, should be designed for efficient outcomes to be achieved. More generally, we have shed light on policy relationships in oligopolies that are characterized by relevant dynamic aspects and constraints on fundamental inputs.

Our investigation has revealed that, when market conditions vary over time, there is a flexibility value to adjusting policy instruments on a per-period basis. Flexible (per-period) taxation can be made more vigorous as long as clean processes can be called upon as substitutes for dirty ones. Particularly, in our framework, this hinges on water availability. Moreover, taxation cannot be too severe in price rigid markets to prevent important market power effects. Hence, specifically in electricity sectors, it is relatively milder at peak than at off-peak times. In turn, thanks to the interaction with the tax, flexible price control allows the Regulator to focus on the untaxed process only, unless the participation of the taxed operator is to be ensured. In the industry under scrutiny, this means that prices can be regulated through an intertemporal cap calibrated on the hydraulic activity only. Interestingly enough, this is reminiscent of "traditional" price regulation of multiproduct monopoly, despite the mechanism suits a Cournot framework where it applies industry-wide.

In sharp contrast, regulatory bodies need to maintain a global view of the concerned sector whenever contracts for water differences, as targeted to the hydraulic generator only, are jointly adopted with pollution taxation. Our analysis has highlighted that, indeed, in scenarios of this kind, neither Regulator can exclusively focus on the target activity. In this perspective, regulating hydraulic production seems to be more burdensome than regulating market prices.

Lastly, our study has led to the prediction that, under specific conditions, charging a tax on overall electricity production at yearly constant rate allows to discourage power overusage and improve environmental quality. Such a tax fails to affect the water allocation rule because it works as a fixed royalty fee *vis-à-vis* the hydraulic generator. Nevertheless, it does extract (some) hydraulic rents.

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A Derivation of (6), (7) and (8)

Firm T's first-order condition with respect to q_1^T writes as

$$p_1 - \frac{\partial c}{\partial q_1^T} + q_1^T \frac{\partial p_1}{\partial Q_1} = 0.$$

Differentiate both sides with respect to q_1^H. This yields

$$\frac{\partial p_1}{\partial Q_1}\left(1 + \frac{dq_1^T}{dq_1^H}\right) - \frac{\partial^2 c}{\partial \left(q_1^T\right)^2}\frac{dq_1^T}{dq_1^H} + \frac{\partial p_1}{\partial Q_1}\frac{dq_1^T}{dq_1^H} + q_1^T \frac{\partial^2 p_1}{\partial Q_1^2}\left(1 + \frac{dq_1^T}{dq_1^H}\right) = 0$$

or, equivalently,

$$\frac{dq_1^T}{dq_1^H}\left(2\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial \left(q_1^T\right)^2} + q_1^T \frac{\partial^2 p_1}{\partial Q_1^2}\right) = -\left(\frac{\partial p_1}{\partial Q_1} + q_1^T \frac{\partial^2 p_1}{\partial Q_1^2}\right),$$

which in turn leads to (6).

Firm T's first-order condition with respect to q_2^T writes as

$$p_2 - \frac{\partial c}{\partial q_2^T} + q_2^T \frac{\partial p_2}{\partial Q_2} = 0.$$

Differentiate both sides with respect to q_1^H. This returns

$$-\frac{\partial p_2}{\partial Q_2}\left(1 + \frac{dQ_2^T}{dq_2^H}\right) + \frac{\partial^2 c}{\partial \left(q_2^T\right)^2}\frac{dQ_2^T}{dq_2^H} - \frac{\partial p_2}{\partial Q_2}\frac{dQ_2^T}{dq_2^H} - q_2^T \frac{\partial^2 p_2}{\partial Q_2^2}\left(1 + \frac{dQ_2^T}{dq_2^H}\right) = 0$$

or, equivalently,

$$-\frac{dQ_2^T}{dq_2^H}\left(2\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial \left(q_1^T\right)^2} + q_1^T \frac{\partial^2 p_1}{\partial Q_1^2}\right) = \frac{\partial p_2}{\partial Q_2} + q_2^T \frac{\partial^2 p_2}{\partial Q_2^2},$$

where $\left(-dQ_2^T/dq_2^H\right) = \left(dQ_2^T/dq_2^H\right)\left(dq_2^H/dq_1^H\right)$. This ultimately yields (7).

Adding up returns

$$\begin{aligned}\frac{dQ^T}{dq_1^H} &= \frac{dq_1^T}{dq_1^H} + \frac{dQ_2^T}{dq_2^H}\frac{dq_2^H}{dq_1^H} \\ &= \frac{\frac{\partial p_1}{\partial Q_1} + q_1^T\frac{\partial^2 p_1}{\partial Q_1^2}}{-\left(2\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial\left(q_1^T\right)^2} + q_1^T\frac{\partial^2 p_1}{\partial Q_1^2}\right)} + \frac{-\left(\frac{\partial p_2}{\partial Q_2} + q_2^T\frac{\partial^2 p_2}{\partial Q_2^2}\right)}{-\left(2\frac{\partial p_2}{\partial Q_2} - \frac{\partial^2 c}{\partial\left(q_2^T\right)^2} + q_2^T\frac{\partial^2 p_2}{\partial Q_2^2}\right)}\end{aligned}$$

and so

$$\begin{aligned}\frac{dQ^T}{dq_1^H} = \; & \frac{-2\frac{\partial p_1}{\partial Q_1}\frac{\partial p_2}{\partial Q_2} - 2q_1^T\frac{\partial^2 p_1}{\partial Q_1^2}\frac{\partial p_2}{\partial Q_2} + \frac{\partial p_1}{\partial Q_1}\frac{\partial^2 c}{\partial\left(q_2^T\right)^2} + q_1^T\frac{\partial^2 p_1}{\partial Q_1^2}\frac{\partial^2 c}{\partial\left(q_2^T\right)^2} - q_2^T\frac{\partial p_1}{\partial Q_1}\frac{\partial^2 p_2}{\partial Q_2^2} - q_1^T q_2^T\frac{\partial^2 p_1}{\partial Q_1^2}\frac{\partial^2 p_2}{\partial Q_2^2}}{\sum_t\left(2\frac{\partial p_t}{\partial Q_t} + q_t^T\frac{\partial^2 p_t}{\partial Q_t^2} - \frac{\partial^2 c}{\partial\left(q_t^T\right)^2}\right)} \\ & + \frac{2\frac{\partial p_1}{\partial Q_1}\frac{\partial p_2}{\partial Q_2} + 2q_2^T\frac{\partial p_1}{\partial Q_1}\frac{\partial^2 p_2}{\partial Q_2^2} - \frac{\partial p_2}{\partial Q_2}\frac{\partial^2 c}{\partial\left(q_1^T\right)^2} - q_2^T\frac{\partial^2 p_2}{\partial Q_2^2}\frac{\partial^2 c}{\partial\left(q_1^T\right)^2} + q_1^T q_2^T\frac{\partial^2 p_1}{\partial Q_1^2}\frac{\partial^2 p_2}{\partial Q_2^2} + q_1^T\frac{\partial^2 p_1}{\partial Q_1^2}\frac{\partial p_2}{\partial Q_2}}{\sum_t\left(2\frac{\partial p_t}{\partial Q_t} + q_t^T\frac{\partial^2 p_t}{\partial Q_t^2} - \frac{\partial^2 c}{\partial\left(q_t^T\right)^2}\right)},\end{aligned}$$

which ultimately reduces to (8).

Proceeding similarly, we are able to establish that, when firm T is subject to a Pigouvian tax, we have instead

$$\begin{aligned}\frac{dQ^T}{dq_1^H} = \; & \frac{-q_1^T\frac{\partial^2 p_1}{\partial Q_1^2}\left(\frac{\partial p_2}{\partial Q_2} - \frac{\partial^2 c}{\partial\left(q_2^T\right)^2} - \tau\frac{\partial^2 e}{\partial\left(q_2^T\right)^2}\right) + q_2^T\frac{\partial^2 p_2}{\partial Q_2^2}\left(\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial\left(q_1^T\right)^2} - \tau\frac{\partial^2 e}{\partial\left(q_1^T\right)^2}\right)}{\sum_t\left(2\frac{\partial p_t}{\partial Q_t} + q_t^T\frac{\partial^2 p_t}{\partial Q_t^2} - \frac{\partial^2 c}{\partial\left(q_t^T\right)^2} - \tau\frac{\partial^2 e}{\partial\left(q_t^T\right)^2}\right)} \\ & + \frac{\frac{\partial p_1}{\partial Q_1}\left(\frac{\partial^2 c}{\partial\left(q_2^T\right)^2} + \tau\frac{\partial^2 e}{\partial\left(q_2^T\right)^2}\right) - \frac{\partial p_2}{\partial Q_2}\left(\frac{\partial^2 c}{\partial\left(q_1^T\right)^2} + \tau\frac{\partial^2 e}{\partial\left(q_1^T\right)^2}\right)}{\sum_t\left(2\frac{\partial p_t}{\partial Q_t} + q_t^T\frac{\partial^2 p_t}{\partial Q_t^2} - \frac{\partial^2 c}{\partial\left(q_t^T\right)^2} - \tau\frac{\partial^2 e}{\partial\left(q_t^T\right)^2}\right)}.\end{aligned}$$

B Derivation of (23)

The unit penalty is computed as

$$\nu = \left|q_1^H\frac{\partial p_1}{\partial Q_1} - q_2^H\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}\right|.$$

To re-express ν in terms of prices, quantities and elasticities, one should first notice that it is $q_t^T = Q_t^T\left(p_t\right)$, where the function $Q_t^T\left(.\right)$ depends on the considered period, since so does $p_t\left(Q_t\right) = \partial u_t\left(Q_t\right)/\partial Q_t$. The function $Q_t^T\left(p_t\right)$ is the thermal supply function, whereas $p_t\left(Q_t\right)$ is the (inverse) demand function. When varying its production at a given period, the hydro-generator has an impact on the price of this period, hence on the production of the thermal operator (still at this same period). More precisely, it is

$$\frac{dQ_t^T}{dq_t^H} = \frac{dQ_t^T}{dp_t}\frac{dp_t}{dQ_t}\frac{dQ_t}{dq_t^H}.$$

Since $\left(dQ_t/dq_t^H\right)$ writes as $1+\left(dQ_t^T/dq_t^H\right)$, it follows that

$$\frac{dQ_t^T}{dq_t^H} = \frac{\left(dQ_t^T/dp_t\right)\left(dp_t/dQ_t\right)}{1-\left(dQ_t^T/dp_t\right)\left(dp_t/dQ_t\right)}.$$

Let us next introduce the price elasticity of the period-2 thermal supply, which specifies as $\theta_2^T \equiv \left(p_2/q_2^T\right)\left(dQ_2^T/dp_2\right)$ respectively. Relying upon this and the other elasticities defined in the main text, we get

$$\begin{aligned}
\frac{dQ_2^T}{dq_2^H} &= \frac{-\left(q_2^T/Q_2\right)\left(\theta_2^T/\eta_2\right)}{1+\left(q_2^T/Q_2\right)\left(\theta_2^T/\eta_2\right)} \\
&= \frac{-\left(q_2^T\theta_2^T\right)/\left(Q_2\eta_2\right)}{1+\left(q_2^T\theta_2^T\right)/\left(Q_2\eta_2\right)}
\end{aligned}$$

and so

$$\begin{aligned}
\frac{dQ_2}{dq_2^H} &= 1+\frac{dQ_2^T}{dq_2^H} \\
&= 1+\frac{-\left(q_2^T\theta_2^T\right)/\left(Q_2\eta_2\right)}{1+\left(q_2^T\theta_2^T\right)/\left(Q_2\eta_2\right)} \\
&= \frac{1}{1+\left(q_2^T\theta_2^T\right)/\left(Q_2\eta_2\right)} \\
&= \frac{Q_2\eta_2}{Q_2\eta_2+q_2^T\theta_2^T}.
\end{aligned}$$

Using this result together with the definition of s_t^i and η_t, $i=H,T$, $t=1,2$, (23) is ultimately obtained.

C Energy Taxation without Economic Regulation

Differentiating both sides of (24) with respect to τ^E we obtain

$$\frac{\partial p_t}{\partial Q_t}\frac{dQ_t}{d\tau_E}+\frac{dq_t^T}{d\tau_E}\frac{\partial p_t}{\partial Q_t}+q_t^T\frac{\partial^2 p_t}{\partial Q_t^2}\frac{dQ_t}{d\tau_E}-\frac{\partial^2 c}{\partial\left(q_t^T\right)^2}\frac{dq_t^T}{d\tau_E}-1=0,$$

which can then be rearranged as

$$\frac{dq_t^T}{d\tau_E} = \frac{\left(\frac{\partial p_t}{\partial Q_t}+q_t^T\frac{\partial^2 p_t}{\partial Q_t^2}\right)\frac{dQ_t}{d\tau_E}-1}{\frac{\partial^2 c}{\partial\left(q_t^T\right)^2}-\frac{\partial p_t}{\partial Q_t}}.$$

On the other hand, differentiating both sides of (5) with respect to τ^E returns

$$\frac{dq_1^H}{d\tau_E} = \frac{-\left(\frac{\partial p_1}{\partial Q_1} + q_1^H \frac{\partial^2 p_1}{\partial Q_1^2}\right)\frac{dQ_1}{d\tau_E} + \left(\frac{\partial p_2}{\partial Q_2} + q_2^H \frac{\partial^2 p_2}{\partial Q_2^2}\frac{dQ_2}{dq_2^H}\right)\frac{dQ_2}{d\tau_E} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{dq_2^H d\tau_E}}{\frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{d\left(q_2^H\right)^2}}.$$

We can then compute

$$\begin{aligned}
\frac{dQ_1}{d\tau_E} &= \frac{dq_1^H}{d\tau_E} + \frac{dq_1^T}{d\tau_E} \\
&= \frac{-\left(\frac{\partial p_1}{\partial Q_1} + q_1^H \frac{\partial^2 p_1}{\partial Q_1^2}\right)\frac{dQ_1}{d\tau_E} + \left(\frac{\partial p_2}{\partial Q_2} + q_2^H \frac{\partial^2 p_2}{\partial Q_2^2}\frac{dQ_2}{dq_2^H}\right)\frac{dQ_2}{d\tau_E} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{dq_2^H d\tau_E}}{\frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{d\left(q_2^H\right)^2}} \\
&\quad + \frac{1 - \left(\frac{\partial p_1}{\partial Q_1} + q_1^T \frac{\partial^2 p_1}{\partial Q_1^2}\right)\frac{dQ_1}{d\tau_E}}{\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial\left(q_1^T\right)^2}}.
\end{aligned}$$

This can be developed to achieve the condition

$$\begin{aligned}
0 &= -\left[\left(2\frac{\partial p_1}{\partial Q_1} + q_1^H \frac{\partial^2 p_1}{\partial Q_1^2} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{d\left(q_2^H\right)^2}\right)\left(\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial\left(q_1^T\right)^2}\right)\right. \\
&\quad \left. + \left(\frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{d\left(q_2^H\right)^2}\right)\left(\frac{\partial p_1}{\partial Q_1} + q_1^T \frac{\partial^2 p_1}{\partial Q_1^2}\right)\right]\frac{dQ_1}{d\tau_E} \\
&\quad + \left(\frac{\partial p_2}{\partial Q_2} + q_2^H \frac{\partial^2 p_2}{\partial Q_2^2}\frac{dQ_2}{dq_2^H}\right)\left(\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial\left(q_1^T\right)^2}\right)\frac{dQ_2}{d\tau_E} \\
&\quad + q_2^H \frac{\partial p_2}{\partial Q_2}\left(\frac{\partial p_1}{\partial Q_1} - \frac{\partial^2 c}{\partial\left(q_1^T\right)^2}\right)\frac{d^2 Q_2^T}{dq_2^H d\tau_E} + \frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} + q_2^H \frac{\partial p_2}{\partial Q_2}\frac{d^2 Q_2^T}{d\left(q_2^H\right)^2}.
\end{aligned}$$

Let us next assume that it is

$$\frac{\partial^2 p_1}{\partial Q_1^2} = \frac{\partial^2 p_2}{\partial Q_2^2} = \frac{\partial^2 c}{\partial\left(q_1^T\right)^2} = \frac{\partial^2 c}{\partial\left(q_2^T\right)^2} = \frac{d^2 Q_2^T}{dq_2^H d\tau_E} = \frac{d^2 Q_2^T}{d\left(q_2^H\right)^2} = 0.$$

Then the expression previously found becomes

$$\begin{aligned}
\frac{\partial p_1}{\partial Q_1}\left(3\frac{\partial p_1}{\partial Q_1} + 2\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}\right)\frac{dQ_1}{d\tau_E} &= \frac{\partial p_1}{\partial Q_1}\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{d\tau_E} \\
&\quad + \frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H},
\end{aligned}$$

where we have

$$\begin{aligned}
\frac{\partial p_1}{\partial Q_1}\left(3\frac{\partial p_1}{\partial Q_1}+2\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}\right) &> 0 \\
\frac{\partial p_1}{\partial Q_1}\frac{\partial p_2}{\partial Q_2} &> 0 \\
\frac{\partial p_1}{\partial Q_1}+\frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} &< 0.
\end{aligned}$$

Proof of Lemma 1

Two different possible scenarios need be considered. Take first $dQ_1/d\tau_E < 0$. In this case, the left-hand side of the equality above is negative. Hence, also the right-hand side must be negative. This happens with

$$\frac{dQ_2}{d\tau_E} < 0 \quad \vee \quad \frac{dQ_2}{d\tau_E} \in \left(0, \frac{-\frac{\partial p_1}{\partial Q_1} - \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}}{\frac{\partial p_1}{\partial Q_1}\frac{\partial p_2}{\partial Q_2}}\right).$$

Next take $dQ_1/d\tau_E > 0$. In this case, the left-hand side of the equality above is positive. Thus the right-hand side must be positive as well. This solely occurs with $dQ_2/d\tau_E > 0$.

Under the above set of assumptions, we have

$$\frac{dq_t^T}{d\tau_E} = -\frac{dQ_t}{d\tau_E} + \frac{1}{\partial p_t/\partial Q_t},\ t = 1, 2.$$

Hence, with $dQ_t/d\tau_E < 0$, we get $dq_t^T/d\tau_E < 0$ as long as it is $(\partial p_t/\partial Q_t)(dQ_t/d\tau_E) > 1$ and $dq_t^T/d\tau_E > 0$ otherwise. On the other hand, with $dQ_t/d\tau_E > 0$, it is necessarily $dq_t^T/d\tau_E < 0$.

We can now rule out the possibility that it is $dQ_1/d\tau_E > 0$ jointly with $dQ_2/d\tau_E > 0$. Indeed, if these inequalities hold true at once, we must simultaneously have $dq_1^T/d\tau_E < 0$ and $dq_2^T/d\tau_E < 0$. Altogether these inequalities would require that the hydraulic output raise in both periods, which is not feasible as long as the resource constraint is binding. Thus a situation where energy taxation causes an increase in total electricity in each period does not materialize in our environment.

Furthermore, we can write

$$\frac{dq_1^H}{d\tau_E} = \frac{-\frac{\partial p_1}{\partial Q_1}\frac{dQ_1}{d\tau_E} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{d\tau_E}}{\frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}}.$$

Provided that it is

$$\frac{\partial p_1}{\partial Q_1} + \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H} < 0,$$

we ultimately have

$$sign\left(\frac{dq_1^H}{d\tau_E}\right) = sign\left(\frac{\partial p_1}{\partial Q_1}\frac{dQ_1}{d\tau_E} - \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{d\tau_E}\right).$$

Take first the situation where it is $dQ_1/d\tau_E < 0$ together with $dQ_2/d\tau_E < 0$. We then have $(\partial p_1/\partial Q_1)(dQ_1/d\tau_E) > 0$ as well as $(-\partial p_2/\partial Q_2)(dQ_2/d\tau_E) < 0$. It follows that it is $dq_1^H/d\tau_E > 0$ if and only if we have $(\partial p_1/\partial Q_1)(dQ_1/d\tau_E) > (\partial p_2/\partial Q_2)(dQ_2/d\tau_E)$ and $dq_1^H/d\tau_E < 0$ otherwise. Take next the case where it is $dQ_1/d\tau_E < 0$ and $dQ_2/d\tau_E \in \left(0, \left(-\frac{\partial p_1}{\partial Q_1} - \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{dq_2^H}\right) \Big/ \left(\frac{\partial p_1}{\partial Q_1}\frac{\partial p_2}{\partial Q_2}\right)\right)$. We then have $(-\partial p_2/\partial Q_2)(dQ_2/d\tau_E) > 0$ and so necessarily $dq_1^H/d\tau_E > 0$.

This last result allows us to rule out the possibility that it be $dQ_1/d\tau_E < 0$ together with $dQ_2/d\tau_E > 0$. Indeed, if we have $dq_1^H/d\tau_E > 0$, then it must also be the case that $dq_2^H/d\tau_E < 0$. Therefore, for Q_2 to increase as the tax rate is raised, we need to have $dq_2^T/d\tau_E > 0$. However, we know that it is

$$\frac{dq_2^T}{d\tau_E} = -\frac{dQ_2}{d\tau_E} + \frac{1}{\partial p_2/\partial Q_2},$$

which is negative with $dQ_2/d\tau_E > 0$ and $\partial p_2/\partial Q_2 < 0$. It follows that the only possible case is the one where total industry output is reduced under energy taxation in either period $(dQ_t/d\tau_E < 0,\ \forall t)$. Lemma 1 thus entails.

Proof of Lemma 2

Let us next investigate how the output of either generator reacts the energy taxation under those same assumptions. At time t, firm T's production reduces as the tax rate is raised whenever it is

$$-\frac{dQ_t}{d\tau_E} + \frac{1}{\partial p_t/\partial Q_t} < 0 \ \Leftrightarrow \frac{\partial p_t}{\partial Q_t}\frac{dQ_t}{d\tau_E} < 1,$$

meaning that, for q_t^T to be decreased, a unit raise in the tax rate should induce a sub-unitary increase in the energy price through quantity reduction. On the other hand, firm H's output at period 1 (resp., 2) reduces (resp., increases) as long as it is

$$\frac{\partial p_1}{\partial Q_1}\frac{dQ_1}{d\tau_E} < \frac{\partial p_2}{\partial Q_2}\frac{dQ_2}{d\tau_E},$$

meaning that, for q_1^H to reduce (resp., for q_2^H to increase), it must be the case that the power price raises more in period 2 than it does in period 1, following to a unit increase in tax rate, which triggers an output decrease.

Let us finally put things together to draw a conclusion about firms' quantities. To begin with, suppose that $dq_1^H/d\tau_E > 0$. As previously mentioned, this happens whenever it is $(\partial p_1/\partial Q_1)(dQ_1/d\tau_E) > (\partial p_2/\partial Q_2)(dQ_2/d\tau_E)$. Moreover, this calls for $dq_2^H/d\tau_E < 0$. Provided that it is $dQ_1/d\tau_E < 0$, it must be the case that $dq_1^T/d\tau_E < 0$, which requires that $(\partial p_1/\partial Q_1)(dQ_1/d\tau_E) < 1$. It follows that we as well have $(\partial p_2/\partial Q_2)(dQ_2/d\tau_E) < (\partial p_1/\partial Q_1)(dQ_1/d\tau_E) < 1$, which in turn involves $dq_2^T/d\tau_E < 0$.

Let us now consider the case where $dq_1^H/d\tau_E < 0$, which materializes whenever it is $(\partial p_1/\partial Q_1)(dQ_1/d\tau_E) < (\partial p_2/\partial Q_2)(dQ_2/d\tau_E)$ and also calls for $dq_2^H/d\tau_E > 0$. Since it is $dQ_2/d\tau_E < 0$, it must be the case that $dq_2^T/d\tau_E < 0$, which requires that $(\partial p_2/\partial Q_2)(dQ_2/d\tau_E) < 1$. Finally, with $(\partial p_1/\partial Q_1)(dQ_1/d\tau_E) < (\partial p_2/\partial Q_2)(dQ_2/d\tau_E) < 1$, it follows that $dq_1^T/d\tau_E < 0$. Lemma 2 thus entails.