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NETWORKS AND PRICE FORMATION IN A GAS BILATERAL DUOPOLY

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Networks and Price Formation in a Gas Bilateral Duopoly

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1 Motivation

On January 1st 2006, New Year's celebrations in western Europe were shadowed by the news that gas extracting company *Gazprom*, backed by the Russian government (whose leaders often coincide with in *Gazprom*'s top executives), began cutting off gas in pipelines to Ukraine. European Union countries were concerned about such an exacerbation of Ukraine-Russia conflict specially because about 80% of Russian gas exports to Western Europe were actually made through Ukraine.

Actually, Russia sold only about 8% of Ukraine's annual gas requirement, supplied at subsidized prices: about 50 US dollars, compared to an average international rate of around 230 US dollars, per 1000 cubic meters. From 80 billion cubic meters of natural gas consumed every year by Ukraine, 20 billion came from its own production, about 36 billion were bought from Turkmenistan, and as many as 17 billion were received from Russia as a transit fee for gas *Gazprom* passed through Ukrainian pipeline network to Western Europe. In fact, from 2001 on, Ukraine reveived a payment in form of gas corresponding to the 15% of the gas passing through its pipes, a figure estimated around 100 billion cubic meters.

Clearly *Gazprom*'s decision to reduce pressure in the pipelines did not help Russia and Ukraine to reach a compromise in their on-going negotiations over the revision of both the price of supplied gas and the transit fee. At the contrary, until January 11th, after many European countries saw an immediate drop in the supply of gas (from 14% in Poland to 40% in Hungary) Russia accused Ukraine to siphon off gas and Ukraine accused Russia to undersupply gas and falsely accuse of siphoning.

Actually, several other times *Gazprom* and Russia used exclusive access to distribution networks as a (definitely credible) threat to enhance their bargaining power during negotiations over the gas price to European countries.

Actually, looking at the map of the material web of gas pipelines connecting Russia to western european countries, one is tempted to believe a commonly quoted idea: Gazprom strategically builds gas networks in order not only to enhance its own bargaining position in negotiations, but also to weaken the power of countries - with large domestic gas consumption - which are either reluctant to accept its price conditions, or attracted by the possibility to diversify the portfolio of gas-extracting countries.

For instance, it is sensible to believe that *Gazprom* signed with *E.On Ruhrgas* and *Basf* a partnership in Germany to build a direct gas pipeline - called *North Stream* - in the Northern Sea, in order to by-pass Poland and Belarus. One can also observe how effectively *Gazprom* has managed to exploit the inability of European countries to set up a common energy purchasing agency, and even to exacerbate competition between european states by signing up individual contracts. Finally, it has been argued that the main reason beyond the agreement signed by *Gazprom* and *ENI* to build in partnership the *South Stream* pipeline from Russia to Bulgary under the Black Sea, is *Gazprom*'s aim at blocking any other project - such as the EU-funded Nabucco - of gas networks directly connecting Europe with alternative gas-extracting countries, like Turkmenistan, Azerbaijan or Iran.

Indeed, on the one hand, it is true that in *any network* infrastructure, a government owning (or having the exclusive right to build) an important branch of the network passing in its country directed to other domestic markets has clearly a better position than a terminal node of a foreign-owned pipeline. On the other hand, however, this issue becomes much more serious in the specific case of the gas market. This is due to two further features of the international gas market. Firstly, the one of gas is not a competitive market, but, at the contrary, shows all the salient characteristics of a *bilateral oligopoly*: a thin market where a very limited number of traders on both sides are likely to strategically affect both the formation of price and the choice of their trading partners[1]. Secondly, and consequently, in the international gas markets, prices are not reflecting the daily trading in an organized financial institution, but are the outcomes of bilateral contracts and complicate decentralized negotiations.

It seems interesting to explore at which extent the negotiations depend on the shape of the distribution network: *what are the interrelations between a trader's bargaining power and its position in the gas network?*

The issue can be very intricate[2]. Here we just focus on the simpler case of a small buyers-sellers network with heterogenous traders, in which fully decentralized negotiations take place. It can be seen as an exploratory analysis: of course, there remains a lot more to investigate[3].

[1] Europe, for instance, depends crucially on gas supplies by just two national extracting companies, Gazprom, from Russia, and Sonatrach, from Algery.

[2] For a fairly updated survey of the potential insights to gas markets from economic models of bargaining and networks, see Galizzi (2006a).

[3] Specially if one aims at being invited by the Kremlin to sit in the board of Gazprom.

2 Discussion of the model hypothesis and related literature

Investigating how prices form and trade takes place in decentralized markets is still one of the most intriguing and challenging areas of both theoretical and empirical research. Micro-economists have never been ashamed of reckoning how far most markets in the real world behave from the cornerstone paradigm of a Walrasian auction. It is only recently, however, that some fresh analysis has explicitly sprung out from the observation that economics has not yet fully uncovered price formation in *thin markets*.

Thin markets are characterized by a small number of traders on each side and are estimated to represent up to 90 percent of the intermediate goods markets. *Bilateral oligopolies*, in particular, show both sides of the market typically concentrated, and endowed with market power at such an extent that both buyers and sellers are able to affect the prices at which they trade. Furthermore, due to the absence of serious searching costs, traders in such thin markets are usually able to affect at some extent the choice of their trading partners.

Examples of bilateral oligopolies may be found in the international gas market, in some of the basic commodities markets - such as the ones for the coffee, tobacco, hazelnuts or some minerals - and, as mentioned, in most the intermediate goods markets: just to name some, the aerospace, aircrafts and shipping industries, the gigantic-size mechanical and electro-mechanical engineering, the infrastructural plants, the defence or pharmaceutical hi-tech. Furthermore, examples of thin markets emerge every time the stocks or derivatives markets are systematically characterized by a restricted number of traders.

As few pioneering studies (Bjornerstedt and Stennek (2004), Hendricks and McAfee (2005)), have recently pointed out, it is very unlikely that the traders on any side of a thin market may behave as price-takers. Rather, it seems reasonable to think at the price formation as the outcome of a complex of negotiations among traders. The mentioned studies have argued that bilateral oligopolies may be reduced to a collection of many bilateral monopolies: the prices, thus, may emerge as the outcome of many simultaneous Nash-bargaining cooperative solutions, or of many simultaneous bilateral negotiations each involving an exogenously matched pair of one seller and one buyer.

In this paper, on the contrary, we explore an alternative approach, by focusing on non-cooperative interdependent bargaining solutions. The aim of this work, in particular, is to investigate the *role of communication networks on endogenous price formation in a thin market*.

In the literature on non-cooperative bargaining in decentralized markets, in fact, it is traditionally assumed that buyers and sellers are pair-wise matched through some random procedure, and that the order in which agents can make or respond to price offers is exogenously given. However, as Chatterjee and Dutta (1998) observe, while these assumptions are acceptable when modelling large anonymous markets, they are less appropriate in thin markets where the search costs are usually low, and, particularly when agents are heterogeneous, traders may have interest in choosing their partner.

Chatterjee and Dutta (1998) provides a first insight into the role of competition for trading partners on the price prevailing in a thin market. They, in fact, investigate three main models of interdependent bargaining among two identical sellers and two heterogeneous buyers. All the models are based on a bargaining procedure with alternating offers between sellers and buyers, and differ just as the communication structure is concerned. In particular, the strategic interaction among traders is cast on three exogenously designed frames where offers are, respectively, public, privately targeted but publicly known or, finally, privately targeted and secret. The equilibria of the negotiation game typically imply multiple prices and delay.

The analysis of Chatterjee and Dutta (1998) raises two interesting, closely related, research questions.

The first concerns the opportunity of modelling negotiations in thin markets with an alternating order of proposers. Clearly, alternating offers is the most natural specification for any exclusive bilateral bargaining. On the other hand, the hypothesis of a random order of proposals has been typically adopted for the analysis of bargaining in large decentralized markets (see for instance Osborne and Rubinstein (1990), Gale (1986), De Fraja and Sakovics (2001)) in the specific sense that, at any instant of time, either side of the market could, equally likely, be entitled to make proposals. Infact, it is usually claimed that such a stylized mechanism closely mimicks the neutral anonimity of markets and enables to draw a direct comparison with the outcome of a Walrasian competitive framework.

Here, in contrast, we argue that the probably most peculiar features shown by thin markets are the negligible, almost inexistent, transaction frictions and the sheer role played by the identity of each individual trader. Therefore, it is difficult to reject the conjecture that the *traders* themselves, rather than the sides of the market, should be endowed by an *ex ante* identical capability to strategically affect price formation. Therefore, in our thin market we imagine that, at any instant of time, any individual trader is equally likely to start a negotiation, by being selected to announce a proposal to the counterparts on the opposite side of the market.

The second question opened by Chatterjee and Dutta (1998) is the investigation of which communication structure, shaping strategic negotiations among traders, is more likely to emerge in thin markets. In fact, any possible set of communication constraints can equivalently be thought as a network of potential links among agents: the existence of a communication link enables a pair of agents to negotiate.

The existence of physical infrastructural networks, altogether with their shape, in fact, play a crucial role in the distribution of bargaining power and in the feasibility of the implementation of trades among companies both in international gas, oil and electricity thin markets. Furthermore, most the intermediate markets are endowed with an immaterial web of communication, reputation and trust links which is very likely to affect business relationships and negotiations.

Therefore, we aim in particular at drawing a preliminary picture of the *interrelations among bargaining* in thin markets with heterogeneous traders *and* specific architectures of the *buyers-sellers networks*.

The issue of endogenous formation of trading links has been already tackled by Kranton and Minehart (2001). On the other hand, sound descriptions of the negotiations' outcome given a fixed network structure has been provided by the works of Calvò-Armengol (2001, 2002, 2003a, 2003b) and Corominas-Bosch (2004). From this perspective, then, our work may be seen as lying at the crossroads between these two approaches, as concerns the case of decentralized thin markets. With respect to the first work, our paper introduces an explicit analysis of a structured bargaining process with interdependent strategic negotiations. With respect to those in the second group, on the other hand, our work contributes to extend the analysis of the interaction between network architectures and negotiations to the case of markets with *heterogeneous* traders and *fully decentralized* bargaining procedures, beyond the specific case of alternating offers with identical traders.

We study a simple model of endogenous price formation in a thin market where trading is restricted by the shape of the formed bipartite networks. In particular, we consider completely decentralized negotiations with random order of proposers in the simplest case of a *bilateral duopoly with heterogeneous buyers*: trade of a homogeneous asset between a seller and a buyer is possible only when a link is present between them.

The rest of the paper is organised as follows. Section 3 is a description of the model. In Section 4 we fully characterize the equilibria of the negotiations game within any fixed network structure. Section 5 contains a comparison of the bargaining position of each trader across networks, a discussion of our results and some considerations on the issues of network formation and experimental validation.

3 The Model

3.1 The market

In our bilateral duopoly two identical sellers, S_1 and S_2 each own one identical indivisible asset - such as a gas bundle. Both sellers have the same zero reservation value for the asset. We can think of them as the national exporting companies from two major gas extracting countries (for instance, the russian *Gazprom* and the algerian *Sonatrach*) endowed by comparable industrial strength in terms of financial means, extracting volumes, market shares and so on. We will refer to sellers as females.

In the thin market there are two heterogeneous buyers, B_1 and B_2, each of whom demands one single asset. The buyers' valuations are $v_1 = 1$ and $v_2 = \lambda$, respectively, with $1 > \lambda > 0$. Analogously, we can think at them as two gas purchasing and distributing companies which are (almost) monopolists in two asymmetric national final markets (say, *ENI* in Italy, and *Gaz de France* in France), or as two asymmetric competitors in a domestic market (say, *ENI* and *Edison,* or the newborn *AEM-ASM*, in Italy). In the following, we will refer to buyers as males, and to B_1 and B_2 as the strong and the weak buyer, respectively.

Importantly, we assume that all the valuations are common knowledge.

The prices at which the goods are exchanged if trade takes place, are exclusively determined by endogenous bargaining among the players. In particular, we assume that all the traders in the thin market negotiate according to a public offers bargaining procedure with random order of proposers. Moreover it is assumed there is no possibility of price discrimination: the same price is intended to be addressed to all partners on the opposite side of the market.

The key feature of the model, however, is that trade may only take place between a buyer and a seller who are directly linked each other. That is when an agent i on one side of the market has to respond to a price offer from traders belonging to the opposite side, he - or she - may only accept or reject a proposal from j such that $g_{ij} = 1$, where, as usual, g_{ij} denotes the existence of a connection among agents i and j. Analogous restrictions hold for proposal of price offers by agent i, which are intended to be directed exclusively to counterparts j such that $g_{ij} = 1$. It is then helpful to denote with $L(i)$ the set of traders on the opposite side of the market linked with agent i. A network is said to be *connected* if there exists a *path* among any possible pair of traders.

It is immediate to see, however, that in our thin market, only seven non-empty network architectures can emerge. In fact they are the *exclusive trade* networks, where each agent on any side of the market is linked only with a single partner (a) - which nests all networks where *just one* buyer-seller *pair* is exclusively connected - the *supply-short-side* networks where only one seller is linked to both buyers (b), the *demand-short-side* networks where one, either strong or weak, buyer is connected to both the sellers $(c\text{-}d)$, the two *asymmetrically connected* structures where either the high-valuation or the low-valuation buyer is linked with both sellers, while the other buyer is connected only with one exclusive partner $(e\text{-}f)$, and, finally, the *complete* connected bipartite graph (g).

At a first glance, only the three latter *connected* network architectures (e,f,g) may embed non trivial bargaining issues, as for the others negotiations can possibly reduce either to a collection of several independent bilateral negotiations à la Binmore-Rubinstein (with random order of proposers), or to some combination of bargaining and bidding where the short side of the market should manage to extract (almost) all the surplus from the trade.

To attempt to shed some light on the price formation in such cases we, then, need to specify some model for the bargaining process.

3.2 The negotiations

We imagine that in the *negotiation stage*, at every round $t \in \{1, 2, ...\}$, one trader within the thin market is randomly selected to propose offers: each trader, independently of history of play, is selected to launch a proposal with equal probability $\frac{1}{n}$, where $n = 1, ..., 4$, is the number of traders still active in the

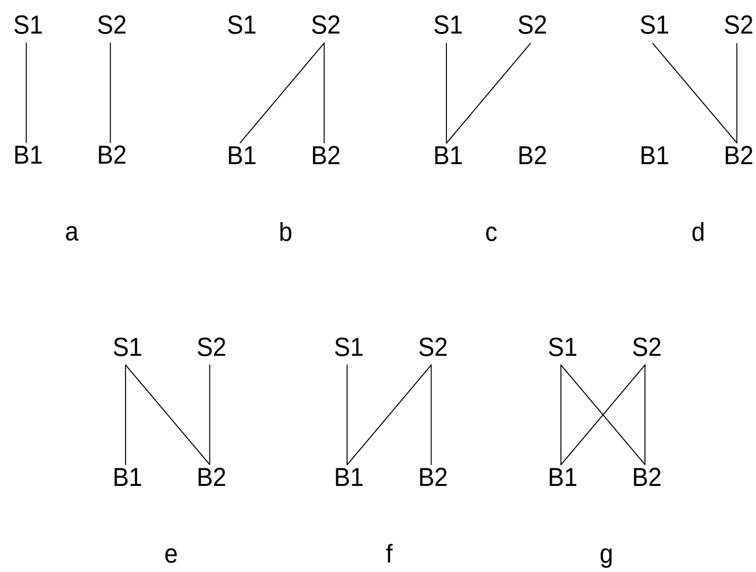

Figure 1: Networks in a bilateral duopoly

market.[4] Traders are considered as active as long as there are still pairs of linked partners bargaining in the thin market.

Any round of the negotiation stage is composed by two phases. First takes place the *price-offer phase*: the agent who has been selected - say buyer B_2 - announces a price, $p_{B_2} \in [0, \infty]$ he is willing to pay for one unit of the asset, from any linked seller $j = S_1, S_2 \in L(B_2)$, i.e. such that $g_{B_2 j} = 1$. Notice that the announced price is intended to be identical for all the linked partners, so that price discrimination is explicitly ruled out.

Thereafter, the *price-response phase* occurs. Each seller $j = S_1, S_2 \in L(B_2)$ responds, simultaneously and independently, to any price offer from B_2. A response is simply either acceptance or rejection of the buyer's latest announced offer p_{B_2}.

Therefore, if *just one* trader on the opposite side - namely S_1 - is in fact *linked* with B_2, the response phase reduces to an individual decision whether to accept or reject p_{B_2} as in a standard bilateral negotiation.

If, at the contrary, *both* traders on the opposite side - say S_1 and S_2 - are indeed *linked* with the proposer, the response phase is modelled as a 2×2 simultaneous moves games. Given the announced price p_{B_2}, in the response game both sellers simultaneously and independently choose whether to accept or to reject p_{B_2}. As a result of such a 2×2 simultaneous moves game, sellers can end out in one of the following situations: either one seller accepts p_{B_2} while the other rejects it, or they both accept p_{B_2}, or, finally, both reject it.

First, if just *one* of the linked sellers *accepts* offer from B_2, she is matched with the weak buyer to trade at p_{B_2}, while the strong buyer and the remaning seller just enter a new round of negotiations *if they are linked together.*

In fact, the peculiar feature of our framework implies that, once a buyer and a seller leave the market after trading, all the links connecting them with any of the other traders are immediately removed by the bipartite graph[5]. If, after such a removal, only isolated agents remain in the market, they all get automatically zero payoffs and the game ends.

If, at the contrary, a connected pair remains unmatched at the end of period t, then in period $t+1$ they enter a further round of the negotiation stage, starting with a fresh random selection of the trader entitled to make proposals. Such a procedure is repeated so long as there are connected traders in the market.

It is worthwhile to underline a consequence of our bargaining procedure. Once just a single buyer and a single seller trade and leave the market, if the two remaining traders are linked each other, the subsequent negotiation stage reduces to a standard bilateral bargaining with random order of proposers. Therefore, from the following bilateral trade, the remaining players expect a surplus of approximately $\frac{1}{2}$ each in case the strong buyer is still in the market, or, alternatively, a surplus of $\frac{\lambda}{2}$ whenever the weak buyer is, which correspond to the Rubinstein bilateral bargaining payoffs.

[4] In real thin markets, in fact, at any period of negotiations, any trader has some chance to formulate the price offers to sell or to buy the asset at.

[5] Alike in Corominas-Bosch (2004).

If, on the contrary, *both* sellers $j = S_1, S_2 \in L(B_2)$ *accept* B_2 's offer of p_{B_2}, then they access a random tie-breaking selection to sort out who is going to trade with B_2 at the announced price p_{B_2}: any of them is randomly picked with $\frac{1}{2}$ probability and matched to trade with B_2. As above, the strong buyer and the seller who has not been selected in the tie-break just enter a new round of negotiations only *if they are linked together*. If, at the contrary, *they are not linked together,* they are forced to leave the market with zero payoffs.

Finally, if *both* sellers *reject* p_{B_2} all traders access a further round of negotiations with a new selection of the player entitled to make offers. In modelling individual strategic choice in the response game, we also assume the tie-breaking hypothesis by which, if any trader is perfectly indifferent about accepting or rejecting an offer, she (he) accepts it.

Which one of the four above situations occurs only depends upon which Nash equilibrium of the 2×2 simultaneous moves game is reached. In general, traders' optimal response correspondences are such that in theory any of the four outcomes can be supported as a Nash equilibrium of the response game.

Clearly, a trader's *best response function* depends upon her (his) continuation payoff and on her outside option in case of a rejection (or a random tie-break) as depending on her position in the graph.

In fact, each of the four outcomes of the 2×2 simultaneous moves game can occur within a specific *set of conditions* on the level of the announced price p_{B_2} in terms of the responders' continuation payoffs and outside options and, ultimately, of values assumed by the (δ, λ) primitive parameters.

By looking at the specific best response functions in the response games, it can happen that some of such sets of conditions are manifestly contraddictory and, then, impossible. On the other hand, sometimes it is possible to order such restrictions in a mutually exclusive way so that a given level of the announced price p_{B_2} may be corresponding to one and just one Nash equilibrium in the response game.

Some other times, however, *multiple equilibria* for a given p_{B_2} may arise in the response game. Typically, it may be the case that B_2 proposes an offer p_{B_2} which is simultaneously compatible with more than one set of conditions and mutual best responses: thus, for instance, two alternative Nash equilibria coexist, one where both sellers accept p_{B_2}, the other where both sellers reject it.

One possible way out of this multiple equilibria result relies on arguing that such buyers' behavior would be inconsistent and contraddictory, and that therefore all the cases giving rise to it should be better discarded. An alternative way is to have recourse to a specific assumption: for instance by assuming that, if, after any proposal from B_2, such multiplicity of equilibria arises, the focus would exclusively be on the Nash equilibrium that shows no delay in trade. After all, it can be argued that, while not altering the very structure of the model, this hypothesis simply rules out economically irrelevant outcomes: in fact, in the tree representation of the game, this just corresponds to cut all those collateral branches diverting from the paths leading to some trade.

Hereafter, however, we adopt an alternative, supposedly more rigorous and less arbitrary, approach. In fact we will always provide a *full characterization* of all the resulting multiple equilibria in the response game and we will look for the equilibria, if any, inducing each corresponding alternative equilibrium in the response game. Notice that, as it is theoretically possible that in some of such equilibria all the traders go back to further negotiations, this would eventually mean to provide a description of equilibria with delayed trade.

Moreover, it is extremely unlikely, but still possible that there are levels of the announced price p_{B_2} for which, by looking at the best response functions, no pure-strategies Nash equilibrium can be guaranteed in the response game. Hence, in such a case, we need to specify what happens in the negotiations game. Although the finiteness of the game clearly ensures the existence of a mixed strategies equilibrium, our exclusive focus on *pure strategies equilibria* implies we need to describe what follows any price-offer node with a proposal p_{B_2} out of all the sets of conditions defining mutual best responses by traders in the response game.

Henceforth, we assume that whenever p_{B_2} is such that no set of conditions for a pure-strategies Nash equilibrium is matched, all the traders just enter a further bargaining stage with a new draw of the proposer. That is, making an offer which can not substain pure strategies Nash equilibrium strategies in the response phase is fully equivalent to making unacceptable offers, as simply implies *accessing a further round of negotiations.* Therefore, in such a case all the traders just get their own continuation payoffs.

Thus, the different levels of the announced price p_{B_2} imply the occurrence of different, possibly multiple, Nash equilibria of the response game. Hence, given any expected equilibrium behaviour in the response game, it is then possible to move back to the *price offer phase* and to work out the proposer's optimal choice. In general, if proposer is a buyer (seller), the optimal choice will be the lowest (highest), price offer implying a Nash equilibrium in which *at least one of the traders on the opposite side accepts that price* in the subsequent response phase, provided it can guarantee at least the proposer's continuation payoff.

We assume impatience, so that all agents have a common discount rate $\delta \in [0, 1)$. Thus, if one unit of the good is exchanged in period t between the buyer i and the seller j at the price p, then the payoff of the buyer will be $\delta^{t-1}(v_i - p)$ and the payoff of the seller $\delta^{t-1}p$.

3.3 The solution

The overall negotiations game among traders is solved given a fixed network structure. In particular, the negotiation game is an infinite horizon dynamic game of complete and imperfect information: in fact, players' payoff functions are common knowledge and, although at each move in the game the players knows the full history of the play thus far, the price-response phases in the negotiation stage are simultaneous moves games. Therefore in the following analysis we solve the negotiations game for its subgame-perfect Nash equilibria using backward induction: for any selection of the trader making offers, we first look at the Nash equilibria in the response phase given any possible proposal

p_i and then at the equilibrium p_i^* in the price offer phase. More precisely, we will look for those players' strategies describing a complete plan of proposals in the price-offers phases and of decisions of either acceptance or rejection in the response phases, which constitute a Nash equilibrium in every subgame and, in particular, generate a Nash equilibrium in the immediately subsequent price-response phases.

Furthermore, given the overall complexity of the present game, we will only focus on the subgame-perfect Nash equilibria in *pure* and *stationary* strategies *(PSSPN equilibria)*. That is, we will only consider equilibria where traders adopt pure strategies at every move, and whose strategies exclusively depend on the number of traders still active in the market and on which phase of the negotiation stage the players are. Therefore, players' strategies are not allowed to be mixed, behavioral or history dependent: any trader always proposes the same price at every equivalent node where he or she has to make an offer, and he or she always behaves in the same way whenever facing identical proposals in the price-response phase.

3.4 The main results

Our main findings are the following. First, we are able to provide a complete characterization of all the *PSSPN* equilibria of the negotiations game within any *fixed* network structure. Some networks only present a single equilibrium for all values of λ and δ. This is the case not only for the *exclusive trade* network, but also for the *asymmetric weak* architecture. On the other hand, while both the B_1 and B_2*-short side* structures show two coexisting equilibria, the *supply-short-side* network presents one equilibrium (*SS2*) for high, one (*SS1*) for intermediate, and one (*SS3*) for low values of λ.

Similar is the case of the *complete* network where, for δ high enough, one equilibrium (*C1)* is defined for high values of λ, another (*C2)* for medium levels, and two alternative equilibria (*C2* and *C3)* co-exist for low values of λ. Even richer is set of *PSSPN* equilibria for the *asymmetric strong* network. When the intertemporal discount factor takes sufficiently high (and realistical) values, as many as eight *PSSPN* equilibria are defined. In fact, three equilibria (*AS6*, *AS5* and *AS1*) are defined for low levels of λ. Medium values of λ are instead cover, in different ranges, by three other equilibria (*AS7*, *AS2* and *AS8)*. Finally, for high levels of λ only one equilibrium (*AS3)* exists, while for extremely high values, an alternative equilibrium (*AS4)* is defined.

In general, in several equilibria an offer is accepted by both traders on a side of the market, so that a random tie-break takes place. This is the case for both equilibria in the B_1 and B_2*-short side* architectures, for two out of three equilibria in the *supply-short-side* network, for all but one equilibria in the *asymmetric strong* network (excluding *AS4*) and even in two of the three equilibria in the *complete* network. In some of such equilibria, then, trade occurs with delay, while in other equilibria, with half probability the least connected traders end up leaving the market without trading at all. In the former case, moreover, *different* prices usually form in the thin market. Therefore, *inefficiency*, both in terms of delay in trade and of impossibility to achieve full exploitation of all potential surplus from trade, can not be ruled out from equilibrium outcomes.

Secondly, we draw direct *comparisons* of traders' payoffs across different network architectures. In particular, we focus such comparisons on the buyers' surplus in order to confirm or reject *two conjectures.*

First, one can guess that B_2, who is clearly in weaker original conditions to start negotiations, if embedded in favourable network configurations, should be able to counterbalance, at some extent, the overwhelming natural advantage of the strong buyer. To seek confirmation of such a guess, one should look at the expected equilibrium payoffs in a *given network* structure to compare the surplus experienced by the two buyers.

We show, however, that such intuitive *guess is rejected by the model's predicted payoffs.* In fact, the only network architectures where B_2 is unambiguously better off than the strong buyer is just the obvious case of the B_2*-short side* network (besides the *weak-couple* one). We show in fact that the strong buyer experiences sistematically higher surplus not only in an *asymmetric strong* network, but also in the *asymmetric weak* and *complete* architectures.

The second conjecture, instead, is related to the surplus of a *given buyer* across different networks. In fact, intuition may suggest that any buyer should always be in a better trading position toward the sellers whenever he is located in a *more connected node than the competing buyer.* In other words, one can argue that, say, the strong buyer would manage to extract better trading opportunities from being not only in a *complete* or *asymmetric strong* network rather than in an *asymmetric weak*, but also in a *asymmetric strong* rather than a *complete* structure. The idea, in fact, is that being connected with more potential partners than the competitor enables a player to enjoy better trading conditions than the rival.

By comparing the *strong buyer*'s equilibrium surplus across networks, we find a quite interesting result: while in an *asymmetric strong* network B_1 gets equilibrium payoffs always as high as in an *asymmetric weak*, direct computations reject the hypothesis that the strong buyer would always be better off in an *asymmetric strong* than in a *complete* network. Indeed, while for λ high enough, B_1 is unambiguously better off within an *asymmetric strong* network, this is no longer true for lower values of the weak buyer's reservation price: at the contrary, for intermediate levels of λ the strong buyer is always strictly better off within a *complete* network.

To shed some light on this surprising result, we attempt a possible explanation. In fact, consider an *asymmetric strong* network where the weak buyer, is exclusively linked with seller S_2. Intuitively, the fact that is linked with an exclusive relationship with the weak buyer provides seller S_2 a *safe outside option* she can always rely on, in the sense that, whenever the weak buyer is selected to make offers, S_2 benefits from having an exclusive partnership with B_2 in terms of high trading prices. Hence, the existence of such alternative trading opportunity implies that, when bargaining with B_1, seller S_2 would never accept any proposal making her worse off with respect to such outside option.

Such a possibility of exclusive dealing with B_2 indirectly provides a *lower* *bound* for competition between the two sellers when fighting for serving the strong buyer in an *asymmetric strong* network. In fact, even S_1 has no interest in proposing the strong buyer something more favourable than S_2's outside option. Thus, both sellers have no incentives to compete too fiercely for the strong buyer, by proposing prices below what S_2 can get from the weak buyer. The existence of such implicit lower bound for sellers' competition in an *asymmetric strong* network clearly hurts the strong buyer who is not able to extract as large trading surplus from negotiations as in a *complete* network.

There is a limit, however, to such B_1's preference towards the *complete* network. In fact, as λ approaches high levels, buyers become more similar in terms of attractiveness for the sellers. Thus, while in a *complete* network, competition to serve the strong buyer becomes less fierce as both sellers can sustain high prices selling to the weak buyer, in the *asymmetric strong* network, B_1 is able to take advantage of the possibility that S_2 exclusively deals with B_2, by obtaining from S_1 prices similar to the one emerging in bilateral negotiations, which, in turn, are now significantly lower than λ.

Hence, we provide evidence that the strongest competing purchaser may genuinely prefer a market structure where communication and trading opportunities are less constrained to one with exclusive partnerships. This result seems counterintuitive, though, and is susceptible of interesting regulation policy implications.

As concerns the *weak buyer*, on the other hand, it turns out that B_2 prefers to bargain in a *complete* network only when λ is high enough, while he is better off in an *asymmetric weak* architecture for lower levels of λ. Intuitively, better connections can help B_2 to overcome significant disadvantages in the original trading capability of the weak buyer, while the protection of a more central node from the competitive pressure of B_1's outside option is no longer a sufficient trading guarantee when this weakness is less pronounced. Therefore, the weak buyer would prefer negotiating in a complete network exactly for levels of λ for which the strong buyer would not.

Moreover, a tension between buyers' interests emerges. In fact, the strong buyer prefers to be embedded within a complete network when λ takes low and medium values, while within an asymmetric strong for high levels of λ. On the contrary, the weak buyer prefers to negotiate within an asymmetric weak architecture when λ is low and within a complete network when his reservation price is high. The emergence of such a prominent conflict of interests among buyers can be regarded as a fascinating prelude to the the investigation of the endogenous strategies of link formation by the traders. As already mentioned, this goal is beyond the scope of the present paper. However, the results obtained insofar, looking at the Pairwise Stable Nash Equilibria (in the spirit of Calvo-Armengol and Ilkilic, 2004) of the endogenous network formation game are encouraging. In fact, only the *complete* and the *asymmetric strong* network emerges as potential candidate equilibria in the network formation game. This, on the one hand, may suggest that also asymmetrically connected graph can represent equilibrium communication structures where decentralized negotiations can take place; on the other hand, it adds to the already present multeplicity of equilibria further complexity and therefore calls for an experimental validation. Both network formation and experimental validation issues are tackled in a companion work (Galizzi, 2007).

4 Bargaining in a Fixed Network

Here we solve for the bargaining sequential game between the traders in the thin markets, given the existence of a fixed bipartite network structure.

Note that the case of the *exclusive trade* networks (a) intuitively can be thought as a minor variant of the model of bilateral negotiations with random order of proposers: in fact, as in a Rubinstein bilateral negotiation, in the limit case $\delta \longrightarrow 1$, we should expect that the strong buyer and his matched seller always get in expected terms a payoff of $\frac{1}{2}$, while the weak buyer and his matched seller each earn an expected surplus of $\frac{\lambda}{2}$.

Thus, in the following we will start describing the negotiation game in the case one single pair of traders is linked, then gradually moving, through more and more connected bipartite graphs in which traders can still be isolated or asymmetrically connected, up to the complete network where any pair of traders is linked together.

At a first glance, it can be argued that, at least within any network where the strong buyer is not isolated, there can not be *PSSPNE* in which the bargaining process keeps on going on forever. Indeed, as the discounted payoffs of all the traders would be zero in such a case, there is certainly a profitable deviation at least by the strong buyer. In fact, whenever he is selected to make an offer, B_1 can always propose a price equal to δ, which, being the highest price both sellers may ever gain in the following rounds, will be immediately accepted in the subsequent response phase. In turn, being $\delta < 1$ ensures the strong buyer a strictly positive payoff, and then a profitable deviation from the perpetual disagreement situation.

Finally, notice that, by a standard argument by theory of infinite horizon dynamic games of complete information (see for instance Osbourne and Rubinstein (1990), Fudenberg and Tirole (1996)), a stationary dynamic game may be fully characterized by describing any of its strategically equivalent subgames.

In particular, define S_i-games the subgames of the original game of negotiations among the four traders in a given network, starting whenever the seller S_i is randomly selected to make offers. Analogously define B_i-games the subgames of the original game that start when buyer B_i is randomly selected to make offers. Hence, being for any given network structure, all the S_i-games and all the B_i-games strategically equivalent by the stationarity hypothesis, the analysis of the *PSSPN* equilibria in the original overall game perfectly corresponds to the investigation of the *PSSPN* equilibria in any of the S_1-games, S_2-games, B_1-games and B_2-games.

In the following sections, we will provide a full description of the equilibria, and their relative proofs, for two network configurations only. This is just to provide an insight of the way we have proceeded in the argumentation. For the remaining networks, we have just proceeded in the same spirit and, being the the steps and the proofs quite mechanical[6], they are omitted for the sake of brevity. Of course they are fully provided in the complete version of the paper (Galizzi, 2006b) and can be asked from the author.

4.1 The ***Exclusive-trade*** network

The *exclusive-trade* network (a) clearly corresponds to a market configuration where two separate pairs of traders negotiate in a mutually exclusive partnership: as if *Gazprom* has an exclusive partnership with *ENI*, while *Sonatrach* deals with *Gaz de France* only. In particular, seller S_1 is linked with the strong buyer only, while S_2 is exclusively connected with the weak buyer. Although, in our framework each trader, at any round, has an identical $\frac{1}{4}$ probability to make offers, this situation is substantially corresponding to a case where two parallel bilateral negotiations are taking place simultaneously and independently, since any trader has just a potential partner to trade with. It is not surprising, then, that the equilibrium outcome of the bargaining process within each pair of traders is basically equivalent to the one of a pairwise Rubinstein negotiation with random order of proposers.

In fact, by an usual argument in bargaining theory it immediately turns out that the only possible equilibrium offer by any trader is to propose her (his) exclusive potential partner just his (her) continuation payoff. For instance, imagine the strong buyer has been selected to make offers. In a *PSSPN* equilibrium, B_1 should propose S_1 a price exactly identical to her own continuation value $\delta V(S_1)$, which, is clearly accepted by S_1.[7] Analogously, consider the $\frac{1}{4}$ probability seller S_1 has been selected to make offers. Clearly, the only possible equilibrium offer by seller S_1 is to propose B_1 a price such to leave him exactly indifferent between accepting and rejecting, thus earning his own continuation value $\delta W(B_1)$, that is a price $p_{S_1} = 1 - \delta W(B_1)$ which is clearly accepted by B_1. In both cases, once either S_1 or B_1, being selected, has proposed such an offer and the relative counterpart has accepted it, the pair trade at the agreed price and leave the market with the corresponding payoffs.

The remaining traders S_2 and B_2 are still in the market and, being connected each other, are allowed to carry on further negotiations. After S_1 and B_1 have traded and left the market, the latter negotiations are exactly equivalent to a standard bilateral bargaining à la Rubinstein with random order of proposers.

[6] A heuphemistical expression for "boring".

[7] In fact, higher price from B_1 would still be accepted but would clearly represent strictly dominated strategies. On the other hand, lower offers from the strong buyer will certainly be rejected by S_1, thus delivering the former just his own continuation value $\delta W(B_1)$. However, it can be reckoned that the proposal $p_{B_1} = \delta V(S_1)$ gives the strong buyer a surplus $1-\delta V(S_1)$ as good as the payoff guaranteed by any, alternative, unacceptable offer: in fact, intuitively, $W(B_1) \leq 1 - \delta V(S_1)$ is always holding as the maximum surplus the strong buyer can ever manage to extract from negotiations in an *exclusive trade* network is exactly what left from his own reservation price once he has paid S_1, his only potential partner, her continuation payoff, and therefore $\delta W(B_1) \leq W(B_1) \leq 1 - \delta V(S_1)$ must a fortiori be holding. Therefore, the above proposal $p^*_{B_1} = \delta V(S_1)$ is in fact the optimal strategy by the strong buyer among all, acceptable and unacceptable, offers.

In fact, with identical $\frac{1}{2}$ probability both S_2 and the weak buyer expect to be selected to make offers. By the usual arguments, in a *PSSPN* equilibrium, whenever selected to make offers, B_2 proposes S_2 a price exactly identical to her own continuation value $\delta V(S_2)$, which, is clearly accepted by S_2. The same holds for S_2. Therefore, it is immediately reckoned that, by accessing bilateral negotiations after S_1 and B_1 have traded and left the market, the weak buyer and S_2 just expect to get a surplus $\frac{\delta\lambda}{2}$ each, that is the discounted value of the equilibrium payoff in a standard bilateral negotiation over λ. Of course, reverse but analogous arguments hold for the cases in which either S_2 or B_2 have been selected to make offers. Therefore, it is possible to state the followings.

Lemma 1 *Within an exclusive-trade network, the unique PSSPNE of the game following a random selection of the strong buyer B_1 implies, in the price-offer phase, B_1 always proposing S_1 a price $p^*_{B_1} = \delta V(S_1)$, where $\delta V(S_1)$ is the discounted value of the expected continuation payoff by S_1 from entering a new negotiation stage, and, in the response phase, S_1 accepting $p^*_{B_1}$. After B_1 and S_1 have traded at $p^*_{B_1}$ and left the market, B_2 and S_2 enter further bilateral negotiations. The equilibrium payoffs for the traders whenever B_1 has been selected as proposer, is then*

$$\begin{cases} \Pi(B_1) = 1 - \delta V(S_1) \\ \Pi(B_2) = \frac{\delta\lambda}{2} \\ \Pi(S_1) = \delta V(S_1) \\ \Pi(S_2) = \frac{\delta\lambda}{2} \end{cases}$$

Lemma 2 *Within an exclusive-trade network, the unique PSSPNE of the game following a random selection of seller S_1 implies, in the price-offer phase, S_1 always proposing B_1 a price $p^*_{S_1} = 1 - \delta W(B_1)$, where $\delta W(B_1)$ is the discounted value of the expected continuation payoff by B_1 from entering a new negotiation stage, and, in the response phase, B_1 accepting $p^*_{S_1}$. After B_1 and S_1 have traded at $p^*_{S_1}$ and left the market, B_2 and S_2 enter further bilateral negotiations. The equilibrium payoffs for the traders whenever either seller has been selected as proposer, is then*

$$\begin{cases} \Pi(B_1) = \delta W(B_1) \\ \Pi(B_2) = \frac{\delta\lambda}{2} \\ \Pi(S_1) = 1 - \delta W(B_1) \\ \Pi(S_2) = \frac{\delta\lambda}{2} \end{cases}.$$

Lemma 3 *Within an exclusive-trade network, the unique PSSPNE of the game following a random selection of the weak buyer B_2 implies, in the price-offer phase, B_2 always proposing S_2 a price $p^*_{B_2} = \delta V(S_2)$, where $\delta V(S_2)$ is the discounted value of the expected continuation payoff by S_2 from entering a new negotiation stage, and, in the response phase, S_2 accepting $p^*_{B_2}$. After B_2 and S_2 have traded at $p^*_{B_2}$ and left the market, B_1 and S_1 enter further bilateral negotiations. The equilibrium payoffs for the traders whenever B_2 has been* *selected as proposer, is then*

$$\begin{cases} \Pi\left(B_1\right) = \frac{\delta}{2} \\ \Pi\left(B_2\right) = \lambda - \delta V(S_2) \\ \Pi\left(S_1\right) = \frac{\delta}{2} \\ \Pi\left(S_2\right) = \delta V(S_2) \end{cases} .$$

Lemma 4 *Within an exclusive-trade network, the unique PSSPNE of the game following a random selection of seller S_2 implies, in the price-offer phase, S_2 always proposing B_2 a price $p^*_{S_2} = \lambda - \delta W(B_2)$, where $\delta W(B_2)$ is the discounted value of the expected continuation payoff by B_2 from entering a new negotiation stage, and, in the response phase, B_2 accepting $p^*_{S_2}$. After B_2 and S_2 have traded at $p^*_{S_2}$ and left the market, B_1 and S_1 enter further bilateral negotiations. The equilibrium payoffs for the traders whenever either seller has been selected as proposer, is then*

$$\begin{cases} \Pi\left(B_1\right) = \frac{\delta}{2} \\ \Pi\left(B_2\right) = \delta W(B_2) \\ \Pi\left(S_1\right) = \frac{\delta}{2} \\ \Pi\left(S_2\right) = \lambda - \delta W(B_2) \end{cases} .$$

We can now try to summarize the above findings. In fact, at the moment we have just provided a description of all the possible subgame-perfect Nash equilibria in pure and stationary strategies which may arise in the negotiation game for *any* random selection of the trader entitled to make offers. However, we should now characterize all the possible expressions for the expected value of the continuation payoffs, by combining each possible equilibrium outcome for any $\frac{1}{4}$ probability selection of the trader making offers. In fact, at a new round of the negotiation game, the traders expect that:

- with $\frac{1}{4}$ probability B_1 is selected to make offers and, in a *PSSPN* equilibrium, B_1 always offers S_1 a price $p^*_{B_1} = \delta V(S_1)$, which in the response phase is accepted by S_1, thus delivering the following equilibrium payoffs

$$\begin{cases} \Pi\left(B_1\right) = 1 - \delta V(S_1) \\ \Pi\left(B_2\right) = \frac{\delta\lambda}{2} \\ \Pi\left(S_1\right) = \delta V(S_1) \\ \Pi\left(S_2\right) = \frac{\delta\lambda}{2} \end{cases}$$

- with $\frac{1}{4}$ probability B_2 is selected to make offers and in a *PSSPN* equilibrium, B_2 always offers S_2 a price $p^*_{B_2} = \delta V(S_2)$, which in the response phase is accepted by S_2, thus delivering the following equilibrium payoffs

$$\begin{cases} \Pi\left(B_1\right) = \frac{\delta}{2} \\ \Pi\left(B_2\right) = \lambda - \delta V(S_2) \\ \Pi\left(S_1\right) = \frac{\delta}{2} \\ \Pi\left(S_2\right) = \delta V(S_2) \end{cases} .$$

- and so on with the other random selection.

Therefore, we can compute the exact expressions for the expected continuation payoffs in a *PSSPN* equilibrium characterized by the above strategies for any possible random selection of the proposer. In fact, by taking each above equilibrium payoff as weighted with $\frac{1}{4}$ probability, we can write the expected continuation payoffs of the traders in such *PSSPN* equilibrium as

$$\begin{cases} W(B_1) = \frac{1}{4}\left(1-\delta V(S_1)\right) + \frac{1}{4}\delta W(B_1) + \frac{1}{2}\left(\frac{\delta}{2}\right) \\ W(B_2) = \frac{1}{2}\left(\frac{\delta\lambda}{2}\right) + \frac{1}{4}\left(\lambda - \delta V(S_2)\right) + \frac{\delta}{4}W(B_2) \\ V(S_1) = \frac{1}{4}\delta V(S_1) + \frac{1}{4}\left(1-\delta W(B_1)\right) + \frac{1}{2}\left(\frac{\delta}{2}\right) \\ V(S_2) = \frac{1}{2}\left(\frac{\delta\lambda}{2}\right) + \frac{\delta}{4}V(S_2) + \frac{1}{4}\left(\lambda - \delta W(B_2)\right) \end{cases}$$

which can be solved as as system, returning the final expressions

$$\begin{cases} W(B_1) = \frac{1+\delta}{4} \\ W(B_2) = \frac{(1+\delta)\lambda}{4} \\ V(S_1) = \frac{1+\delta}{4} \\ V(S_2) = \frac{(1+\delta)\lambda}{4} \end{cases}$$

that, in the limit case as $\delta \longrightarrow 1$, clearly approach the values

$$\begin{cases} W(B_1) \longrightarrow \frac{1}{2} \\ W(B_2) \longrightarrow \frac{\lambda}{2} \\ V(S_1) \longrightarrow \frac{1}{2} \\ V(S_2) \longrightarrow \frac{\lambda}{2} \end{cases}.$$

We can then summarize the previous results in the following statement:

Proposition 5 *For any discount rate δ and reservation price λ, there exists a unique PSSPN equilibrium of the negotiation game in the* exclusive trade *network where*

- *B_1, whenever is selected to make offer, proposes S_1 a price $p^*_{B_1} = \delta V(S_1)$, which is accepted by S_1,*
- *B_2, whenever is selected to make offer, proposes S_2 a price $p^*_{B_2} = \delta V(S_2)$, which is accepted by S_2,*
- *S_1, whenever is selected to make offer, proposes B_1 a price $p^*_{S_1} = 1 - \delta W(B_1)$, which in the response phase is accepted by B_1,*
- *S_2, whenever is selected to make offer, proposes B_2 a price $p^*_{S_2} = \lambda - \delta W(B_2)$, which in the response phase is accepted by B_2.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{1+\delta}{4} \\ W(B_2) = \frac{(1+\delta)\lambda}{4} \\ V(S_1) = \frac{1+\delta}{4} \\ V(S_2) = \frac{(1+\delta)\lambda}{4} \end{cases}$$

that, in the limit case as $\delta \longrightarrow 1$*, approach the values* $W(B_1) \longrightarrow V(S_1) \longrightarrow \frac{1}{2}$, $W(B_2) \longrightarrow V(S_2) \longrightarrow \frac{\lambda}{2}$.

Notice that the *exclusive trade* network naturally nests two other subgraphs. In fact, by straight adaptations of the above arguments, it is possible to characterize the *PSSPN* equilibria of the negotiations game within the *strong* and the *weak couple* network configurations. By the former, in fact, we simply mean a market in which only the strong buyer and a seller - say S_1 - are connected, while the weak buyer and the remaining seller - S_2 - are isolated traders. The latter case is the analogous graph where S_2 is connected with the weak buyer while the remaining traders are isolated[8].

4.2 The *Supply-short-side* network

In the *supply-short-side* networks (b) only one seller S_i with $i = 1, 2$ is linked to both buyers while S_{-i} is an isolated trader. This trading structure mimicks all the market configurations where an exclusive seller is naturally endowed by the capability to elicit a significant extraction of surplus from two competing buyers. The leading market position of *Gazprom*, which provides gas to all the european countries, is perhaps the better example.[9]

However, as discussed above, a peculiar trait of the present bargaining framework is that, as long as some not isolated agents still remain in the market, any trader is entitled with an identical probability to propose offers. Hence, in such a communication network - alike in the *(c)* and *(d)* below - agents incur delays in trade with at least $\frac{1}{4}$ probability, namely, at least after any S_{-i} selection.

Another peculiar feature of the present *supply-short-side* structure is that the *buyers* are in a *symmetric* position concerning the number of accessible connections. In fact, in such a network both buyers can access an identical number of partners. Thus, it is with no lack of generality and likelihood of the analysis that henceforth we ask the strong buyer's continuation payoff to not differ from the weak buyer's by more than some upper bound, equal to the original difference in their reservation prices, in the way expressed by *Condition k*,

$$\delta\left[W\left(B_1\right) - W\left(B_1\right)\right] \leq 1 - \lambda,$$

by which the discounted value of the difference in the buyers' expected continuation payoffs can not exceed the relative distance between their primitive reservation prices. The positive difference $1 - \lambda$ thus represents an upper bound meeting the relative advantage on the surplus expected by the strong buyer, and avoids the latter explosively diverges beyond any sensible initial dissimilarity on the traders' *ex-ante* bargaining positions.

It is worthwhile to clarify that we are not actually forcing the equilibrium payoffs to satisfy this property. Rather we impose it at the beginning of our

[8] See the original version of the paper.

[9] A similar model of negotiations with random order of proposers in such thin market is presented in Galizzi (2006c), even though in a different environment of endogenous coalition formation.

analysis and we then check it *a posteriori*, selecting the equilibria whose computed continuation values actually satisfy it.

Finally, given the symmetry across sellers, we hereafter characterize the equilibrium offers within a *supply-short-side* network where S_2 *is linked with both buyers while* S_1 *is isolated*, like in (b), to easily extend the corresponding findings to the reverse positions of the sellers. We now describe one by one each subgame of the negotiations game, starting whenever any from the four traders is randomly selected to make offers.

4.2.1 B_1 proposes offers

The peculiar shape of the supply-short-side network implies that, whenever in a round either buyer has been selected to make proposals, which occurs with identical $\frac{1}{4}$ probability each, they both face no other alternatives but making offers to the only linked seller S_2.

In particular, by an usual argument it immediately turns out that the only possible equilibrium offer by the strong buyer is to propose S_2 a price exactly identical to her own continuation value $\delta V(S_2)$, which, in equilibrium is clearly accepted by S_2.[10]

Therefore, the following immediately derive:

Lemma 6 *Within a supply-short-side network, the unique PSSPNE of the game following a random selection of the strong buyer* B_1 *implies, in the price-offer phase,* B_1 *always proposing* S_2 *a price* $p^*_{B_1} = \delta V(S_2)$*, where* $\delta V(S_2)$ *is the discounted value of the expected continuation payoff by* S_2 *from entering a new negotiation stage, and, in the response phase,* S_2 *accepting* $p^*_{B_1}$*. The equilibrium payoffs for the traders whenever* B_1 *has been selected as proposer, are then*

$$\begin{cases} \Pi(B_1) = 1 - \delta V(S_2) \\ \Pi(B_2) = 0 \\ \Pi(S_1) = 0 \\ \Pi(S_2) = \delta V(S_2) \end{cases}$$

[10] In fact, higher price from B_1 would still be accepted but would clearly represent strictly dominated strategies. On the other hand, lower offers from the strong buyer will certainly be rejected by S_2, thus delivering the former just his own continuation value $\delta W(B_1)$. However, it can be reckoned that the proposal $p_{B_1} = \delta V(S_2)$ gives the strong buyer a surplus $1-\delta V(S_2)$ as good as the payoff guaranteed by any, alternative, unacceptable offer: in fact, intuitively, $\delta W(B_1) \leq W(B_1) \leq 1 - \delta V(S_2)$ is always holding as the maximum surplus the strong buyer can ever manage to extract from negotiations in an supply-short-side network is indeed what left from his own reservation price once he has paid S_2, his only potential partner, her continuation payoff. The fact that $\delta V(S_2)$ can in no way exceed the unit maximum surplus of the strong buyer guarantees the expression $1 - \delta V(S_2)$ always takes non negative values. Therefore, the above proposal $p^*_{B_2} = \delta V(S_2)$ is in fact the optimal strategy by the strong buyer among all, acceptable and unacceptable, offers. As a consequence, being S_2 then matched with the strong buyer to trade at the agreed price, both S_1 and the weak buyer would leave the market with no trade and zero payoffs.

4.2.2 Seller S_2 proposes offers

Consider finally the $\frac{1}{4}$ probability that S_2 has been selected to propose offers to both connected buyers. By using a standard argument of backward induction, we first describe, for any given proposed price p_{S_2}, the set of all the possible Nash Equilibria in the response game played by the two linked buyers, and we then look for the optimal pricing strategy by S_2 in the offer phase.

The outcomes of such a game are symmetric for buyers in that both rely on zero payoffs as their outside options. In fact, only if one of them have been announcing to accept p_{S_2}, he is going to exit negotiations with a surplus equal to the difference between his reservation price and p_{S_2}, while the remaining buyer, getting isolated, has no other possibility but exiting the market with a zero payoff.

The same logic applies to the outcome of the random tie-break selecting with identical probability which buyer is entitled to trade at p_{S_2} with S_2, and which one, instead, leaves market with a zero surplus. Finally, as usual, whenever both buyers reject p_{S_2}, all the traders simply enter a further stage of negotiations with a new random selection of the proposer.

From the above discussion follows the payoff matrix of the buyers response game, for a given proposal p_{S_2}, as reported in Figure 2.

		B_2: **Accept p_{S2}**	B_2: **Reject p_{S2}**
B_1	**Accept p_{S2}**	$\frac{1}{2}(1 - p_{S2})$; $\frac{1}{2}(\lambda - p_{S2})$	$1 - p_{S2}$; 0
	Reject p_{S2}	0 ; $\lambda - p_{S2}$	$\delta W(B_1)$; $\delta W(B_2)$

Figure 2: Buyers' response game in a supply-short-side network

Looking at such matrix, is possible to identify the *Best Response functions* for each buyer. The set of strategies available to each buyer has just two elements: either strategy *Accept* p_{S_2} or strategy *Reject* p_{S_2}. For instance, given that B_2 chooses *Reject* p_{S_2}, it is better for the strong buyer to *Accept* p_{S_2}, if and only if $1-p_{S_2} \geq \delta W(B_1)$, while, given that B_2 plays *Accept* p_{S_2}, it is better for B_1 to also accept it if and only if $\frac{1}{2}(1-p_{S_2}) \geq 0$, that is, if $p_{S_2} \leq 1$. On the other hand, given that B_1 *accepts* p_{S_2}, it is optimal for B_2 to also accept it if and only if $\frac{1}{2}(\lambda - p_{S_2}) \geq 0$, that is, if $p_{S_2} \leq \lambda$. Finally, given that B_1 *Rejects*

p_{S_2}, it is optimal for B_2 to accept it if and only if $\lambda - p_{S_2} \geq \delta W(B_2)$.[11]

The next step is then to look for the candidate *pure-strategies Nash equilibria* of the response game, by working out whether the strategies played by the two buyers may be mutually best responding within some range of the relevant parameters. In particular, we aim at providing a full description of the set of conditions necessary to hold in order to observe any possible pure-strategies Nash equilibrium.

To start with, the combination of $[B_1, B_2]$ pure strategies $[\textit{Accept } p_{S_2}, \textit{Accept } p_{S_2}]$ is a Nash equilibrium of the response game if and only if $\begin{cases} p_{S_2} \leq 1 \\ p_{S_2} \leq \lambda \end{cases}$, that is, whenever $p_{S_2} \leq \lambda$.

On the other hand, pure strategies $[\textit{Accept } p_{S_2}, \textit{Reject } p_{S_2}]$ are a Nash equilibrium of the response game if $\begin{cases} p_{S_2} \leq 1 - \delta W(B_1) \\ p_{S_2} > \lambda \end{cases}$, while $[\textit{Reject } p_{S_2}, \textit{Accept } p_{S_2}]$ are a pure-strategies Nash equilibrium if $\begin{cases} p_{S_2} > 1 \\ p_{S_2} \leq \lambda - \delta W(B_2) \end{cases}$.

Finally, $[\textit{Reject } p_{S_2}, \textit{Reject } p_{S_2}]$ are a pure-strategies Nash equilibrium if $\begin{cases} p_{S_2} > 1 - \delta W(B_1) \\ p_{S_2} > \lambda - \delta W(B_2) \end{cases}$, that is if $p_{S_2} > \max\{1 - \delta W(B_1),\, \lambda - \delta W(B_2)\}$. By the above discussed *Condition k,* $\delta\left[W(B_1) - W(B_1)\right] \leq 1 - \lambda$, the latter condition reduces to $p_{S_2} > 1 - \delta W(B_1)$.

Furthermore, it is easily reckoned that offer p_{S_2} can *never verify* the set of restrictions $\begin{cases} p_{S_2} > 1 \\ p_{S_2} \leq \lambda - \delta W(B_2) \end{cases}$, as, by definition, $\lambda \leq 1$ and B_2's continuation payoff is surely non negative, $\delta W(B_2) \geq 0$. Hence, there is no offer p_{S_2} verifying $\lambda - \delta W(B_2) \geq p_{S_2} > 1$. Intuition suggests, in fact, that it can never exist an equilibrium of the buyers' response game in such a network where the strong buyer rejects what is accepted by the weak buyer, thus earning a zero surplus. As if in a standard auction, whenever traded in such a network, the asset should be definitely bought by the strong buyer.

Therefore, only three possible combinations of strategies can represent Nash equilibria of the response game under some range of restrictions on the price offer p_{S_2}: either $[\textit{Accept } p_{S_2}, \textit{Accept } p_{S_2}]$ if $p_{S_2} \leq \lambda$, or $[\textit{Accept } p_{S_2}, \textit{Reject } p_{S_2}]$ are a Nash equilibrium of the response game if $\begin{cases} p_{S_2} \leq 1 - \delta W(B_1) \\ p_{S_2} > \lambda \end{cases}$, or, finally, $[\textit{Reject } p_{S_2}, \textit{Reject } p_{S_2}]$ if $p_{S_2} > 1 - \delta W(B_1)$. This leads to one consideration.

In fact, it may be the case that, for some range of p_{S_2}, multiple Nash equilibria in the response game arise. In particular, this is possible only whenever S_2 proposes an offer p_{S_2} such that both $p_{S_2} \leq \lambda$ and $p_{S_2} > 1 - \delta W(B_1)$ hold. In such a case two alternative Nash equilibria coexist: one where both buyers accept p_{S_2}, the other where both buyers reject it. As discussed above, we will always provide a full characterization of all the resulting multiple equilibria in

[11] Hereafter, we just take advantage of standard tie-breaking assumptions by which if a trader is perfectly indifferent between accepting or rejecting an offer she accepts it, while, if a trader is ever indifferent between proposing acceptable or unacceptable offers, she makes the acceptable one.

the response game and we will look for the *PSSPN* equilibria, if any, inducing each corresponding alternative equilibrium in the response game.

We are able to state the following characterization of the equilibrium offers by the seller.

Proposition 7 *Whenever seller S_2 is selected to make offers within a supply-short-side network, three PSSPN equilibria can arise, according to the levels of the continuation payoffs. Denote $\delta V(S_i)$ and $\delta W(B_i)$ the discounted value of the expected continuation payoff by S_i and B_i, respectively, from entering a new negotiation stage. Hence,*

- *If $\delta W(B_1) < 1 - \lambda$ is satisfied, then there exists a PSSPN equilibrium where, in the price offer phase, S_2 offers a price $p^*_{S_2} = 1 - \delta W(B_1)$ and in the response phase, only B_1 accepts $p^*_{S_2} = 1 - \delta W(B_1)$, while B_2 rejects it and then leaves the market without trading. Hence, whenever S_2 is selected to make offers, traders' expected payoffs from such PSSPN* I-*equilibrium are*

$$\begin{cases} \Pi(B_1) = \delta W(B_1) \\ \Pi(B_2) = 0 \\ \Pi(S_1) = 0 \\ \Pi(S_2) = 1 - \delta W(B_1) \end{cases}$$

- *If, instead, the following set of conditions holds*

$$\begin{cases} \delta W(B_1) \geq 1 - \lambda \\ \delta V(S_2) \leq \lambda \end{cases}$$

then there exists a PSSPN equilibrium where, in the price offer phase, S_2 offers a price $p^*_{S_2} = \lambda$ and in the response phase, both B_1 and B_2 accept $p^*_{S_2} = \lambda$ as well as any offer within the range $(1 - \delta W(B_1), \lambda]$. In such equilibrium, thus, buyers enter a random selection to determine which one trades with seller S_2 at $p^*_{S_2}$ and which, instead, leaves the market. Hence, whenever S_2 is selected to make offers, traders' expected payoffs from such PSSPN IIa-*equilibrium are*

$$\begin{cases} \Pi(B_1) = \frac{1-\lambda}{2} \\ \Pi(B_2) = 0 \\ \Pi(S_1) = 0 \\ \Pi(S_2) = \lambda. \end{cases}$$

- *Finally, if just $W(B_1) \geq 1 - \lambda$ holds, then there exists a PSSPN equilibrium where, in the price offer phase, S_2 offers a price $p^*_{S_2} = 1 - \delta W(B_1)$ and in the response phase, both B_1 and B_2 accept $p^*_{S_2} = 1 - \delta W(B_1)$ while they both reject any offer within the range $(1 - \delta W(B_1), \lambda]$. In such equilibrium, thus, buyers enter a random selection to determine which one trades with seller S_2 at $p^*_{S_2}$ and which, instead, leaves the market. Hence,* *whenever S_2 is selected to make offers, traders' expected payoffs from such PSSPN* IIr-*equilibrium are*

$$\begin{cases} \Pi\left(B_1\right) = \frac{\delta W(B_1)}{2} \\ \Pi\left(B_2\right) = \frac{\delta W(B_1)}{2} + \frac{\lambda-1}{2} \\ \Pi\left(S_1\right) = 0 \\ \Pi\left(S_2\right) = 1 - \delta W(B_1) \end{cases}$$

Proof. In Appendix. ■

4.2.3 B_2 proposes offers

Analogously, consider the weak buyer B_2. The only possible equilibrium acceptable offer by the weak buyer is to propose S_2 a price exactly identical to her own continuation value $\delta V(S_2)$, which, in equilibrium is clearly accepted by S_2. [12] However, we need here to apply specific conditions on the weak buyer's continuation payoff. In particular, the weak buyer faces a dychotomic choice. If conditon $\delta W(B_2) \leq \lambda - \delta V(S_2)$ is satisfied, his optimal behaviour entails making the lowest possible acceptable offer, that is a proposal $p^*_{B_2} = \delta V(S_2)$. Otherwise, if $\delta W(B_2) > \lambda - \delta V(S_2)$, his optimal strategy is at the contrary to propose any offer strictly below $p^*_{B_2} = \delta V(S_2)$ such that is certainly going to be rejected. In such a case all the traders access a new round of negotiations, which guarantees the weak buyer higher payoffs than from any acceptable offer.

Intuitively, in fact, whenever λ is too low, the weak buyer strictly prefers to stay out of negotiations as paying the seller's continuation payoff is excessively costly to him. This is equivalent to state the following.

Lemma 8 *Within a supply-short-side network, if condition $\delta W(B_2) \leq \lambda - \delta V(S_2)$ holds, the unique* Acc-*PSSPNE of the game following a random selection of the weak buyer B_2 implies*

- *in the price-offer phase, B_2 always proposing S_2 a price $p^*_{B_2} = \delta V(S_2)$, where $\delta V(S_2)$ is the discounted value of the expected continuation payoff by S_2 from entering a new negotiation stage, and,*

[12] In fact, higher price, although still accepted, would represent strictly dominated strategies. Therefore, in an equilibrium where B_2 proposes *acceptable* offers, the weak buyer should definitely propose the strictly dominant $p^*_{B_2} = \delta V(S_2)$. As a consequence, being S_2 then matched with the weak buyer to trade at the agreed price, both S_1 and the strong buyer would leave the market without trading. However, this is not yet sufficient to show that such best acceptable proposal $p^*_{B_2} = \delta V(S_2)$, giving the weak buyer a surplus of $\lambda - \delta V(S_2)$ is necessarily an optimal strategy for B_2. In particular, $p^*_{B_2} = \delta V(S_2)$ should indeed also be a better strategy than making *unacceptable* offers. This occurs only if $p^*_{B_2} = \delta V(S_2)$ guarantees a surplus at least as good as the payoff conveyed by the latter strategy, equal to his own continuation value $\delta W(B_2)$, that is, if $\delta W(B_2) \leq \lambda - \delta V(S_2)$. However, there is no reason why this condition should be satisfied exclusively in view of the assumptions discussed above. In particular, $\lambda - \delta V(S_2)$ may well be a negative expression, thus implying an impossible upper bound for the weak buyer's continuation payoff. In general, in fact, neither from *Condition k* nor from any other of the above assumptions we are able to guarantee that condition $\delta V(S_2) \leq \lambda - \delta W(B_2)$ should always be satisfied in equilibrium.

- *in the response phase, S_2 accepting $p^*_{B_2}$.*

The equilibrium payoffs for the traders whenever B_2 has been selected as proposer, are then $\Pi(B_1) = \Pi(S_1) = 0$, $\Pi(B_2) = \lambda - \delta V(S_2)$ and $\Pi(S_2) = \delta V(S_2)$.

Lemma 9 *Within a supply-short-side network, if condition $\delta W(B_2) > \lambda - \delta V(S_2)$ holds, instead, the unique* Unacc-*PSSPNE of the game following a random selection of the weak buyer B_2 implies*

- *in the price-offer phase, B_2 always proposing S_2 any unacceptable price $p'_{B_2} < \delta V(S_2)$, where $\delta V(S_2)$ is the discounted value of the expected continuation payoff by S_2 from entering a new negotiation stage, and,*
- *in the response phase, S_2 rejecting any such p'_{B_2}.*

The equilibrium payoffs for the traders whenever B_2 has been selected as proposer, are then just their discounted continuation values.

4.2.4 Seller S_1 proposes offers

As discussed above, our bargaining framework implies that, as long as some not isolated agents still remain in the market, any trader is selected at any round with an identical probability to make offers. Therefore, in the present supply-short-side market, even the isolated seller S_1 has the chance to ask for some price at which, however, she would never be able to trade at.[13] However, for whatever price would be actually announced by seller S_1, the final outcome of any $\frac{1}{4}$ selection of S_1 can never be anything else but a further round of negotiations all the traders are forced to access. Therefore, whenever the isolated seller S_1 is selected to make offers, all the traders just expect their continuation values.

4.2.5 Description of the equilibria

We now need to summarize the above findings. In fact, up to here we have provided a full characterization of all the possible subgame-perfect Nash equilibria in pure and stationary strategies which may arise in the negotiation game for *any* random selection of the trader entitled to make offers. Each equilibrium has been described with a companion set of conditions on the traders' expected continuation payoffs that restricts the compatible range of the primitive parameters in which such equilibrium is possible.

However, we should now characterize the expected continuation values, by combining each possible equilibrium outcome for any $\frac{1}{4}$ probability selection of the trader making offers. Clearly, each set of expected continuation payoffs is only possible within a particular set of restrictions, namely the ones resulting

[13] This modelling issue implies that all agents incur delays in trade with $\frac{1}{4}$ probability, at least any time S_1 is selected, and captures the peculiar inefficient feature of having traders bargaining within incompletely connected graphs, thus prevented from a full exploitation of the maximum potential surplus from trading.

from the conditions characterizing the equilibrium outcome under the selection of B_2 and S_2 as proposers. As these restrictions hold simultaneously, whenever an expression for a trader's expected continuation payoff violates any of them, the corresponding combination of equilibrium outcomes following each $\frac{1}{4}$ probability selection can not be viewed as a sensible candidate equilibrium, and should instead be discarded. In other words, there may exist equilibrium outcomes following particular traders' selection which are not mutually compatible.

Such a repeated process will finally eliminate all the possible equilibria whose expected payoffs are not consistent with any from the compatibility restrictions, thus providing a full characterization of all the possible *PSSPN* equilibria of the negotiation game within the *supply-short-side network* as the ones which survive such a feasibility check.

For instance, consider a new round of the negotiation game in which all the traders expect that

- with $\frac{1}{4}$ probability B_1 is selected to make offers, in a *PSSPN* equilibrium, B_1 always offers S_2 a price $p^*_{B_1} = \delta V(S_2)$, which in the response phase is accepted by S_2, thus delivering the following equilibrium payoffs $\Pi(B_1) = 1 - \delta V(S_2)$, $\Pi(S_2) = \delta V(S_2)$, $\Pi(B_2) = \Pi(S_1) = 0$.

- with $\frac{1}{4}$ probability B_2 is selected to make offers, and, as condition $\delta W(B_2) \leq \lambda - \delta V(S_2)$ holds, in the unique *Acc-PSSPN* equilibrium B_2 always offers a price $p^*_{B_2} = \delta V(S_2)$ which, in the response phase is accepted by seller S_2. Hence the traders' equilibrium payoffs are $\Pi(B_1) = \Pi(S_1) = 0$, $\Pi(B_2) = \lambda - \delta V(S_2)$ and $\Pi(S_2) = \delta V(S_2)$.

- with $\frac{1}{4}$ probability S_1 is selected to make offers and in a *PSSPN* equilibrium, for any offer proposed by S_1, the expected payoffs for the traders are just their continuation payoffs.

- with $\frac{1}{4}$ probability S_2 is selected to make offers, and, as condition $\delta W(B_1) < 1 - \lambda$ is satisfied, then in the *PSSPN I*-equilibrium S_2 offers a price $p^*_{S_2} = 1 - \delta W(B_1)$ which in the response phase is accepted only by B_1, while B_2 rejects it and then leaves the market without trading. Thus, expected payoffs by the traders from such *PSSPN I*-equilibrium are $\Pi(B_1) = \delta W(B_1)$, $\Pi(S_2) = 1 - \delta W(B_1)$, $\Pi(B_2) = \Pi(S_1) = 0$.

Therefore, under the conditions obtained from the restrictions above, namely the one holding when S_2 have been selected to make offers, we can compute the exact expressions for the expected continuation payoffs in a *PSSPN* equilibrium characterized by the above strategies for any possible random selection of the proposer.

In fact, by taking each above equilibrium payoff as weighted with $\frac{1}{4}$ probability, we can write the expected continuation payoffs of the traders in such

PSSPN equilibrium as

$$\begin{cases} W(B_1) = \frac{1}{4}\left(1-\delta V(S_2)\right) + \frac{\delta}{2} W(B_1) \\ W(B_2) = \frac{1}{4}\left(\lambda-\delta V(S_2)\right) + \frac{\delta}{4} W(B_2) \\ V\left(S_1\right) = 0 \\ V\left(S_2\right) = \frac{3}{4}\delta V(S_1) + \frac{1}{4}\left(1-\delta W(B_1)\right) \end{cases}$$

which can be solved as as system, returning the final expressions

$$\begin{cases} W(B_1) = \frac{4(1-\delta)}{5\delta^2-20\delta+16} \\ W(B_2) = \frac{-3\delta^2-5\delta^2\lambda+4\delta+20\delta\lambda-16\lambda}{5\delta^3-40\delta^2+96\delta-64} \\ V\left(S_1\right) = 0 \\ V\left(S_2\right) = \frac{4-3\delta}{5\delta^2-20\delta+16} \end{cases}$$

that, in the limit case as $\delta \longrightarrow 1$ approach the values $W(B_1) \longrightarrow V\left(S_1\right) \longrightarrow 0$, $V\left(S_2\right) \longrightarrow 1$ and $W(B_2) \longrightarrow \frac{1}{3}\left(\lambda-1\right)$.

We now need to check whether all the found expressions for the expected continuation payoffs are fully compatible with the above restrictions $\delta W(B_2) \leq \lambda - \delta V(S_2)$ and $\delta W(B_1) < 1-\lambda$. In this regard, we help our analysis by means of simple numerical simulations over the primitive parameters of the model, namely, the intertemporal discount rate δ and the reservation price of the weak buyer λ, both contained by definition within a range $(0,1)$.

Simulations, in fact, suggest that the strong buyer's expected continuation payoff is such that condition $\delta W(B_1) < 1-\lambda$ is verified for *not extremely high* value of the $\lambda < 1$ parameter, say, (when $\delta = 0.9$) for approximately any $\lambda < \overline{\lambda} \cong 0.825$.

Moreover, the weak buyer's and seller S_2's payoffs are such that condition $\delta W(B_2) \leq \lambda - \delta V(S_2)$ is satisfied for values of λ *high* enough, in particular, for λ approximately such that (when $\delta = 0.9$) $\lambda \geq \widehat{\lambda} \cong 0.5$, a lower bound which is fully compatible with the previous constraint.

Therefore, we can conclude that, for any value of the discount rate $\delta \neq 1$ and for *high, but not extremely high* value of the weak buyer reservation price λ, the expected continuation payoffs in the above characterized equilibrium are perfectly consistent with the restriction holding for a *PSSPN* equilibrium raising whenever S_2 is selected to make offers. This, in turn, allows us to state the following.

Proposition 10 *For any discount rate $\delta \neq 1$ and reservation price $\widehat{\lambda} \leq \lambda < \overline{\lambda}$, there exists a PSSPN equilibrium* SS1 *of the negotiation game in the supply-short-side network where*

- *B_1, whenever is selected to make offer, proposes S_2 a price $p^*_{B_1} = \delta V(S_2)$ which, in the response phase, is accepted by S_2*
- *B_2, whenever is selected to make offer, proposes seller S_2 a price $p^*_{B_2} = \delta V(S_2)$ which, in the response phase is accepted by seller S_2*

- *for any offer proposed by S_1, whenever is selected, the expected payoffs for the traders are just their continuation payoffs*
- *S_2, whenever is selected to make offer, proposes both buyers a price $p^*_{S_2} = 1 - \delta W(B_1)$ which, in the response phase, is accepted only by B_1.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{4(1-\delta)}{5\delta^2-20\delta+16} \\ W(B_2) = \frac{-3\delta^2-5\delta^2\lambda+4\delta+20\delta\lambda-16\lambda}{5\delta^3-40\delta^2+96\delta-64} \\ V(S_1) = 0 \\ V(S_2) = \frac{4-3\delta}{5\delta^2-20\delta+16} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, would approach the values $W(B_1) \longrightarrow V(S_1) \longrightarrow 0$, $V(S_2) \longrightarrow 1$ and $W(B_2) \longrightarrow \frac{1}{3}(\lambda - 1)$.

The analysis of the traders' payoffs in the limit case of absence of impatience confirms that, in such equilibrium, the connected seller is able to fully exploit most the surplus from the negotiation, leaving both buyers with payoffs very close to zero. Also notice that, although the expected continuation payoff by B_2 gets negative as $\delta \longrightarrow 1$, for any $\delta \neq 1$ it is always non-negative as $\widehat{\lambda} \leq \lambda < \overline{\lambda}$.

An analogous line of reasoning lies behind the process of elimination of any combination of equilibrium strategies inconsistent with the restrictions of (δ, λ) parameters necessary for their characterization. Indeed, direct calculations supported by numerical simulations show another possible equilibrium holding for λ *extremely high*, the complementary case $\lambda \geq \overline{\lambda} \cong 0.825$ (when $\delta = 0.9$).

Proposition 11 *For any discount rate δ and reservation price $\lambda \geq \overline{\lambda}$, there exists a PSSPN equilibrium* SS2 *of the negotiation game in the supply-short-side network where*

- *B_1, whenever is selected to make offer, proposes S_2 a price $p^*_{B_1} = \delta V(S_2)$ which, in the response phase, is accepted by S_2*
- *B_2, whenever is selected to make offer, proposes seller S_2 a price $p^*_{B_2} = \delta V(S_2)$ which, in the response phase is accepted by seller S_2*
- *for any offer proposed by S_1, whenever is selected, the expected payoffs for the traders are just their continuation payoffs*
- *S_2, whenever is selected to make offer, proposes both buyers a price $p^*_{S_2} = \lambda$ which, in the response phase, is accepted by both buyers.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{-9\delta+\delta\lambda+12-4\lambda}{2(3\delta^2-16\delta+16)} \\ W(B_2) = \frac{4\lambda(1-\delta)}{3\delta^2-16\delta+16} \\ V(S_1) = 0 \\ V(S_2) = \frac{\lambda}{4-3\delta} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values $W(B_1) \longrightarrow \frac{1-\lambda}{2}$, $V(S_2) \longrightarrow \lambda$, $W(B_2) \longrightarrow V(S_1) \longrightarrow 0$.

It is interesting to notice that in the present equilibrium the connected seller is not able to fully exploit all the potential surplus from the trade. At the contrary, she can just appropriate from negotiations at most the weak buyer's reservation price, leaving the strong buyer with a positive surplus tending to half the difference between the reservation prices.

Finally, it can be shown that, for relatively *low* values of λ, namely within the other complementary case of $\lambda < \widehat{\lambda} \cong 0.5$, there exists an alternative equilibrium where the weak buyer decides indeed to make unacceptable offer so to avoid paying excessively onerous prices. More precisely,

Proposition 12 *For any reservation price $\lambda < \widehat{\lambda}$, there exists a PSSPN equilibrium* SS3 *of the negotiation game in the supply-short-side network where*

- *B_1, whenever is selected to make offer, proposes S_2 a price $p^*_{B_1} = \delta V(S_2)$ which, in the response phase, is accepted by S_2*
- *B_2, whenever is selected to make offer, proposes seller S_2 any unacceptable price $p'_{B_2} < \delta V(S_2)$ which, in the response phase is rejected by seller S_2*
- *for any offer proposed by S_1, whenever is selected, the expected payoffs for the traders are just their continuation payoffs*
- *S_2, whenever is selected to make offer, proposes both buyers a price $p^*_{S_2} = 1 - \delta W(B_1)$ which, in the response phase, is accepted only by B_1.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{1}{2(2-\delta)} \\ W(B_2) = 0 \\ V(S_1) = 0 \\ V(S_2) = \frac{1}{2(2-\delta)} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, would approach the values $W(B_1) \longrightarrow V(S_2) \longrightarrow \frac{1}{2}$, $W(B_2) \longrightarrow V(S_1) \longrightarrow 0$.

Hence, when asymmetry among buyers is particularly sharp, the weak buyer chooses to abstain from active trading so that negotiations mimick in fact bilateral bargaining among the seller and the strong buyer only. Intuitively this equilibrium outcome is due to the fact that the seller's continuation payoff is so high compared to λ that the weak buyer is better off choosing not to compete with the strong buyer.

The above findings can be correspondingly extended to the symmetric case of a supply-short-side network where S_1 is connected with both buyers and S_2 is instead isolated.

4.3 The B_1-*short-side* network

In the B_1-*short-side* networks (c) only the strong buyer B_1 is linked to both sellers while the weak buyer is an isolated trader. This trading network captures all the market structures where an exclusive large purchaser is naturally endowed by the power to exploit the existing competition between two homogeneous sellers.[14]

For this and the following networks, for the lack of brevity, we just provide the full description of the equilibria. Details and proofs are in the original version of the paper.

Proposition 13 *For any discount rate δ and reservation price λ, there exists a PSSPN* A-*equilibrium of the negotiation game in the B_1-short-side network where*

- *B_1, whenever is selected to make offer, proposes the sellers a price, $p^*_{B_1} = 0$, accepted by both sellers*
- *for any offer proposed by B_2, whenever is selected, the expected payoffs for the traders are just their continuation payoffs*
- *S_1, whenever is selected to make offer, proposes the strong buyer a price $p^*_{S_1} = 1 - \delta W(B_1)$ which is accepted by B_1.*
- *S_2, whenever is selected to make offer, proposes the strong buyer a price $p^*_{S_2} = 1 - \delta W(B_1)$ which is accepted by B_1.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{1}{4-3\delta} \\ W(B_2) = 0 \\ V(S_1) = \frac{1}{4-\delta}\left(1 - \frac{\delta}{4-3\delta}\right) \\ V(S_2) = \frac{1}{4-\delta}\left(1 - \frac{\delta}{4-3\delta}\right) \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values $W(B_1) \longrightarrow 1$, $W(B_2) \longrightarrow V(S_1) \longrightarrow V(S_2) \longrightarrow 0$.

Proposition 14 *For any discount rate δ and reservation price λ, there also exist two symmetric PSSPN* R-*equilibria of the negotiation game in the B_1-short-side network where*

- *B_1, whenever is selected to make offer, proposes the sellers a price $p^*_{B_1} = \min\{\delta V(S_1), \delta V(S_2)\}$ accepted by both sellers*

[14] The case may be probably thought as the secret dream of *ENI*, the national incumbent gas-distributor, when it speaks about the ambition to trasform Italy into a gas distribution *hub* for the Mediterranean Sea.

- *for any offer proposed by B_2, whenever is selected, the expected payoffs for the traders are just their continuation payoffs*

- *S_1, whenever is selected to make offer, proposes the strong buyer a price $p^*_{S_1} = 1 - \delta W(B_1)$ which is accepted by B_1.*

- *S_2, whenever is selected to make offer, proposes the strong buyer a price $p^*_{S_2} = 1 - \delta W(B_1)$ which is accepted by B_1.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{8-5\delta}{7\delta^2-36\delta+32} \\ W(B_2) = 0 \\ V(S_1) = \frac{8(1-\delta)}{7\delta^2-36\delta+32} \\ V(S_2) = \frac{8(1-\delta)}{7\delta^2-36\delta+32} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values $W(B_1) \longrightarrow 1$, $W(B_2) \longrightarrow V(S_1) \longrightarrow V(S_2) \longrightarrow 0$.

Notice that, although implying different traders' payoffs, both equilibria converge to the same values at the limit case of absence of impatience: as intuition suggests, the strong buyer is able to extract all the potential surplus from the trade.

4.4 The B_2-*short-side* network

In the B_2-*short-side* networks (d), instead, only the weak buyer B_2 is linked to both sellers while the strong buyer is an isolated trader. It can be reckoned how straight the arguments in the previous section can be adapted to this case. Analogous considerations in fact show the following results.

Proposition 15 *For any discount rate δ and reservation price λ, there exists a PSSPN* a-*equilibrium of the negotiation game in the B_2-short-side network where*

- *for any offer proposed by B_1, whenever is selected, the expected payoffs for the traders are just their continuation payoffs*

- *B_2, whenever is selected to make offer, proposes the sellers a price, $p^*_{B_2} = 0$, accepted by both sellers*

- *S_1, whenever is selected to make offer, proposes the strong buyer a price $p^*_{S_1} = \lambda - \delta W(B_2)$ which is accepted by B_2.*

- *S_2, whenever is selected to make offer, proposes the strong buyer a price $p^*_{S_2} = \lambda - \delta W(B_2)$ which is accepted by B_2.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = 0 \\ W(B_2) = \frac{\lambda}{4-3\delta} \\ V(S_1) = \frac{\lambda}{4-\delta}\left(1 - \frac{\delta}{4-3\delta}\right) \\ V(S_2) = \frac{\lambda}{4-\delta}\left(1 - \frac{\delta}{4-3\delta}\right) \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values $W(B_2) \longrightarrow \lambda$, $W(B_1) \longrightarrow V(S_1) \longrightarrow V(S_2) \longrightarrow 0$.

Proposition 16 *For any discount rate δ and reservation price λ, there also exist two symmetric PSSPN* r-*equilibria of the negotiation game in the B_2-short-side network where*

- *for any offer proposed by B_1, whenever is selected, the expected payoffs for the traders are just their continuation payoffs*
- *B_2, whenever is selected to make offer, proposes the sellers a price, $p^*_{B_2} = \min\{\delta V(S_1), \delta V(S_2)\}$, accepted by both sellers*
- *S_1, whenever is selected to make offer, proposes the strong buyer a price $p^*_{S_1} = \lambda - \delta W(B_2)$ which is accepted by B_2.*
- *S_2, whenever is selected to make offer, proposes the strong buyer a price $p^*_{S_2} = \lambda - \delta W(B_2)$ which is accepted by B_2.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = 0 \\ W(B_2) = \frac{(8-5\delta)\lambda}{7\delta^2-36\delta+32} \\ V(S_1) = \frac{8\lambda(1-\delta)}{7\delta^2-36\delta+32} \\ V(S_2) = \frac{8\lambda(1-\delta)}{7\delta^2-36\delta+32} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values $W(B_2) \longrightarrow \lambda$, $W(B_1) \longrightarrow V(S_1) \longrightarrow V(S_2) \longrightarrow 0$.

Notice that, although implying different traders' payoffs, both equilibria converge to the same values at the limit case of absence of impatience: clearly the weak buyer is able to extract all the potential surplus from the trade.

4.5 The *Asymmetric Weak* Network

We now investigate the equilibrium prices and outcomes as emerging from negotiations within network architectures which are asymmetrically connected as the buyers are concerned, like *(e)*. In particular, we first consider the network where the *weak buyer is connected to both sellers*, while the *strong buyer is only linked to seller* S_2. As a consequence, both the strong buyer and seller S_1 are exclusively connected with, respectively, S_2 and B_2. Clearly the negotiation game within such a network is strategically equivalent to the one taking place in the homologous graph obtained re-labelling the nodes by switching S_1 with S_2.

Alike for the previous network configurations (and for the complete network characterized below) where the buyers are in a symmetric position as the number of accessible connections is regarded, even in the present *asymmetric weak* network we may think at *Condition k*,

$$\delta \left[W \left(B_1 \right) - W \left(B_1 \right) \right] \leq 1 - \lambda,$$

as a convenient restriction.

In fact, if *Condition k* is a reasonable restriction on buyers' expected continuation payoffs for any symmetric structure, it must *a fortiori* be a necessary feature of the latter within an asymmetric weak network. In fact, in such a case, the payoff advantage enjoyable by the strong buyer should be even less remarkable as the weak buyer is in fact endowed with a relatively sounder bargaining position concerning the number of accessible partners. Therefore, also in the present network configuration we assume and check *a posteriori* that the buyers' continuation payoffs are as suggested by *Condition k*: $1 - \delta W \left(B_1 \right) \geq \lambda - \delta W \left(B_2 \right)$. An analogous consideration on the supposedly better trading opportunities entailed by traders in more connected nodes lies behind a corresponding assumption on the sellers' continuation payoffs, $V \left(S_2 \right) \geq V \left(S_1 \right)$.

Proposition 17 *For any discount rate δ and reservation price λ, there exists a unique PSSPN equilibrium* AW *of the negotiation game in the asymmetric weak network where*

- *B_1, whenever is selected to make offer, proposes S_2 a price $p^*_{B_1} = \delta V(S_2)$ which, in the response phase, is accepted by S_2*
- *B_2, whenever is selected to make offer, proposes both sellers a price $p^*_{B_2} = \delta V(S_1)$ which, in the response phase, is accepted only by S_1*
- *S_1, whenever is selected to make offer, proposes B_2 a price $p^*_{S_1} = \lambda - \delta W(B_2)$, which, in the response phase, is accepted by B_2*
- *S_2, whenever is selected to make offer, proposes both buyers a price $p^*_{S_2} = 1 - \delta W(B_1)$ which, in the response phase, is accepted only by B_1.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{1+\delta}{4} \\ W(B_2) = \frac{\lambda(1+\delta)}{4} \\ V\left(S_1\right) = \frac{\lambda(1+\delta)}{4} \\ V\left(S_2\right) = \frac{1+\delta}{4} \end{cases}.$$

which, in the limit case $\delta \longrightarrow 1$*, approach the values* $W(B_1) \longrightarrow V(S_2) \longrightarrow \frac{1}{2}$*,* $W(B_2) \longrightarrow V(S_1) \longrightarrow \frac{\lambda}{2}$*.*

The traders' payoffs of the unique equilibrium in the negotiations game within an asymmetric weak network are interesting. Somehow contrarily to what one could expect, they suggest that the weak buyer is never able to take advantage of his most connected location in order to get better trading opportunities than the strong buyer. In fact, for any impatience rate and weak buyer's reservation price, the strong buyer is always better off than the weak.

4.6 The *Asymmetric Strong* Network

We then consider the equilibrium prices and outcomes emerging from negotiations within the alternative asymmetrically connected network architecture (f). In particular, we now consider the network where the *strong buyer is connected to both sellers*, while the *weak buyer is only linked to seller* S_2. As a consequence, both the weak buyer and seller S_1 are exclusively connected with, respectively, S_2 and B_1.[15] This market configuration fits very well the case of most domestic gas markets, where the incumbent is usually endowed by a wider set of energetic sources than the smaller competitors.

It is immediately reckoned that the previous arguments in favour of the assumption of *Condition k* as a neutral and realistic simplifying restriction on buyers' continuation payoffs can no longer keep their validity when we move to this asymmetric strong network. In fact, in the present case, B_1 reinforces his original strength due to the higher reservation value with the trading advantages conveyed by his central position in the graph. Thus, we should at the contrary argue that the strong buyer reasonably expects a surplus from the trade which may well be beyond any upper bound as implied by *Condition k*. Therefore, all along the following analysis, we will separately consider both the case where $1 - \delta W(B_1) \geq \lambda - \delta W(B_2)$ and the one in which $1 - \delta W(B_1) \leq \lambda - \delta W(B_2)$. We now describe one by one each subgame of the negotiations game, starting whenever any from the four traders is randomly selected to make offers.

Proposition 18 *For high enough discount rate* $\delta \geq \overline{\overline{\overline{\delta}}}$ *and low reservation price* $\lambda \leq \overline{\overline{\overline{\lambda}}}$*, there exists a PSSPN equilibrium* AS1 *of the negotiation game in the asymmetric strong network where*

- B_1*, whenever is selected to make offer, proposes both sellers a price* $p^*_{B_1} = \frac{\delta\lambda}{2}$ *which, in the response phase, is accepted by both* S_1 *and* S_2
- B_2*, whenever is selected to make offer, proposes* S_2 *a price* $p^*_{B_2} = \delta V(S_2)$ *which, in the response phase, is accepted by* S_2

[15] Again the negotiation game in such a network is strategically equivalent to the one taking place in the homologous graph obtained re-labelling the nodes after having switched S_1 with S_2.

- S_1, *whenever is selected to make offer, proposes* B_1 *a price* $p^*_{S_1} = 1 - \delta W(B_1)$*, which, in the response phase, is accepted by* B_1

- S_2, *whenever is selected to make offer, proposes both buyers a price* $p^*_{S_2} = 1 - \delta W(B_1)$ *which, in the response phase, only* B_1 *accepts while* B_2 *rejects it.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{1}{2(2-\delta)} + \frac{\delta(1-\lambda)}{4(2-\delta)} \\ W(B_2) = \frac{\delta^3+6\delta^3\lambda+6\delta^2-22\delta^2\lambda-8\delta+32\lambda}{16(2-\delta)(4-\delta)} \\ V(S_1) = \frac{-3\delta^2-2\delta+2\delta\lambda+8}{16(2-\delta)} \\ V(S_2) = \frac{-\delta^2-3\delta^2\lambda-6\delta+8\delta\lambda+8}{4(2-\delta)(4-\delta)} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$*, approach the values*

$$\begin{cases} W(B_1) \longrightarrow \frac{1}{2} + \frac{1-\lambda}{4} \\ W(B_2) \longrightarrow \frac{\lambda}{3} - \frac{1}{48} \\ V(S_1) \longrightarrow \frac{3}{16} + \frac{\lambda}{8} \\ V(S_2) \longrightarrow \frac{1}{12} + \frac{5}{12}\lambda \end{cases}.$$

In fact, the above equilibrium holds for the primitive parameters such that $\delta \geq \overline{\overline{\overline{\delta}}} \cong 0.65$ and for $\lambda \leq \overline{\overline{\overline{\lambda}}} \cong 0.4$.

Proposition 19 *For high discount rate* $\delta \geq \overline{\delta}$ *and intermediate reservation price* $\underline{\lambda} \leq \lambda \leq \overline{\lambda}$*, there exists a PSSPN equilibrium* AS2 *of the negotiation game in the asymmetric strong network where*

- B_1, *whenever is selected to make offer, proposes both sellers a price* $p^*_{B_1} = \frac{\delta\lambda}{2}$ *which, in the response phase, is accepted by both* S_1 *and* S_2

- B_2, *whenever is selected to make offer, proposes* S_2 *a price* $p^*_{B_2} = \delta V(S_2)$ *which, in the response phase, is accepted by* S_2

- S_1, *whenever is selected to make offer, proposes* B_1 *a price* $p^*_{S_1} = 1 - \delta W(B_1)$*, which, in the response phase, is accepted by* B_1

- S_2, *whenever is selected to make offer, proposes both buyers a price* $p^*_{S_2} = 1 - \frac{\delta}{2}$ *which, in the response phase, is accepted by both buyers.*

As a results, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{1+\delta}{4-\delta} - \frac{\delta\lambda}{2(4-\delta)} \\ W(B_2) = \frac{\delta^2-7\delta^2\lambda+2\delta+6\delta\lambda-8+24\lambda}{16(4-\delta)} \\ V(S_1) = \frac{-7\delta^2+\delta^2\lambda+4\delta+4\delta\lambda+16}{16(4-\delta)} \\ V(S_2) = \frac{1+\delta\lambda}{4-\delta} - \frac{\delta}{2(4-\delta)} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values

$$\left\{\begin{array}{c} W(B_1) \longrightarrow \frac{2}{3} - \frac{\lambda}{6} \\ W(B_2) \longrightarrow \frac{23}{48}\lambda - \frac{5}{48} \\ V(S_1) \longrightarrow \frac{13}{48} + \frac{5}{48}\lambda \\ V(S_2) \longrightarrow \frac{1}{6} + \frac{\lambda}{3} \end{array}\right. .$$

In fact, the above equilibrium holds for parameters varying within a range such that, approximately, $\delta \geq \overline{\delta} \cong 0.85$ and $0.55 \cong \underline{\lambda} \leq \lambda \leq \overline{\lambda} \cong 0.65$.

Proposition 20 *For not extremely high values both of the discount rate $\delta \leq \overline{\overline{\delta}}$ and for mildly high values of the reservation price $\underline{\underline{\lambda}} \leq \lambda \leq \overline{\overline{\lambda}}$, there exists a PSSPN equilibrium* AS3 *of the negotiation game in the asymmetric strong network where*

- *B_1, whenever is selected to make offer, proposes both sellers a price $p^*_{B_1} = \delta V(S_1)$ which, in the response phase, is accepted only by S_1, while S_2 rejects it*
- *B_2, whenever is selected to make offer, proposes S_2 a price $p^*_{B_2} = \delta V(S_2)$ which, in the response phase, is accepted by S_2*
- *S_1, whenever is selected to make offer, proposes B_1 a price $p^*_{S_1} = 1 - \delta W(B_1)$, which, in the response phase, is accepted by B_1*
- *S_2, whenever is selected to make offer, proposes both buyers a price $p^*_{S_2} = 1 - \frac{\delta}{2}$ which, in the response phase, is accepted by both buyers.*

As a results, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\left\{\begin{array}{c} W(B_1) = \frac{-7\delta^2+8\delta+16}{32(2-\delta)} \\ W(B_2) = \frac{\delta^2-8\delta^2\lambda+2\delta+10\delta\lambda-8+24\lambda}{16(4-\delta)} \\ V(S_1) = \frac{-7\delta^2+4\delta+16}{32(2-\delta)} \\ V(S_2) = \frac{1+\delta\lambda}{4-\delta} - \frac{\delta}{2(4-\delta)} \end{array}\right.$$

which, in the limit case $\delta \longrightarrow 1$, approach the values

$$\left\{\begin{array}{c} W(B_1) \longrightarrow \frac{17}{32} \\ W(B_2) \longrightarrow \frac{13}{24}\lambda - \frac{5}{48} \\ V(S_1) \longrightarrow \frac{13}{32} \\ V(S_2) \longrightarrow \frac{1}{6} + \frac{\lambda}{3} \end{array}\right. .$$

In fact, this equilibrium holds for parameters varying within a range such that, approximately, $\delta \geq \overline{\overline{\delta}} \cong 0.9$ and $0.8 \cong \underline{\underline{\lambda}} \leq \lambda \leq \overline{\overline{\lambda}} \cong 0.9$.

Proposition 21 *For extremely high values of the reservation price $\lambda \geq \widetilde{\lambda}$, there exists a PSSPN equilibrium* AS4 *of the negotiation game in the asymmetric strong network where*

- B_1, *whenever is selected to make offer, proposes both sellers a price* $p^*_{B_1} = \delta V(S_1)$ *which, in the response phase, is accepted only by* S_1*, while* S_2 *rejects it*

- B_2, *whenever is selected to make offer, proposes* S_2 *a price* $p^*_{B_2} = \delta V(S_2)$ *which, in the response phase, is accepted by* S_2

- S_1, *whenever is selected to make offer, proposes* B_1 *a price* $p^*_{S_1} = 1 - \delta W(B_1)$*, which, in the response phase, is accepted by* B_1

- S_2, *whenever is selected to make offer, proposes both buyers a price* $p^*_{S_2} = \lambda - \delta W(B_2)$ *which, in the response phase, is accepted only by* B_2*, while* B_1 *rejects it.*

As a results, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{1+\delta}{4} \\ W(B_2) = \frac{1+\delta}{4}\lambda \\ V(S_1) = \frac{1+\delta}{4} \\ V(S_2) = \frac{1+\delta}{4}\lambda \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$*, of course approach the values* $\begin{cases} W(B_1) \longrightarrow \frac{1}{2} \\ W(B_2) \longrightarrow \frac{\lambda}{2} \\ V(S_1) \longrightarrow \frac{1}{2} \\ V(S_2) \longrightarrow \frac{\lambda}{2} \end{cases}$.

In particular, this equilibrium holds for extremely high reservation price by the weak buyer, approximately, $\lambda \geq \widetilde{\lambda} \cong 0.9$.

Proposition 22 *For low values of the reservation price* $\lambda \leq \widetilde{\widetilde{\lambda}}$ *and high enough values of the discount rate* $\delta \geq \widetilde{\widetilde{\delta}}$ *there exists a PSSPN equilibrium* AS5 *of the negotiation game in the asymmetric strong network where*

- B_1, *whenever is selected to make offer, proposes both sellers a price* $p^*_{B_1} = \delta V(S_2)$ *which, in the response phase, is accepted by both sellers, while they both reject any other offer* $p_{B_1} \in \left[\delta V(S_2), \frac{\delta\lambda}{2}\right)$.

- B_2, *whenever is selected to make offer, proposes* S_2 *a price* $p^*_{B_2} = \delta V(S_2)$ *which, in the response phase, is accepted by* S_2

- S_1, *whenever is selected to make offer, proposes* B_1 *a price* $p^*_{S_1} = 1 - \delta W(B_1)$*, which, in the response phase, is accepted by* B_1

- S_2, *whenever is selected to make offer, proposes both buyers a price* $p^*_{S_2} = 1 - \delta W(B_1)$ *which, in the response phase, is accepted only by* B_1*, while* B_2 *rejects it.*

As a results, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{-3\delta^2(1+\lambda)-2\delta+16}{8(\delta^2+8-7\delta)} \\ W(B_2) = \frac{\delta^3+6\delta^3\lambda+6\delta^2-23\delta^2\lambda\ -8\delta-4\delta\lambda+32\lambda}{16(\delta^2+8-7\delta)} \\ V(S_1) = \frac{3\delta^3-12\delta^2+3\delta^2\lambda\ -16\delta+32}{16(\delta^2+8-7\delta)} \\ V(S_2) = \frac{-\delta^2-3\delta^2\lambda-6\delta+6\delta\lambda+8}{4(\delta^2+8-7\delta)} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values

$$\begin{cases} W(B_1) \longrightarrow \frac{11}{16} - \frac{3}{16}\lambda \\ W(B_2) \longrightarrow \frac{11}{32}\lambda - \frac{1}{32} \\ V(S_1) \longrightarrow \frac{7}{32} + \frac{3}{32}\lambda \\ V(S_2) \longrightarrow \frac{1}{8} + \frac{3}{8}\lambda \end{cases}$$

In fact, this equilibrium holds for parameters varying within a range such that, approximately, $\delta \geq \tilde{\tilde{\delta}} \cong 0.85$ and $\lambda \leq \tilde{\tilde{\lambda}} \cong 0.3$.

Proposition 23 *For very low values of the reservation price $\lambda \leq \tilde{\tilde{\tilde{\lambda}}}$ there exists a PSSPN equilibrium* AS6 *of the negotiation game in the asymmetric strong network where*

- *B_1, whenever is selected to make offer, proposes both sellers a price $p^*_{B_1} = \delta V(S_1)$ which, in the response phase, is accepted by both sellers, while they both reject any other offer $p_{B_1} \in \left[\delta V(S_1),\ \frac{\delta\lambda}{2}\right)$.*
- *B_2, whenever is selected to make offer, proposes S_2 a price $p^*_{B_2} = \delta V(S_2)$ which, in the response phase, is accepted by S_2*
- *S_1, whenever is selected to make offer, proposes B_1 a price $p^*_{S_1} = 1 - \delta W(B_1)$, which, in the response phase, is accepted by B_1*
- *S_2, whenever is selected to make offer, proposes both buyers a price $p^*_{S_2} = 1 - \delta W(B_1)$ which, in the response phase, is accepted only by B_1, while B_2 rejects it.*

As a results, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{-3\delta^2+2\delta+16}{4(5\delta-8)} \\ W(B_2) = \frac{\delta^3+15\delta^3\lambda+12\delta^2-44\delta^2\lambda-16\delta-8\delta\lambda+64\lambda}{8(5\delta^2-28\delta+32)} \\ V(S_1) = \frac{-3\delta^2-2\delta+8}{4(5\delta-8)} \\ V(S_2) = \frac{-2\delta^2-15\delta^2\lambda-24\delta+24\delta\lambda+32}{4(5\delta^2-28\delta+32)} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values $\begin{cases} W(B_1) \longrightarrow \frac{5}{8} \\ W(B_2) \longrightarrow \frac{3}{8}\lambda - \frac{1}{24} \\ V(S_1) \longrightarrow \frac{1}{4} \\ V(S_2) \longrightarrow \frac{1}{6} + \frac{\lambda}{4} \end{cases}$

In particular, this equilibrium holds for parameters varying within a range such that, approximately, $\lambda \leq \tilde{\tilde{\tilde{\lambda}}} \cong 0.25$.

Proposition 24 *For medium values of the reservation price* $\widehat{\lambda} \leq \lambda \leq \widehat{\widehat{\lambda}}$ *and high enough values of the discount rate* $\delta \geq \widehat{\delta}$ *there exists a PSSPN equilibrium* AS7 *of the negotiation game in the asymmetric strong network where*

- B_1*, whenever is selected to make offer, proposes both sellers a price* $p^*_{B_1} = \frac{\delta\lambda}{2}$ *which, in the response phase, is accepted by both sellers, while they both accept any other offer* $p_{B_1} \in \left[\min\left\{\delta V(S_1),\ \delta V(S_2)\right\},\ \frac{\delta\lambda}{2}\right)$.
- B_2*, whenever is selected to make offer, proposes* S_2 *a price* $p^*_{B_2} = \delta V(S_2)$ *which, in the response phase, is accepted by* S_2
- S_1*, whenever is selected to make offer, proposes* B_1 *a price* $p^*_{S_1} = 1 - \delta W(B_1)$*, which, in the response phase, is accepted by* B_1
- S_2*, whenever is selected to make offer, proposes both buyers a price* $p^*_{S_2} = 1 - \delta W(B_1)$ *which, in the response phase, is accepted by both buyers, while they both reject any other offer* $p_{S_2} \in \left[1 - \delta W(B_1),\ \min\left\{\lambda,\ 1 - \frac{\delta}{2}\right\}\right)$.

As a results, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{-3\delta+2\delta\lambda-4}{2(3\delta-8)} \\ W(B_2) = \frac{3\delta^3+19\delta^3\lambda+22\delta^2-82\delta^2\lambda+24\delta-24\delta\lambda+192\lambda-64}{16(3\delta^2-20\delta+32)} \\ V(S_1) = \frac{15\delta^2-\delta^2\lambda-4\delta-8\delta\lambda+32}{16(3\delta-8)} \\ V(S_2) = \frac{-3\delta^2-4\delta^2\lambda-10\delta+16\delta\lambda-16}{2(3\delta^2-20\delta+32)} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$*, approach the values* $\begin{cases} W(B_1) \longrightarrow \frac{7}{10} - \frac{\lambda}{5} \\ W(B_2) \longrightarrow \frac{7}{16}\lambda - \frac{1}{16} \\ V(S_1) \longrightarrow \frac{21}{80} + \frac{9}{80}\lambda \\ V(S_2) \longrightarrow \frac{1}{10} + \frac{2}{5}\lambda \end{cases}$

In fact, this equilibrium holds for parameters varying within a range such that, approximately, $\delta \geq \widehat{\delta} \cong 0.85$ and $0.45 \cong \widehat{\lambda} \leq \lambda \leq \widehat{\widehat{\lambda}} \cong 0.65$.

Proposition 25 *Finally, either for medium values of the reservation price* $\overline{\widehat{\lambda}} \leq \lambda \leq \overline{\widehat{\widehat{\lambda}}}$ *or for both mildly high values of the reservation price* $\overline{\widehat{\widehat{\lambda}}} \leq \lambda \leq \widehat{\widehat{\widehat{\lambda}}}$ *and very high values of the discount rate* $\delta \geq \widehat{\widehat{\widehat{\delta}}}$ *there exists a PSSPN equilibrium* AS8 *of the negotiation game in the asymmetric strong network where*

- B_1*, whenever is selected to make offer, proposes both sellers a price* $p^*_{B_1} = \delta V(S_1)$ *which, in the response phase, is accepted by* S_1 *only, while* S_2 *rejects it*

- B_2*, whenever is selected to make offer, proposes* S_2 *a price* $p^*_{B_2} = \delta V(S_2)$ *which, in the response phase, is accepted by* S_2
- S_1*, whenever is selected to make offer, proposes* B_1 *a price* $p^*_{S_1} = 1 - \delta W(B_1)$*, which, in the response phase, is accepted by* B_1
- S_2*, whenever is selected to make offer, proposes both buyers a price* $p^*_{S_2} = 1 - \delta W(B_1)$ *which, in the response phase, is accepted by both buyers, while they both reject any other offer* $p_{S_2} \in \left[1 - \delta W(B_1),\ \min\left\{\lambda,\, 1 - \frac{\delta}{2}\right\}\right)$.

As a results, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{-3\delta^2+2\delta+8}{\delta^2-20\delta+32} \\ W(B_2) = \frac{3\delta^4+4\delta^4\lambda+11\delta^3-85\delta^3\lambda-32\delta^2+216\delta^2\lambda-80\delta+80\delta\lambda-384\lambda+128}{8(\delta^3-24\delta^2+112\delta-128)} \\ V(S_1) = \frac{-15\delta^2+4\delta+32}{4(\delta^2-20\delta+32)} \\ V(S_2) = \frac{-3\delta^3-\delta^3\lambda+\delta^2+20\delta^2\lambda+28\delta-32\delta\lambda+32}{\delta^3-24\delta^2+112\delta-128} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$*, approach the values* $\begin{cases} W(B_1) \longrightarrow \frac{7}{13} \\ W(B_2) \longrightarrow \frac{13}{24}\lambda - \frac{5}{52} \\ V(S_1) \longrightarrow \frac{21}{52} \\ V(S_2) \longrightarrow \frac{2}{13} + \frac{\lambda}{3} \end{cases}$

This equilibrium, finally, holds for parameters varying within two ranges: either for, approximately, any $0.5 \cong \overline{\hat{\lambda}} \leq \lambda \leq \overline{\hat{\hat{\lambda}}} \cong 0.65$ or, approximately, for $\delta \geq \hat{\hat{\delta}} \cong 0.95$ and $0.65 \cong \overline{\overline{\hat{\hat{\lambda}}}} \leq \lambda \leq \hat{\hat{\hat{\lambda}}} \cong 0.75$.

Therefore, the negotiations within an *asymmetric strong* network show a rich multiplicity of equilibria. In fact, for low levels of λ as many as three equilibria are defined: *AS6*, up to $\lambda \cong 0.25$, *AS5*, up to $\lambda \cong 0.3$, *AS1*, up to $\lambda \cong 0.4$. Medium values of λ are instead cover, in different ranges, by equilibria *AS7*, *AS2* and *AS8*. For high levels of λ equilibrium *AS3* exists, while for extremely high values, equilibrium *AS4* is defined.

Furthermore, it can be reckoned that, in equilibrium offers are often accepted by both traders on a side of the market, so that a random tie-break takes place. This is the case for all but one equilibria, with the only exception of *AS4*. As a consequence, in some of such equilibria, then, trade occurs with delay, while in other equilibria, with half probability the least connected traders end up leaving the market without trading at all. In the former case, moreover, *different* prices usually form in the thin market. Therefore, *inefficiency*, both in terms of delay in trade and of impossibility to achieve full exploitation of all the potential surplus from trade, can not be ruled out in an *asymmetric strong* network.

4.7 The *Complete* Network

In the complete bipartite network (g), each buyer is connected with both the sellers. The existence of links between all the possible buyer-seller pairs should enable the exploitation of any potential trade in the thin market.

The case of a bilateral duopoly connected by a complete bipartite graph corresponds to the market structure studied by the *public offers* model in Chatterjee and Dutta (1998). In fact, in such a case, our model of negotiations differs from the latter only in two, though crucial, aspects. First, while in Chatterjee and Dutta (1998) the bargaining procedure follows an alternating order of proposers between the supply and the demand side, in our model the negotiation entails a random order of proposers with identical odds for any trader.

Second, in our model there is an explicit formalization of the strategic interaction occurring between traders of the same side of the market competing when responding to an offer. In fact, unlike Chatterjee and Dutta (1998), we explicitly model a simultaneous moves 2×2 game between buyers (sellers) in the price response phase.

Furthermore, our model of negotiations in such a case may be seen as an extension of the model by Corominas-Bosch (2004) to the case of heterogeneous buyers and random selection of traders (rather than sides of the market).

Our model of negotiations among traders in such a complete bipartite graph implies that after a single buyer and a single seller have been matched to trade, have left the market and all their links have been removed, the two remaining traders have *always* the chance to stay in the market to carry on further negotiations. In fact they automatically access a standard bilateral bargaining with random order of proposers, whose *PSSPN* equilibrium payoffs are the one described by a standard Rubinstein model: if the strong buyer is still in the market, each of the remaining traders expects from the following bargaining rounds a surplus of $\frac{1}{2}$, alternatively, whenever the weak buyer is the one left, they expect a surplus of $\frac{\lambda}{2}$.

Alike for the above networks where the buyers are in a symmetric position concerning the number of accessible connections, also in the complete connected graph we will take advantage of *Condition k*,

$$\delta \left[W \left(B_1 \right) - W \left(B_1 \right) \right] \leq 1 - \lambda,$$

by which the discounted value of the difference in the buyers' expected continuation payoffs can never exceed the relative distance between their primitive reservation prices. The positive difference $1 - \lambda$ thus represents an upper bound meeting the relative advantage on the surplus expected by the strong buyer, and avoids that the latter explosively diverges beyond any sensible initial dissimilarity on the traders' *ex-ante* bargaining positions.[16]

Hence, direct computation and numerical simulations allow us to state that, for a I-equilibrium arising when B_2 has been selected to make offers where continuation payoffs are such that $V(S_1) \leq V(S_2)$, only three compatible equilibrium outcomes can emerge.

[16] In fact, in a complete network both buyers can access an identical number of partners therefore being in *symmetric ex-ante* bargaining position. Thus, it is with no lack of generality and likelihood of the analysis that we constrain the strong buyer's continuation payoff to not differ from the weak buyer's by more than some upper bound, equal to the original difference in their reservation prices, in the way expressed by *Condition k*.

Proposition 26 *For high reservation price $\lambda \geq \overline{\lambda}$, there exists a PSSPN equilibrium* C1 *of the negotiation game in the complete network where*

- B_1*, whenever is selected to make offer, proposes both sellers a price* $p^*_{B_1} = \delta V(S_1)$ *which, in the response phase, is accepted by* S_1
- B_2*, whenever is selected to make offer, proposes both sellers a price* $p^*_{B_2} = \delta V(S_1)$ *which, in the response phase, is accepted by* S_1
- S_1*, whenever is selected to make offer, proposes both buyers a price* $p^*_{S_1} = 1 - \delta W(B_1)$*, which, in the response phase, is accepted by* B_1
- S_2*, whenever is selected to make offer, proposes both buyers a price* $p^*_{S_2} = 1 - \delta W(B_1)$ *which, in the response phase, is accepted by* B_1*.*

Consequently, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{-2\delta^2-\delta^2\lambda-2\delta+8}{2(3\delta^2-16\delta+16)} \\ W(B_2) = \frac{\delta^3+11\delta^3\lambda+6\delta^2-46\delta^2\lambda-8\delta+16\delta\lambda+32\lambda}{8(3\delta^2-16\delta+16)} \\ V(S_1) = \frac{-\delta^2-2\delta^2\lambda-6\delta+4\delta\lambda+8}{2(3\delta^2-16\delta+16)} \\ V(S_2) = \frac{5\delta^3+7\delta^3\lambda-8\delta^2-32\delta^2\lambda-24\delta+32\delta\lambda+32}{8(3\delta^2-16\delta+16)} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values

$$\begin{cases} W(B_1) \longrightarrow \frac{2}{3} - \frac{\lambda}{6} \\ W(B_2) \longrightarrow \frac{13}{24}\lambda - \frac{1}{24} \\ V(S_1) \longrightarrow \frac{1}{6} + \frac{\lambda}{3} \\ V(S_2) \longrightarrow \frac{5}{24} + \frac{7}{24}\lambda \end{cases}.$$

In fact, the above equilibrium holds for parameters varying within a range such that, approximately, $\lambda \geq \overline{\lambda} \cong 0.8$.

Proposition 27 *For not too high values of the reservation price $\lambda \leq \widetilde{\lambda}$, there exists a PSSPN equilibrium* C2 *of the negotiation game in the complete network where*

- B_1*, whenever is selected to make offer, proposes both sellers a price* $p^*_{B_1} = \frac{\delta\lambda}{2}$ *which, in the response phase, is accepted by both* S_1 *and* S_2*, who would accept any offer in the range* $\left[\frac{\delta\lambda}{2}, \delta V(S_1)\right)$
- B_2*, whenever is selected to make offer, proposes both sellers a price* $p^*_{B_2} = \delta V(S_1)$ *which, in the response phase, is accepted by* S_1
- S_1*, whenever is selected to make offer, proposes both buyers a price* $p^*_{S_1} = 1 - \delta W(B_1)$*, which, in the response phase, is accepted by* B_1
- S_2*, whenever is selected to make offer, proposes both buyers a price* $p^*_{S_2} = 1 - \delta W(B_1)$ *which, in the response phase, is accepted by* B_1*.*

As a result, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{2+\delta(1-\lambda)}{4(2-\delta)} \\ W(B_2) = \frac{\delta^3+9\delta^3\lambda+6\delta^2-40\delta^2\lambda-8\delta+24\delta\lambda+32\lambda}{16(\delta^2-6\delta+8)} \\ V(S_1) = \frac{-\delta^2-3\delta^2\lambda-6\delta+8\delta\lambda+8}{4(\delta^2-6\delta+8)} \\ V(S_2) = \frac{3\delta^2(1+\lambda)+2\delta-8\delta\lambda-8}{16(\delta-2)} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values

$$\begin{cases} W(B_1) \longrightarrow \frac{3}{4} - \frac{\lambda}{4} \\ W(B_2) \longrightarrow \frac{25}{48}\lambda - \frac{1}{48} \\ V(S_1) \longrightarrow \frac{1}{12} + \frac{5}{12}\lambda \\ V(S_2) \longrightarrow \frac{3}{16} + \frac{5}{16}\lambda \end{cases}.$$

In fact, the above equilibrium holds for parameters varying within a range such that, approximately, $\lambda \leq \widetilde{\lambda} \cong 0.8$.

Proposition 28 *For high values of the discount rate $\delta \geq \overline{\overline{\delta}}$ and low values of the reservation price $\lambda \leq \overline{\overline{\lambda}}$, there exists a PSSPN equilibrium* C3 *of the negotiation game in the complete network where*

- *B_1, whenever is selected to make offer, proposes both sellers a price $p^*_{B_1} = \delta V(S_1)$ which, in the response phase, is accepted by both S_1 and S_2, who would reject any offer in the range $\left[\frac{\delta\lambda}{2}, \delta V(S_1)\right)$*
- *B_2, whenever is selected to make offer, proposes both sellers a price $p^*_{B_2} = \delta V(S_1)$ which, in the response phase, is accepted by S_1*
- *S_1, whenever is selected to make offer, proposes both buyers a price $p^*_{S_1} = 1 - \delta W(B_1)$, which, in the response phase, is accepted by B_1*
- *S_2, whenever is selected to make offer, proposes both buyers a price $p^*_{S_2} = 1 - \delta W(B_1)$ which, in the response phase, is accepted by B_1.*

As a result, the traders' expected continuation payoffs by entering a new stage of the negotiation game are

$$\begin{cases} W(B_1) = \frac{-3\delta^2(1+\lambda)-2\delta+16}{8(\delta^2-7\delta+8)} \\ W(B_2) = \frac{\delta^3+9\delta^3\lambda+6\delta^2-44\delta^2\lambda-8\delta+20\delta\lambda+32\lambda}{16(\delta^2-7\delta+8)} \\ V(S_1) = \frac{-\delta^2-3\delta^2\lambda-6\delta+6\delta\lambda+8}{4(\delta^2-7\delta+8)} \\ V(S_2) = \frac{3\delta^3(1+\lambda)-12\delta^2-18\delta^2\lambda-16\delta+24\delta\lambda+32}{16(\delta^2-7\delta+8)} \end{cases}$$

which, in the limit case $\delta \longrightarrow 1$, approach the values

$$\begin{cases} W(B_1) \longrightarrow \frac{11}{16} - \frac{3}{16}\lambda \\ W(B_2) \longrightarrow \frac{17}{32}\lambda - \frac{1}{32} \\ V(S_1) \longrightarrow \frac{1}{8} + \frac{3}{8}\lambda \\ V(S_2) \longrightarrow \frac{7}{32} + \frac{9}{32}\lambda \end{cases}.$$

In fact, this equilibrium holds for parameters varying within a range such that, approximately, $\delta \geq \overline{\overline{\delta}} \cong 0.85$ and $\lambda \leq \overline{\overline{\lambda}} \cong 0.5$.

Of course, there exist *three other equilibria* corresponding to the case a II-equilibrium arises when the weak buyer has been selected to make offers in which continuation payoffs are such that $V(S_2) \leq V(S_1)$. Such perfectly symmetric equilibria are in fact immediately obtained by switching the labelling for the two sellers. Note that the payoffs for the two buyers remain unaltered.

5 Comparisons across different networks

In this last section, we aim at drawing some preliminary conclusions on the impact of network structures on the bargaining process among traders in a gas bilateral duopoly. Here we conduct our analysis taking as given a particular network architecture and we compare traders' equilibrium payoffs across network configuration.

5.1 Equilibria

In order to carry on some simple comparisons we first need to be able to rank the different equilibria for any network in some sensible way. In fact, we can order all the possible equilibria across different networks in a $(0, 1)$ square box having the weak buyer's reservation price λ on its horizontal axis and the common discount factor δ on its vertical one. Is then possible to draw all the (λ, δ) regions where any equilibrium for a given network is defined and thus see which equilibria are indeed comparable across different architectures. The only drawback of such procedure is that the final graphic representation of such comparisons turns out to be truly cumbersome. However, for providing a hint of our main qualitative findings here we draw an overall picture of how the equilibria can be ranked across networks just according to the values of the weak buyer's reservation price λ, for a fixed level of the discount factor $\delta = 0.85$. Qualitative results, however, are identical for any other realistical value of the impatience rate.

Clearly, some network configurations only present a single equilibrium for all values of λ. This is the case not only for the *exclusive trade* network - and the nested *strong* and *weak couple* architectures - but also for the *asymmetric weak* architecture. On the other hand, while both the B_1 and B_2-*short side* structures show two coexisting equilibria, the *supply-short-side* network presents one equilibrium for values of λ rather high and another for all the remaining values.

Similar is the case of the *complete* network where only equilibrium *C1* is defined for high values of λ, equilibrium *C2* for medium levels, and both equilibria *C2* and *C3* exist for low values of λ.

Even richer is the *asymmetric strong* network. In fact, for low levels of λ as many as three equilibria are defined: *AS6*, up to $\lambda \cong 0.25$, *AS5*, up to $\lambda \cong 0.3$, *AS1*, up to $\lambda \cong 0.4$. Medium values of λ are instead cover, in different ranges,

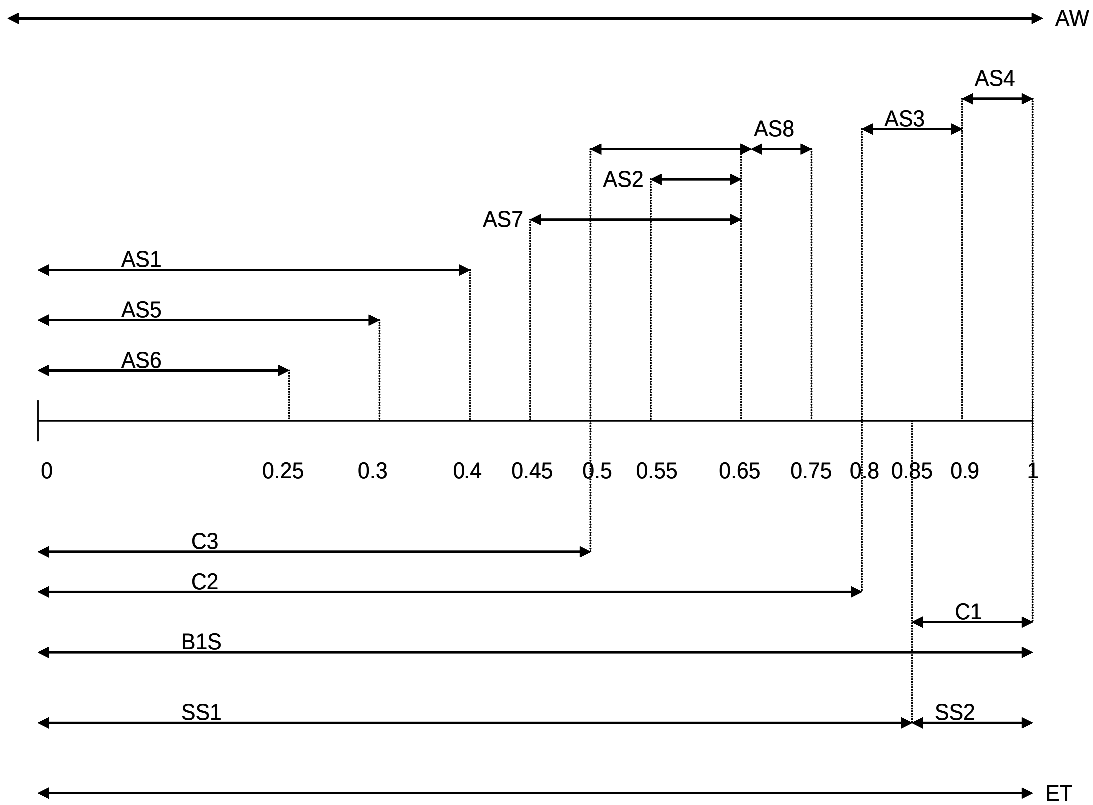

Figure 3: Comparison between equilibria across networks

by equilibria *AS7*, *AS2* and *AS8*. For high levels of λ equilibrium *AS3* exists, while for extremely high values, equilibrium *AS4* is defined.

Some general considerations are in order. In fact, by looking at the most salient features of the above equilibria, it can be reckoned that, offers are often accepted by both traders on a side of the market, so that a random tie-break takes place. This is the case for both equilibria in the B_1 and B_2-*short side* architectures, for two out of three equilibria in the *supply-short-side* network, for all but one equilibria in the *asymmetric strong* network (excluding *AS4*) and even in two of the three equilibria in the *complete* network. In some of such equilibria, then, trade occurs with delay, while in other equilibria, with half probability the least connected traders end up leaving the market without trading at all. In the former case, moreover, *different* prices usually form in the thin market. Therefore, *inefficiency*, both in terms of delay in trade and of impossibility to achieve full exploitation of all the potential surplus from trade, can not be ruled out from the above described equilibria.

Moreover, comparisons are possible across equilibria for different network structures which are defined within compatible values of the primitive parameters δ and λ. In the following, we discuss some of the main results we have found out by comparing, by means of direct computations and simulations, the equilibrium payoffs of the traders across compatible equilibria in different networks. As the primary interest of the paper lies in the investigation of buyers' bargaining power and the model itself is in fact symmetric between sellers, we have limited our comparisons to the payoffs of strong and the weak buyer. Analogous simulations and comparing procedures, however, can be easily extended to sellers too.

In fact, there are two main conjectures one can be interested in confirming or rejecting in view of direct comparisons. First, one can guess that B_2, who is clearly in weaker original conditions to start negotiations, if embedded in favourable network configurations, should be in theory able to counterbalance, at some extent, the overwhelming natural advantage of the strong buyer. To seek confirmation of such a guess, one should look at the expected equilibrium payoffs in a *given network* structure to compare the surplus experienced by the two buyers.

It is immediate to check, however, that such intuitive *guess is rejected by the model's predicted payoffs*. In fact, the only network architectures where B_2 is unambiguously better off than the strong buyer are the obvious cases of the *weak-couple* and the B_2-*short side* network. While it cannot surprise that the strong buyer indeed experiences sistematically higher surplus in an *asymmetric strong* network, this does sound less obvious for the other two salient connected networks. However, direct comparisons clearly show that the weak buyer is always worse off than B_1 even in the unique equilibrium of the *asymmetric weak* structure. Moreover, it turns out that also in a complete network the strong buyer is always strictly better off than the weak, except in the extraordinary case of values of λ so extremely high to approach the limit case $\lambda \rightarrow 1$ of symmetric buyers. In such a case, the relative counterbalance of B_2's surplus seems to be due not only to the closeness of the reservation prices, but also to the fact that in the *C1* equilibrium the weak buyer accesses bilateral negotiations more often than B_1.

The second conjecture, instead, is related to the surplus of a *given buyer* across different networks. In fact, one can intuitively argue that any buyer should always be in a better trading position toward the sellers whenever he is located in a more connected node than the competing buyer. In other words, intuition may suggest that the strong buyer would manage to extract better trading opportunities from being not only, clearly, in a *complete* or *asymmetric strong* network rather than in an *asymmetric weak*, but also in a *asymmetric strong* rather than a *complete* structure. The idea, in fact, is that being connected with more potential partners than the competitor enables a player to enjoy better trading conditions than the rival.

To confirm or discard such a conjecture we need to compare the payoffs for each buyers *across* different network architectures.

5.2 Strong buyer

We start with the *strong buyer*. By direct computations and numerical simulations we find out several results of interest. First, obviously, the highest surplus experienced by the strong buyer is the one attainable in a B_1-*short-side* network, while the worst is clearly the equilibrium payoff in a B_2-*short-side* network. Secondly, the surplus faced within a *strong couple* network is equivalent, for very *low* values of λ, to the equilibrium payoff from the *SS3* equilibrium payoff in a *supply-short-side* network. Third, interestingly, the equilibrium surplus earned by the strong buyer within the *exclusive trade* exactly corresponds to the one gained in equilibrium within the *asymmetric weak* network. Moreover, by direct comparisons, it turns out that such surplus is always *strictly lower* than what the strong buyer can obtain in equilibrium from bargaining either in an *asymmetric strong*, or in a *complete*, so that the following holds:

$$\Pi\left(B_1\right)_{ET} \equiv \Pi\left(B_1\right)_{AW} < \left\{\Pi\left(B_1\right)_{AS}\ ,\ \Pi\left(B_1\right)_{C}\right\} \leq \Pi\left(B_1\right)_{B_1-Short}$$

This confirms, therefore, that any connected bipartite graph makes B_1 strictly better off than within an exclusive bilateral negotiation thus providing a natural incentive to the strong buyer to avoid locking in an exclusive partnership and to rather prefer to be embedded into more connected architectures.

Moreover, as intuition would suggest, it turns out that B_1 always enjoy strictly higher surplus in an *exclusive trade* network than in any equilibrium of the *supply-short-side* architecture, unless for low values of δ when λ is *extremely low* (*SS2*). Thus, it can be checked that, a fortiori, any from the *complete*, the *asymmetric strong* and the *asymmetric weak* network ensures to the strong buyer at least as large equilibrium surplus than the *supply-short-side* structure:

$$\Pi\left(B_1\right)_{Supply-Short} \leq \Pi\left(B_1\right)_{ET} \equiv \Pi\left(B_1\right)_{AW} < \left\{\Pi\left(B_1\right)_{AS}\ ,\ \Pi\left(B_1\right)_{C}\right\}$$

Finally, it is possible to directly compare and rank the equilibrium payoffs which the completely connected graphs convey to the strong buyer. Our findings from computations and simulations are rather interesting.

First, as already seen, it clearly turns out that both in an *asymmetric strong* and in a *complete* network B_1 gets equilibrium payoffs never lower than in *asymmetric weak*. More precisely, the strong buyer is always strictly better off in an *asymmetric strong* network, unless when λ is extremely high, in which case he earns the same surplus in both architectures.

Furthermore, we can carry on a direct check of our second conjecture. In fact, direct computations reject the hypothesis that the strong buyer would always be better off in an *asymmetric strong* network in which he would enjoy more trading links than the weak competitor. Indeed, while for λ high enough B_1 is unambiguosly better off within an *asymmetric strong* network, this is no longer true for lower values of the weak buyer's reservation price: at the contrary, while for extremely low values of λ comparisons among equilibria payoffs are ambiguous, for low and intermediate levels of λ the strong buyer is always strictly better off within a *complete* network.

To shed some light on this surprising result, we provide a tentative explanation. In fact, consider an *asymmetric strong* network where the weak buyer, characterized by a low reservation price, is exclusively linked with seller S_2. Intuitively, the fact that is linked with an exclusive relationship with the weak buyer provides seller S_2 a *safe outside option* she can always rely on, in the sense that, whenever the weak buyer is selected to make offers, S_2 benefits from having an exclusive partnership with B_2 in terms of high trading prices. Hence, the existence of such alternative trading opportunity implies that, when bargaining with B_1, seller S_2 would never accept any proposal making her worse off with respect to such outside option.

In other words, the possibility of exclusive dealing with B_2 indirectly provides a *lower bound* for competition between the two sellers when fighting for serving the strong buyer. In fact, even S_1 knows that S_2 would never accept from the strong buyer any price below a proposal making her indifferent to what she can get from the weak buyer. Therefore, is common knowledge that S_2 would never exert any competitive pressure below that threshold. However, even S_1 has no interest in proposing the strong buyer something more favourable than S_2's outside option. Thus, both sellers have no incentives to compete too fiercely for the strong buyer, by proposing prices below what S_2 can get from the weak buyer. The existence of such implicit lower bound for sellers' competition clearly hurts the strong buyer, as he is not able to extract larger trading surplus from negotiations. This is because, when making offers to B_1, both sellers are likely to ask something comparable to what S_2 can get from the weak buyer.

Therefore, to avoid being hurt by such *price floor* limit to competition, the strong buyer may be better off in a *complete* network. In fact, as long as B_2's reservation price is kept on low or medium levels, B_1 prefers the weak buyer takes part into negotiations from a fully connected, rather than in a less central node. From this point of view, it seems that the strongest competing purchaser genuinely prefers a market structure where communication and trading opportunities are easier and less constrained to one with protected exclusive partnerships. Asymmetry across reservation prices is sharp enough to guarantee the strong buyer getting larger surplus than in an *asymmetric strong* network anyway. This result seems counterintuitive, though, and is susceptible of interesting regulation policy implications.[17]

There is a limit, however, to such B_1's preference towards the *complete* network. In fact, as λ approaches high levels, buyers become more similar in terms of attractiveness for the sellers. Thus, while in a *complete* network, competition to serve the strong buyer becomes less fierce as both sellers can sustain high prices selling to the weak buyer, in the *asymmetric strong* network, B_1 is able to take advantage of the possibility that S_2 exclusively deals with B_2, by obtaining from S_1 prices similar to the one emerging in bilateral negotiations, which, in turn, are now significantly lower than λ.

5.3 Weak buyer

Such a preference for bargaining in a complete architecture is partially common to the *weak buyer* too. Clearly, it immediately turns out that the weak buyer is always striclty better off within a *complete* rather than in an *asymmetric strong* network. Of course, also all the other intuitive results are confirmed for the weak buyer too.[18]

Moreover, interestingly, from direct comparisons it also turns out that B_2 prefers to bargain in a *complete* network only when λ is high enough, while he is better off in an *asymmetric weak* architecture for lower levels of λ. These findings are intuitive too. Infact, better connections can help B_2 to overcome significant disadvantages in the original trading capability of the weak buyer. However, a line of arguments which are the mirror image of the ones discussed for the strong buyer, implies that the protection of a more central node from the competitive pressure of B_1's outside option is no longer a sufficient trading guarantee when this weakness is less pronounced. Therefore, the weak buyer would prefer negotiating in a complete network exactly for levels of λ for which the strong buyer would not.

Moreover, a tension between buyers' interests can be easily reckoned. In fact, the strong buyer prefers to be embedded within a complete network when λ takes low and medium values, while within an asymmetric strong for high levels of λ. On the contrary, the weak buyer prefers to negotiate within an asymmetric weak architecture when λ is low and within a complete network when his reservation price is high.

[17] Can we imagine the italian Antitrust Authority trying to persuade the government and the chairman of *ENI* that *ENI* would make higher profits allowing a small competitor with a more limited portfolio of energetic sources entering the market?

[18] Therefore, B_2 gets its worse equilibrium payoff in a B_1-*short-side* and its best in a B_2-*short-side* network. Again, it turns out that bargaining in an *exclusive trade* network delivers B_2 exactly the same equilibrium payoffs than negotiations in an *asymmetric weak* architecture. Such a positive externality from being better connected than in an exclusive partnership arises, again rather intuitively, also within a *complete network*, but only as λ is high enough.

The emergence of such a prominent conflict of interests among buyers can be regarded as a fascinating prelude to the the investigation of the endogenous strategies of link formation by the traders. As already mentioned, this goal is left for a companion paper.

5.4 Extensions and concluding remarks

We have analyzed the interaction between strategic negotiations and network structures in a bilateral oligopoly with identical sellers and heterogeneous buyers. We have provided a full characterization of all the subgame perfect Nash equilibria in pure and stationary strategies possibly emerging in the negotiations stage in any fixed network architecture. We have then described some salient features of such bargaining equilibria and compare traders' payoffs within and across networks.

The next step, on which we are currently working in a companion paper (Galizzi, 2007), consists on endogenizing the emergence of buyers-sellers bipartite graphs. In fact, it is possible to explicitly model a non-cooperative network formation game in which, given her expected payoff from the negotiations game in any network sructure, any trader, simultaneously and independently, chooses which partners on the other side of the market she wants to be connected. A link is then formed whenever both affected traders have decided to form it, and the corresponding buyers-sellers network consequently emerges. The results obtained insofar, looking at the Pairwise Stable Nash Equilibria (in the spirit of Calvo-Armengol and Ilkilic, 2004) of the endogenous network formation game are encouraging. In fact, only the *complete* and the *asymmetric strong* network emerges as potential candidate equilibria in the network formation game. This, may suggest that also asymmetrically connected graph can represent equilibrium communication structures where decentralized negotiations can take place.

Such analysis could be successfully enriched by including further significant features of thin markets. Above all, two closely related extensions can be immediately specified. A first generalization would be to model *price discrimination* by allowing traders to offer different prices to partners of the opposite side of the market, when making proposals.

The second extension would be to remove the hypothesis of indivisible assets and to generalize the strategic bargaining process both on *price and quantity.* Results in standard price-quantity bilateral bargaining typically present an equilibrium outcome in which quantity are set to a level such to maximize the joint profits, and then prices are strategically bargained in order to share the attained surplus. In interdependent negotiations, however, results are likely to be much more complicated as quantities can be strategically assigned to partners in order to weaken their bargaining position. This conjecture somehow resembles the arguments beyond the richer strategic space available to bidders in auctions of shares.

Incidentally, notice that it is possible to trace a close analogy between both possible extensions and the analysis of bipartite buyers-sellers graphs with *weighted links* along the line already investigated by Bloch and Dutta (2005) for the communication networks.

However, it is worthwhile to conclude this preliminary contribution by underlining the ineluctable emergence of multiple equilibria in the negotiations game. Since multiplicity of equilibria is inherently related to the behavioural strategic interaction of traders in thin markets, the analysis would be extremely enriched if these findings would be verified experimentally[19]. Indeed, an experiment of this model would not only overcome the difficulty of obtaining individual data on strategic behaviour in gas thin markets but, perhaps more importantly, allow for testing the theoretical results under the same controlled conditions as the theory itself. Indeed, experimental testing would allow one to (quoting Hey, 1991) *"...test whether the theory is correct under the "ceteris paribus" conditions and whether the theory survives the transition from the world of the theory to the... real world – the world in which data is gathered "*. This remains indeed the final goal of our current research.

6 Appendix: sample of proofs

6.1 Proof of Proposition 7

In order to show the equilibrium offers described in the Proposition, we first argue that the set of potential pure-strategies Nash equilibria turns out to be even narrower under specific combinations of parameters. In order to characterize it in greater detail, we need to consider all the possible, mutually exclusive, ranking of the thresholds which are relevant for the response game.

In fact, all the above sets of conditions just depend upon the relative size of two levels: λ and $1-\delta W(B_1)$. Consider either ranking, henceforth called cases I and II:

$$\begin{array}{rl} I\text{:} & 1-\delta W(B_1) > \lambda \\ II\text{:} & \lambda \geq 1-\delta W(B_1) \end{array}$$

which corrispond on imposing, respectively

$$\begin{array}{rl} I\text{:} & \delta W(B_1) < 1-\lambda \\ II\text{:} & \delta W(B_1) \geq 1-\lambda \end{array}$$

We now look at them in greater detail.

Case *I* Under case I ranking, the buyers' response game can show any of the three potential pure strategies Nash equilibria within some range of the offered price p_{S_2}: either [*Accept* p_{S_2}, *Accept* p_{S_2}] if $p_{S_2} \leq \lambda$, or [*Accept* p_{S_2}, *Reject* p_{S_2}] if $\begin{cases} p_{S_2} \leq 1-\delta W(B_1) \\ p_{S_2} > \lambda \end{cases}$, or, finally, [*Reject* p_{S_2}, *Reject* p_{S_2}] if $p_{S_2} > 1-\delta W(B_1)$.

[19] Except our current project, in fact, as far as we know, the only previous experiment on bargaining in buyers-sellers networks is the one by Charness, Corominas-Bosch and Frechette (2005).

It can be reckoned that the latter conditions are not only such that the whole range of parameters is covered by some equilibrium, but are also mutually exclusive, therefore ruling out any multiplicity of equilibria.

Therefore, the seller S_2's choice in case I is rather straight. She knows that whenever she charges more than $1 - \delta W(B_1)$ both buyers are going to reject her offer: all the traders would enter further negotiations, and she would get nothing but her own continuation payoff $\delta V\left(S_2\right)$.

On the other hand, S_2 knows that, whenever she proposes any price p_{S_2} not higher than $1 - \delta W(B_1)$, that offer would immediately be accepted either by the strong buyer only, if $1 - \delta W(B_1) \geq p_{S_2} > \lambda$, or by both buyers, if $p_{S_2} \leq \lambda$, thus always delivering her a payoff of p_{S_2}. It is also immediate to realize that, among all such possible acceptable offers, $p_{S_2} = 1 - \delta W(B_1)$ is clearly a dominant strategy by seller S_2, as any lower price, still accepted by some buyer, would return her a strictly lower surplus.

Therefore, the optimal decision rule by seller S_2 is clear-cut: in fact, as long as $\delta V\left(S_2\right) \leq 1 - \delta W(B_1)$, the best strategy for S_2 is unambiguously to propose a price offer $p^*_{S_2} = 1 - \delta W(B_1)$, and, otherwise to make any highest, unacceptable, offer. However condition $\delta V\left(S_2\right) \leq 1 - \delta W(B_1)$ is always verified as $\delta W\left(B_1\right) \leq W\left(B_1\right) \leq 1 - \delta V\left(S_2\right)$ comes from the fact that the most the strong buyer can get from negotiations within such a network is what is left out of his surplus once seller S_2 has been paid her continuation payoff. In fact, from *Condition k*, what she can earn from negotiating with the weak buyer is certainly lower as $\lambda - \delta W(B_2) \leq 1 - \delta W(B_1)$.

Thus, as long as condition $\delta W(B_1) < 1 - \lambda$ holds, we can characterize a *PSSP equilibrium* within case I, whenever seller S_2 is selected to make a proposal.

The equilibrium in case I is as follows. In the price offer phase, S_2 offers a price $p^*_{S_2} = 1 - \delta W(B_1)$, which in the response game is accepted in equilibrium by the strong buyer only, while the weak buyer rejects it and leaves consequently the market with a zero payoff. Hence, whenever seller S_2 is selected to make offers, traders' expected payoffs from case I equilibrium are

$$\begin{cases} \Pi\left(B_1\right) = \delta W(B_1) \\ \Pi\left(B_2\right) = 0 \\ \Pi\left(S_1\right) = 0 \\ \Pi\left(S_2\right) = 1 - \delta W(B_1) \end{cases}$$

It is straight to check that the described pure strategies indeed constitute a subgame perfect equilibrium. First, we can check that $\left[\textit{Reject } p^*_{S_2}, \textit{Accept } p^*_{S_2}\right]$ is in fact a pure strategies equilibrium in the response game. Given that B_1 accepts $p^*_{S_2} = 1 - \delta W\left(B_1\right)$, B_2 cannot profitably deviate by also accepting $p^*_{S_2}$ as it would give him a payoff $\frac{1}{2}\left(\lambda - 1 + \delta W\left(B_1\right)\right)$ which, in fact, is never higher than the zero payoff associated to leaving the market, as condition $\delta W(B_1) < 1 - \lambda$ is always satisfied under case I. On the other hand, given that B_2 rejects, the strong buyer would get exactly the same payoff $\delta W\left(B_1\right)$ if he rejects $p^*_{S_2}$ too. Finally, given the buyers' behavior in the response game, seller S_2 has no way to profitably deviate: in fact, if she propose any lower price $p'_{S_2} = p^*_{S_2} - \varepsilon$, still accepted at least by the strong buyer, she would gain a lower surplus, while for any higher, unaccepted, price $p''_{S_2} = p^*_{S_2} + \varepsilon$, she would just get her own continuation value, which is never better as $\delta V(S_2) \leq 1 - \delta W(B_1)$ is always holding. Therefore, the above described is indeed a pure strategies subgame perfect equilibrium.

Case II Under case *II*, instead, the ranking $\lambda \geq 1 - \delta W(B_1)$ clearly implies that the set of conditions $\begin{cases} p_{S_2} \leq 1 - \delta W(B_1) \\ p_{S_2} > \lambda \end{cases}$ can never identify any non-empty range: in fact, the lower bound on the strong buyer's continuation payoff is clearly incompatible with the conditions necessary to meet a $[Accept\ p_{S_2}, Reject\ p_{S_2}]$ pure strategies Nash equilibrium in the response game.

Even in this case the whole range of parameters is clearly covered by some equilibrium. However, in case *II* the conditions describing the Nash equilibria in the response game are no longer mutually exclusive. In fact, whenever S_2 proposes any offer $\widetilde{p}_{S_2}$ such that $\lambda \geq \widetilde{p}_{S_2} > 1 - \delta W(B_1)$ *two alternative Nash equilibria* co-exist in the response game: either *both buyers accept* $\widetilde{p}_{S_2}$, or they *both reject* it.

Therefore, the overall game can be solved by separately considering each of the equilibria in the response subgame following any offer $\lambda \geq \widetilde{p}_{S_2} > 1 - \delta W(B_1)$.

In fact, consider the equilibrium in the response game where both buyers *accept* any offer $\lambda \geq \widetilde{p}_{S_2} > 1 - \delta W(B_1)$, henceforth called *IIa*. The subsequent random tie-break determines which buyer is entitled to trade with S_2 at $\widetilde{p}_{S_2}$ and which one leaves the market with zero payoff. Following an offer $\lambda \geq \widetilde{p}_{S_2} > 1 - \delta W(B_1)$, the overall game can thus be solved by plugging the corresponding traders' equilibrium payoffs into the final nodes at the response phase induced by such a proposal:

$$\begin{cases} \Pi(B_1) = \frac{1}{2}(1 - \widetilde{p}_{S_2}) \\ \Pi(B_2) = \frac{1}{2}(\lambda - \widetilde{p}_{S_2}) \\ \Pi(S_1) = 0 \\ \Pi(S_2) = \widetilde{p}_{S_2} \end{cases}$$

Moving back to the offer phase, consider now S_2's choice in case *IIa*. She knows that whenever she charges any price $\widetilde{p}_{S_2} \leq \lambda$, her proposal is immediately accepted by both buyers, delivering her a payoff of $\widetilde{p}_{S_2}$. Among all such acceptable offers, $\widetilde{p}_{S_2} = \lambda$ is clearly a dominant strategy for seller S_2, as any lower price, still accepted, would return her a strictly lower surplus. Also she knows that, for any other possible offer above λ, the response game shows a unique equilibrium where both buyers reject that offer, all traders enter a new round of negotiations so that her final payoff is just her own continuation value $\delta V(S_2)$.

Therefore, the optimal decision rule by seller S_2 is simple: in fact, as long as $\delta V(S_2) \leq \lambda$, the best strategy for S_2 is unambiguously to propose an acceptable offer $p^*_{S_2} = \lambda$, and, otherwise to make any highest, unacceptable, offer. Hence, in order to ensure acceptable offers in equilibrium, we need to impose explicit restrictions on seller S_2's continuation payoff. Thus, as long as one of the following sets of conditions holds

$$\begin{cases} \delta W(B_1) \geq 1-\lambda \\ \delta V(S_2) \leq \lambda \end{cases}$$

we can characterize a *PSSP equilibrium* within case *IIa*, whenever seller S_2 is selected to make a proposal. The equilibrium in case *IIa* is as follows. In the price offer phase, S_2 offers a price $p^*_{S_2} = \lambda$, which, in the response game, is accepted by both buyers. The subsequent random tie-break selects which buyer is going to trade with seller S_2 at $p^*_{S_2} = \lambda$, and which, instead, leaves the market with no trade and surplus. Hence, whenever S_2 is selected to make offers, traders' expected payoffs from case *IIa* equilibrium are

$$\begin{cases} \Pi(B_1) = \frac{1-\lambda}{2} \\ \Pi(B_2) = 0 \\ \Pi(S_1) = 0 \\ \Pi(S_2) = \lambda. \end{cases}$$

Again, it is quickly checked that the described pure strategies constitute a subgame perfect equilibrium. First, we can check that $\left[\textit{Accept } p^*_{S_2}, \textit{Accept } p^*_{S_2}\right]$ is in fact a pure strategies equilibrium in the response game. Given that B_2 accepts $p^*_{S_2} = \lambda$, B_1 cannot profitably deviate by rejecting $p^*_{S_2}$ as it would return him a zero payoff, lower as $\lambda < 1$. On the other hand, given that B_1 accepts $p^*_{S_2} = \lambda$, if B_2 deviates by rejecting $p^*_{S_2}$ he would get exactly the same zero surplus. Finally, given the buyers' behavior in the response game, seller S_2 has no way to profitably deviate: in fact, if she proposes any lower price $p'_{S_2} = \lambda - \varepsilon$, still accepted by the buyers even if such that $p'_{S_2} > 1 - \delta W(B_1)$, she would gain a lower payoff, while for any, rejected, higher price $p''_{S_2} = \lambda + \varepsilon$, she would just get her own continuation value, which is never better as condition $\delta V(S_2) \leq \lambda$ holds. Therefore, the above described is indeed a pure strategies subgame perfect equilibrium.

On the other hand, consider the alternative case where, following any offer $\lambda \geq \widetilde{p}_{S_2} > 1 - \delta W(B_1)$ from S_2, the unique equilibrium in the response phase is such that both buyers *reject* $\widetilde{p}_{S_2}$, henceforth called case *IIr*. That implies that, whenever S_2 proposes an offer $\lambda \geq \widetilde{p}_{S_2} > 1 - \delta W(B_1)$, all the traders enter a further round of negotiations. After a proposal $\lambda \geq \widetilde{p}_{S_2} > 1 - \delta W(B_1)$, the overall game can thus be solved by plugging the corresponding traders' continuation payoffs into the final nodes at the response phase induced by such an offer.

Moving back to the offer phase, in fact, consider S_2's choice in case *IIr*. He knows that whenever she charges any price $\lambda \geq \widetilde{p}_{S_2} > 1 - \delta W(B_1)$, her proposal is immediately rejected by both buyers, delivering her a payoff of $\delta V(S_2)$. Also she knows that, for any other possible proposal above λ, the response game shows a unique equilibrium where both buyers reject that offer, all traders enter a new round of negotiations so that the final payoff is again her own continuation value $\delta V(S_2)$. Therefore, any price $\widetilde{p}_{S_2} > 1 - \delta W(B_1)$ would return her nothing but her continuation value. On the other hand, she knows that any price at most as high as $1 - \delta W(B_1)$ would be certainly accepted by both buyers. Among all such possible acceptable offers, $p_{S_2} = 1 - \delta W(B_1)$ is clearly a dominant strategy for S_2, as any lower price, still accepted, would return her a strictly lower earning.

Therefore, as long as $\delta V(S_2) \leq 1 - \delta W(B_1)$, the best strategy for B_1 is clearly to propose a price offer $p^*_{B_1} = 1 - \delta W(B_1)$, and, otherwise to make any unacceptable, offer. However, notice that the latter condition is always verified: in fact $\delta W(B_1) \leq W(B_1) \leq 1 - \delta V(S_2)$ holds from the fact that the most the strong buyer can ever get from negotiations is what remains from his potential surplus once he has paid S_2 her own continuation payoff. Hence, as long as condition $\delta W(B_1) \geq 1-\lambda$ holds, we can characterize a *PSSP equilibrium* within case *IIr*, for the case S_2 is selected to make a proposal.

Case *IIr* equilibrium is as follows. S_2 offers a price $p^*_{S_2} = 1 - \delta W(B_1)$, which, in the response game's equilibrium is accepted by both buyers. The subsequent random tie-break selects which buyer is going to trade with S_2 at $p^*_{S_2} = 1 - \delta W(B_1)$, and which, instead, can, eventually, access further bilateral negotiations with S_1, if linked together. Hence, whenever S_2 is selected to make offers, traders' expected payoffs from case *IIr* equilibrium are:

$$\begin{cases} \Pi(B_1) = \frac{1}{2}\left(1 - (1 - \delta W(B_1))\right) = \frac{\delta W(B_1)}{2} \\ \Pi(B_2) = \frac{1}{2}\left(\lambda - (1 - \delta W(B_1))\right) = \frac{\delta W(B_1)}{2} + \frac{\lambda - 1}{2} \\ \Pi(S_1) = 0 \\ \Pi(S_2) = 1 - \delta W(B_1) \end{cases}$$

Checking that the above described strategies are a pure strategies subgame perfect equilibrium is immediate. First, one can check that $\left[Accept\ p^*_{S_2}, Accept\ p^*_{S_2}\right]$ is in fact a pure strategies equilibrium in the response game. Given that B_2 accepts $p^*_{S_2}$, B_1 cannot profitably deviate by rejecting $p^*_{S_2}$, as it would only give him a zero payoff, which is clearly lower than any positive $\frac{\delta W(B_1)}{2}$. On the other hand, given that B_1 accepts $p^*_{S_2} = 1 - \delta W(B_1)$, if B_2 deviates by rejecting $p^*_{S_2}$ he would take a zero payoff which is clearly never better than $\frac{\delta W(B_1)}{2} + \frac{\lambda-1}{2}$ as in case *IIr* it always holds that $\delta W(B_1) \geq 1 - \lambda$. Then, given such equilibrium behavior in the response game, neither seller S_2 has any way to profitably deviate: in fact, if she deviates by proposing any lower price $p'_{S_2} = p^*_{S_2} - \varepsilon$, still accepted, he would clearly get a smaller earning, while if she deviates by any price p''_{S_2} strictly above $p^*_{S_2}$, rejected by both buyers, she would earn her continuation value, which is never better as $\delta V(S_2) \leq 1 - \delta W(B_1)$ is always true. Therefore, the above described is indeed a pure strategies subgame perfect equilibrium.

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References

[1] Bala, V. and S. Goyal (2000): *A Non-Cooperative Model of Network Formation*, Econometrica, 68, 1181-1229.

[2] Bjornerstedt, J. and J. Stennek (2004): *Bilateral Oligopoly*, WZB Discussion Papers, FS IV 01-08, Berlin.

[3] Bloch, F. and A. Gomes (2006): *Contracting with Externalities and Outside Options*, Journal of Economic Theory, forthcoming.

[4] Bolton, G.E., K. Chatterjee and K.L. McGinn (2003): *How Communication Links Influence Coalition Bargaining: A Laboratory Investigation*, Management Science, 49, 5, 583-598.

[5] Calvó-Armengol A. (2001): *Bargaining Power in Communication Networks*, Mathematical Social Sciences, 41, 69-87.

[6] Calvó-Armengol A. (2002): *On Bargaining Partner Selection when Communication Is Restricted*, International Journal of Game Theory, 30, 503-515.

[7] Calvó-Armengol A. (2003a): *A Decentralized Market with Trading Links*, Mathematical Social Sciences, 45, 83-103.

[8] Calvó-Armengol A. (2003b): *Stable and Efficient Bargaining Networks*, Review of Economic Design, 7, 411-428.

[9] Calvó-Armengol A. and R. Ilkiliç (2004): *Pairwise Stability and Nash Equilibria in Network Formation*, Universitat Autonoma de Barcelona, mimeo

[10] Charness, G., M. Corominas-Bosch and G. Frechette (2005): *Bargaining and Network Structure: An Experiment*, mimeo available at SSRN: http://ssrn.com/abstract=248669.

[11] Chatterjee K. and B. Dutta (1998): *Rubinstein Auctions: on Competition for Bargaining Partners*, Games and Economic Behavior, 23, 119-145.

[12] Corominas-Bosch M. (1999): *On Two-Sided Network Markets*, PhD Dissertation, Universitat Pompeu Fabra, Department of Economics and Business.

[13] Corominas-Bosch M. (2004): *Bargaining in a Network of Buyers and Sellers*, Journal of Economic Theory, 115, 35-77.

[14] De Fraja G. and J. Sakovics (2001): *Walras Retrouvè: Decentralized Trading Mechanisms and the Competitive Price*; Journal of Political Economy, 109, 842-863.

[15] Demange, G. and M. Wooders (2005): *Group Formation in Economics: Networks, Clubs, and Coalitions*, Cambridge University Press.

[16] Dutta, B. and M. Jackson (2003): *On the Formation of Networks and Groups*, in *Networks and Groups: Models of Strategic Formation*, edited by B. Dutta and M.O. Jackson, Springer–Verlag, Heidelberg.

[17] Galbraith, J.K. (1952): *American Capitalism: the Concept of Countervailing Power*. Houghton Mifflin, Boston.

[18] Gale D. (1986): *Bargaining and Competition: Part I*; Econometrica, 54, 785-806.

[19] Galizzi (2006a): Cooperation and Competition in a Thin Market, Chapter 1, PhD dissertation, *Thin Markets, Networks, Bargaining: Essays in Theory and Experiments*, University of York.

[20] Galizzi (2006b): Networks and Price Formation in a Bilateral Duopoly, Chapter 2, PhD dissertation, *Thin Markets, Networks, Bargaining: Essays in Theory and Experiments,* University of York.

[21] Galizzi (2006c): Price Formation in Gas Thin Markets: Insights from Bargaining and Network Models, Working Paper, DEPA; University of Milan.

[22] Galizzi (2007): Network Formation in a Bilateral Duopoly: Equilibria and Experimental Evidence, manuscript, University of Brescia.

[23] Goeree, J.K., A. Riedl and A. Ule (2005): *In Search for Stars: Efficiency, Equity and the Dynamics of Networks Formation*, mimeo, CREED Amsterdam.

[24] Harley, C.B. (1981): *Learning the Evolutionary Stable Strategy*, Journal of Theoretical Biology, 89, 611-633.

[25] Hendricks K. and R. P. McAfee (2005): A *Theory of Bilateral Oligopoly*, Caltech, working papers.

[26] Hey J. (1991): *Experiments in Economics*, Blackwell.

[27] Jackson, M. (2005): *The Economics of Social Networks*, Chapter 1 in Volume I of *Advances in Economics and Econometrics, Theory and Applications: Ninth World Congress of the Econometric Society*, edited by R. Blundell, W. Newey and T. Persson, Cambridge University Press, 2006.

[28] Jackson, M. and A. Wolinsky (1996): *A Strategic Model of Economic and Social Networks*, Journal of Economic Theory, 71, 44-74.

[29] Kranton R.E. and D. Minehart (2001): *A Theory of Buyer-Seller Networks*, American Economic Review, 91(3), 485-508.

[30] Osborne, M. and A. Rubinstein (1990): *Bargaining and Markets*, 1990, Academic Press.