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# SCHOLARSHIPS OR STUDENT LOANS? SUBSIDIZING HIGHER EDUCATION IN THE PRESENCE OF MORAL HAZARD

ALESSANDRO CIGNO AND ANNALISA LUPORINI

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# Scholarships or student loans? Subsidizing higher education in the presence of moral hazard.

Alessandro Cigno[*] and Annalisa Luporini[†]
University of Florence, CESifo and CHILD

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**Abstract**

Given that, in the absence of full contingent markets, a number of school leavers who would otherwise choose to invest in a higher education will not do so, which is the best way to help these young people attend university in the presence of uncertainty about both university and labour market outcomes, and of moral hazard? The straight answer is a scholarship scheme financed by a tax on graduate earnings. The second-best scholarship is the sum of a need-based grant decreasing in parental means, and a merit-based award increasing in academic performance. An income-contingent loan scheme can at best replicate the allocation brought about by a scholarship-*cum*-graduate tax scheme, and only on condition that there is nothing to stop the public authority using tuition fees as if they were taxes. In the presence of an additional restriction, that a loan repayment cannot be greater than the value of the loan capitalized at market interest rates, a loan scheme will exclude not only some rich, but also some poor young people. The paper addresses also the choice of subject mix, and finds that students should be allowed to follow their natural inclination, rather than pushed into the study of subjects that promise a high monetary reward. On the other hand, if any student is denied public support, he will be one who is inclined towards the study of subjects that do not make such a promise.

*Key words:* universities, higher-education externality, degree choice, study effort, moral hazard, multi-agency problem, scholarships, income-contingent loans, graduate tax, tuition fees.

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[*]Email: cigno@unifi.it

[†]Corresponding author. Email: annalisa.luporini@unifi.it

# 1 Introduction

Whether and in which way the government should help young people pay for a higher education is a matter of great practical and theoretical interest. The arguments used to justify government intervention in this sphere are generally three.

The first is based on the assumption of a positive externality.[1] Although there is some controversy over the size of this external benefit,[2] nobody seems to have doubts about its existence. Also beyond dispute is the fact that education is a major factor in economic growth.[3] If growth is deemed to be desirable, there is then an argument for improving access to university, but not necessarily for subsidizing students out of general tax revenue. For the latter to be justified, it would have to be shown that the wage differential between graduates and non-graduates (the "graduate premium") does not fully reflect the private cost of higher education.

The second is a market imperfection argument. A number of persons who, in the presence of a complete system of contingent markets, would have chosen to attend university may not do so because the existing markets do not allow them to (a) borrow against future earnings, and (b) insure against the risk of an unfavourable degree result, or an unfavourable outcome in the graduate labour market. The reasons for these market imperfections are well known. Adverse selection and moral hazard problems make it unwise for insurance companies to offer cover for scholastic and earnings risks. In the absence of insurance, private lending institutions are generally reluctant to lend without suitable collateral. Since human capital cannot be mortgaged, this implies that young people will find it difficult to borrow for educational purposes unless they, or their parents, have conventional assets to offer as collateral.

The third argument for public intervention is that poor parents are less able to pay for their children's higher education out of current income, and less likely to obtain credit and insurance from the market, than rich parents are.[4] The young people excluded from university as a consequence of credit

[1] One possibility is that graduate and non-graduate workers are complementary, and that university education will thus increase everybody's productivity, not just that of those who bear the cost of attending university. Another is that a higher education reduces social costs by increasing social cohesion, or reducing antisocial behaviour. Bynner and Egerton (2000) report evidence of a positive association between higher education and willingness to participate in the democratic process and community activities, egalitaran attitudes, and even good parenting.

[2] The numbers one gets appear to be heavily dependent on the *a priori* assumptions one makes. For the USA, Moretti (1998) puts the effect of a one percent increase in the share of college graduates on the wages of all workers at around 1.6 percent, Acemoglu and Angrist (1999) put the excess of social over private return at less than one percent. Havemann and Wolfe (1984) produce estimates far in excess of those obtained by others, and argue that conventional methods of calculation underestimate the welfare effects of education.

[3] See, for example, Barro (1998), Bassanini and Scarpenta (2001), Jorgenson and Fraumeni (1992).

[4] While there is little doubt that credit constraints are binding for many households, the US-based evidence on the higher educational effects of these constraints is somewhat controversial. Cameron and Heckman (1998), and Carneiro and Heckman (2002), find that credit quotas

and insurance market imperfection are thus likely to come from families at the lower end of the income distribution.[5] This may be interpreted as a horizontal equity argument, if we regard the household as the basic decision unit, or as an equality of opportunity argument, if the decision unit is the potential student.

Granted that there is a case for public intervention, which are the policy instruments? At one extreme, we may think of a scholarship scheme, financed by general tax revenue, covering both the tuition and the maintenance costs of every young person with the right personal characteristics. The good thing about this policy is that it allows risk spreading, and helps to internalize the educational externality (if there is one). The distributional effect, however, is ambiguous. By allowing bright young people to attend university irrespective of family income, the scheme does in fact achieve equality of opportunity. Depending on how progressive the general income tax is, however, the cost may fall partly on the not-so-bright children of poor families who do not qualify for a scholarship. At the other extreme, we may think of a credit scheme, again covering both tuition and maintenance costs, repayable in full by the student at market interest rates. The only definitely good thing one can say about this alternative is that it, too, makes it possible for young people with the right personal characteristics to attend university irrespective of parental support. On the negative side, however, the scheme lacks an insurance element. Therefore, if repayment is strictly enforced in all cases (but it is difficult to see how), the scheme will not appeal to students from poor families.

Real-life student support schemes tend to lie somewhere in between these two extremes. A common arrangement is to have tuition subsidized on a sliding scale according to parental income ("need"), and maintenance costs covered in some measure either by scholarships, or by loans repayable at below market interest rates. Scholarships and subsidized loans are usually conditional on school record, and university performance ("merit"). These in-between arrangements obviate some of the problems, but miss out also some of the benefits, of pure schemes. One way to obviate the possibly regressive effect of scholarships financed by general tax revenue is to have the scheme financed by an income tax surcharge on graduates, but the idea does not appear to have attracted much political support. An idea that is becoming increasing popular, by contrast, is that of income-contingent loans.[6] These differ from straight loans in that the size of the repayment is conditional on the amount that the borrower earns after graduation (we would argue that *all* student loans are *de facto* income-contingent, because human capital cannot be mortgaged, and it is thus difficult to enforce the repayment of a loan on an unsuccessful student). Income-contingent loans

play no significant role in college attendance decisions. Using a sophisticated structural model, however, Keane and Wolpin (2001) argue that this is only because college students have the opportunity to support themselves by working; parental support is thus essentially a determinant of student leisure time.

[5] Dynarski (2003) finds that withdrawing social security tuition support in the US would significantly reduce enrolment and completion by disadvantaged groups.

[6] The idea comes originally from Friedman (1962), but was resuscitated in Barr (1991). It has been fully implemented in Australia. Elements of it figure also in the Netherlands and UK systems. New Zealand and Sweden experimented with it, but then abandoned the idea.

thus have a redistributive, and an insurance element. The problem is that the scheme can break even only if successful graduates are asked to pay back, on average, more than the value of their loan capitalized at market interest rates. As participation is obviously voluntary, bright young people will then accept a loan only if they are credit constrained. As a consequence, young people rich enough to pay for a higher education out of family resources will do so, and will not participate in the loan scheme. As there will then be less money to redistribute, some bright young people from poor families may be denied a loan, and consequently be excluded from university.

The question whether an education policy is justified is addressed in De Fraja (2002). Assuming a utilitarian social welfare function, the author finds that public intervention is indeed justified, and that the second-best policy redistributes in favour of richer and more talented students. The choice of policy instruments is investigated in Garcia-Peñalosa and Wälde (2000), Hanushek *et al.* (2003, 2004), and Del Rey and Racionero (2006). The first of these papers compares four alternative policies: pure loans, an education subsidy financed by general tax revenue, an education subsidy financed by a tax on graduates, and income-contingent loans. Rather than maximizing a social welfare function, the authors rank these policies on the basis of three criteria: Pareto optimality, *ex ante* equality of opportunity, and *ex post* equality of lifetime income. They find that the graduate tax scheme comes out the winner on all scores. The two Hanushek *et al.* papers use a calibrated general equilibrium model. The policies considered are tuition subsidies, need-based grants, merit awards, and income-contingent loans. They find that education subsidies in general perform less well than other forms of redistribution where equity is concerned. If there is an education externality in production, however, the case for education subsidies becomes overwhelming. Need-based grants achieve greater equality than merit-based ones. Income-contingent loans perform rather badly. Del Rey and Racionero adopt output maximization as the optimality criterion. The policy that is optimal by this criterion fully insures the lowest-ability individual included in the scheme, and partially insures those with higher ability.

All but one of these papers allow potential students to differ in their ability to learn. Del Rey and Racionero allow graduates to differ in their productivity (wage rates), but the probability of becoming a graduate is the same for everybody. De Fraja, and Hanushek *et al.*, allow parents to choose how much support to give. Garcia-Peñalosa and Wälde, and Del Rey and Racionero, take such support as exogenous. In De Fraja, there is an adverse selection problem, arising from the assumption that a student's learning ability is known to the student, but not observable by the government. In the other papers, ability is uncertain, but there is no adverse selection problem, because the government is assumed to have the same information as the student. As none of these papers allows university success to depend on individual study effort, there is no moral hazard. There is, furthermore, no choice of degree content.[7]

[7] The latter is examined in connection with graduate migration by Poutvaara (2004). Given the focus of that paper, the relevant distinction is between subjects that contain internationally

Like our predecessors, we start from the premise that, in the absence of a complete system of contingent markets, a number of school leavers who would have otherwise gone to university will go straight into the labour market instead. This implies that it may be possible to raise social welfare by helping at least some of these young persons to attend university. The welfare gain will be larger if there is a positive higher-education externality. Unlike our predecessors, however, we focus on the incentive problem that may arise from the non-observability of individual study effort, and on the subject choice problem associated with differences in individual aptitude for the study of different subjects. We postulate that

(i) parental support for university students depends only on parental means;

(ii) school leavers differ not only in their learning ability, but also in their aptitude for the study of different subjects;

(iii) both the policy maker and the school leaver know the latter's absolute and comparative learning ability;

(iv) the degree result is a random variable, with probability distribution conditional on study effort (non observable);

(v) graduate earnings are a random variable, with probability distribution conditional on subject mix, and degree result;

(vi) individuals are risk averse.

Postulate (i), is the same as in Garcia-Peñalosa and Wälde, and Del Rey and Racionero. It carries the implication that the policy maker can in some way compel parents to contribute to their children's higher education costs in accordance with their means.[8] Postulate (ii) extends the papers mentioned in that it recognizes that people may differ not only in absolute, but also in comparative ability. Postulate (iii), implicit in all the earlier papers except De Fraja, is justified if school records are informative of a school leaver's potential for higher education, and aptitude for different subjects,[9] or admission to university is conditional on passing an informative entry examination. As a simplification, we shall assume that absolute ability can take only two values, high or low, and that a low-ability person would not benefit from a higher education under any circumstances. This will allow us to divide school-leavers into two groups, "university material", and the rest. We similarly assume that high-ability school leavers have an aptitude either for the study of subjects that offer the prospect of a well-paid career ("science"), or for the study of subjects that do not ("arts").[10] Postulate (iv) recognizes that the degree result (not

applicable knowledge, and subjects containing knowledge that can be used only in the country of origin.

[8] That used to be case, at least in theory, in pre-Thatcher UK, when a student admitted to a university course was awarded a nominal grant. This was supposed to be paid in part by the student's local authority, and in part by the student's own parents. Since the part to be paid by the parents increased with their income, this was in effect an income tax. In practice, this tax was widely evaded. Like Garcia-Penalosa and Walde, and Del Rey and Racionero, we are assuming that it cannot.

[9] This in turn implies that primary and secondary education are sufficiently subsidized, and the school curriculum sufficiently broad, to provide such information about all children of the relevant age.

[10] It should be clear that this is only short-hand. For present purposes, accountancy would

just pass-or-fail, as in all papers where university success is uncertain, but the actual mark)[11] has a random component. As effort is not observable, we have then a moral hazard problem, absent in the papers cited. Postulate (v) adds a second layer of uncertainty to the outcome of an educational investment.[12] The probability of labour market success is conditional on the student's choice of subject mix, and on degree result (hence, indirectly, on the student's choice of effort level). Postulate (vi) generates a demand for insurance.

Our primary aim is to find the policy that will achieve the largest expected welfare gain over *laissez faire,* under the constraint that the policy will be subsidized by the general tax payer only to the extent that there is a positive higher-education externality. Rather than starting with a list of policies, and asking which is preferable according to some criterion, we start by characterizing the allocation that maximizes a Benthamite social welfare function, and then look for ways to implement it. Although we are ultimately interested in the second best, we characterize also the first best, because that will allow us to distinguish equity and insurance, from incentive considerations. We find that both the first and the second best can always be implemented using a scholarship scheme financed by a graduate tax. A loan scheme, even an income-contingent one, can at best replicate the allocation generated by a scholarship and graduate tax scheme. If tuition fees are restricted to be no higher than the average total cost of universities,[13] a number of high-ability school leavers from poor families may in fact be excluded from the scheme, and consequently from university. If there is also a "no usury" constraint, that a loan repayment cannot be larger than the loan capitalized at market interest rates, then not only some poor, but also some rich, high-ability school leavers may be excluded from the scheme. Unlike the former, the latter will not be excluded from a university education. The number of excluded agents is a decreasing function of the educational externality.

Our secondary aim is to establish whether students should be allowed to follow their own inclination in the choice of subjects, or pushed towards subjects that promise higher monetary rewards. In this connection, we find that, in the absence of moral hazard, it would be optimal for scientifically-inclined students to specialize completely in science subjects, and for artistically-inclined ones to specialize in arts subjects, but not completely. Given moral hazard, the former will still specialize completely in science subjects, and the latter will be induced to specialize even *further* in arts subjects. We also find that scientifically inclined students put in more effort than artistically inclined ones, and that this is how it should be.

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be classified as science, economics as arts.

[11] In some university systems, such as the British or the Italian one, there is an actual degree result. Where this is not the case, as in the US system, we may think of the degree result as of the average of the grades obtained in individual examinations.

[12] Although graduates earn more than non graduates on average, there is a great deal of variation in expected earnings across subjects. Graduates in certain subjects earn less than some non graduates; see, for example, Blundell, Dearden, Goodman and Reed (2000).

[13] Defined to include the cost of doing research.

## 2 Social optimization as a multi-agency problem

Since the policy maker's objective function does not coincide with that of the students, and given that one of the actions undertaken by the latter can be neither observed nor inferred by the former, the optimization has the structure of a multi-agency problem. In the present context, the agents are all the high-ability school leavers. The principal is a public authority with the power to fix tuition fees, award scholarships, guarantee student loans, and raise an earmarked tax on graduate earnings (the general income tax is assumed given). We shall assume that universities are publicly owned or, equivalently, that the authority in question will make good any difference between tuition fees and tuition costs (so that it does not matter whether the university system is private or public). If there is no higher-education externality, the policy must break-even. Otherwise, it will be subsidized out of general tax revenue by the monetary value of the externality. Both the graduate tax and the loan repayment can be made contingent on graduate earnings, and other observables, but there is a fundamental difference between the two. A student can avoid having to pay a loan back by not accepting one in the first place, but cannot escape the graduate tax by not accepting a scholarship. We shall suppose that loan repayments and graduate taxes are collected at zero marginal cost.[14]

There are only two types of agent, indexed $i = a$, $s$, but "many" agents of each type. Type $a$ has a comparative advantage in the study of the arts, type $s$ in that of science (recall that this is just short-hand for low and high expected graduate earnings). The precise sense in which we speak of comparative advantage in the present context is explained in the next section. In the light of (iii) above, the principal knows whether the agent is an $a$ or an $s$. There are three dates, labelled $t = 0, 1, 2$. The principal announces his policy at date 0. Agents take their decisions at date 1. Degree results become available between dates 1 and 2. Graduate earnings are revealed at date 2. Date-2 money values are discounted back to date 1 using the market rate of interest.

According to the logic of agency problems, if an action is either unobservable or costly to observe, the agents should be provided with the incentive to undertake that action at the level desired by the principal. In the present context, the action that falls into this category is individual study effort. By contrast, if an action is both observable and enforceable at zero cost, it does not make sense for the principal to offer costly incentives. In such a case, the agents will be forced to undertake the action at the level desired by the principal using what is politely called a "forcing contract" (in plain English, by threatening the agent with a penalty high enough to dissuade him from doing otherwise). In the present context, the action falling into this category is the choice of degree type. The principal will design the policy in such a way that it is in the agent's interest to accept the help (scholarship, loan) offered to him, and then make this help conditional on the agent choosing the prescribed degree course.[15]

[14] In Australia, loan repayments are collected at no extra cost together with income tax; see Chapman (1997).

[15] Like all social policy applications of the forcing contract idea, this has a distasteful to-

## 3 Agents

At date 1, an agent can go either to university, or straight into the labour market. If he decides to become a student, he will receive a nonnegative transfer $m_1$ from his parents. If he decides to become a worker, he will receive a lower parental transfer (normalized to zero). In the first case, the agent chooses the type of degree, $d$, and the amount of effort he wants to put into it, $e$. We characterize the degree type by the proportion of science subjects contained in it, such that $d$ takes value 0 if the student takes only arts subjects, 1 if he takes only science ones. Effort varies in the closed interval $E = [\underline{e}, \overline{e}] \in R^{+}$. For a type-$i$ student, $i = a, s$, the disutility of putting $e$ units of effort into the degree $d$ is given by

$$z = z^{i}(e, d).$$

We assume that the $z^{i}(.)$ functions have the following properties.

1. If an agent exerts no effort, his disutility is zero whatever the subject mix,

$$z^{i}(0, .) = 0.$$

2 For any positive effort level, the disutility of a type-$a$ student who studies only arts subjects is equal to the disutility of a type-$s$ student who studies only science,

$$z^{a}(e, 0) = z^{s}(e, 1), \ e > 0. \tag{1}$$

3. Disutility is increasing and convex in effort level,

$$z^{i}_{e}(e, d) > 0, \ z^{i}_{ee}(e, d) > 0. \tag{2}$$

4. For a type-$a$ student, disutility is increasing and convex in science content,

$$z^{a}_{d}(e, d) > 0, \ z^{a}_{dd}(e, d) > 0. \tag{3}$$

5. For a type-$s$ student, disutility is decreasing in science content,

$$z^{s}_{d}(e, d) < 0. \tag{4}$$

6. The marginal disutility of effort is increasing in science content for type $a$, non-decreasing for type $s$,

$$z^{s}_{ed}(e, d) \leq 0 < z^{a}_{ed}(e, d), \tag{5}$$

The final degree result, denoted by $x$,[16] is uncertain. We assume that $x$ is distributed over the closed interval $X = [\underline{x}, \overline{x}] \in R^{+}$, with density $f(x|e)$

talitarian ring. It must be remembered, however, that restrictions on the student's choice of degree course exist in all university systems that practice some form of selection at entry. If selection is by school results, the access to a particular type of degree is typically conditional on the student having achieved certain minimum grades in certain specified school subjects. If selection is by entry examination, the student may have to pass a test common to all degree courses, but typically also one that is specific to that particular type of degree.

[16] Recall that we take this to be an actual mark, not just pass or fail.

conditional on effort level. Studying hard raises a student's chances of obtaining a good degree result in the precise sense that the cumulative distribution of $x$, $F(x|.)$ associated with a higher $e$ first-order stochastically dominates (FOSD) the one associated with a lower $e$,

$$F_e(x|e) \leq 0.$$

The subscript denotes partial differentiation with respect to $e$. For each $e$, there will be some $x$ such that the condition holds as an inequality. We assume that the monotone likelihood ratio (MLR), and convexity of the distribution function (CDF), conditions are satisfied.[17] For the time being, we shall assume that the $x$s of the different agents are i.i.d..[18]

The graduate premium (defined as the difference between the graduate, and the non-graduate wage rate), denoted by $m_2$, is distributed over the closed interval $M_2 = [0, \overline{m_2}] \in R$, with density $g(m_2|x, d)$ conditional on degree type, and degree result. If a student fails his degree, his skill premium will be zero. A degree with a high science content, or with a high mark, makes it more likely that the graduate will attract a large premium. More precisely, the cumulative distribution of $m_2$, $G(m_2|.)$ associated with a higher $(x, d)$ first-order stochastically dominates the one associated with a lower $(x, d)$. Furthermore, an increase in the science content of the degree does not affect the marginal effect of the degree result on the distribution of $m_2$. Using the subscripts $d$ and $x$ to denote partial differentiation with respect to, respectively, subject mix and degree result, we can then write

$$G_d(m_2|x, d) \leq 0, \; G_x(m_2|x, d) \leq 0, \; G_{x,d}(m_2|x, d) = 0.$$

For each $(x,d)$, there is some $m_2$ such that the first two of these restrictions hold as inequalities. We again make the CDF assumption that $G(.)$, is convex in $(x, d)$.[19] For the time being, we assume that $m_2$ is i.i.d.[20]

The lifetime utility of a type-$i$ agent is given *ex post* by

$$U = u_1(c_1) - z^i(d, e) + u_2(c_2), \tag{6}$$

where $c_t$ is his consumption at date $t$. The functions $u_t(.)$ are increasing and concave (implying risk aversion), with $u_t'(0) = \infty$.

If an agent goes straight into the labour market after leaving school, he earns $w_1$ at date 1, and $w_2$ at date 2. These earnings, assumed the same for all non-graduate workers, are defined net of any general income tax. The agent will

[17] These are restrictions on the form of $f(.)$. The former requires that $(f_e/f)$ is increasing in $x$, the latter that the cumulative distribution of $x$ is convex in $e$.

[18] This simplifying assumption is not crucial for our results. The characteristics of the first-best solution would remain unaltered even if we assumed correlation. At the end of section 7, we point out that correlation would only slightly modify the form of the second-best transfers.

[19] Together with the MLR and CDF restrictions imposed on $F(.)$, this standard assumption ensures the concavity of the expected utility function (see Appendix), and thus the uniqueness of the agent's choice of effort. This will allow us to substitute the agent's first-order condition for the incentive-compatibility constraint in the policy optimization problem.

[20] This simplifying assumption will be relaxed at the end of section 7.

then consume

$$c_t = w_t, \ t = 1, 2, \tag{7}$$

and enjoy the *ex-post* utility level

$$\pi_{LM} \equiv u_1 (w_1) + u_2 (w_2) . \tag{8}$$

As he does not face uncertainty, that is also his *ex-ante* utility.

If an agent decides to become a student (recall that, by definition of agent, he has the qualifications required for university admission), he pays a nonnegative tuition fee, $\theta$, possibly dependent on parental means, and personal and degree type. He may also receive from the principal a nonnegative payment, $\chi$, possibly dependent on parental means, university performance, and personal and degree type. Both these payments will occur at date 1.[21] The net payment received,

$$y_1 \equiv \chi - \theta,$$

can take any sign, but cannot be lower than $-\theta$ because $\chi$ is nonnegative. The agent's date-1 consumption is

$$c_1 = m_1 + y_1. \tag{9}$$

At date-2, he will pay the principal a nonnegative amount, $y_2$, possibly conditional on his graduate earnings, personal and degree type, and degree result. His consumption will be

$$c_2 = w_2 + m_2 - y_2. \tag{10}$$

# 4 Laissez faire

In *laissez faire*, $\chi$ and $y_2$ are obviously zero, and $\theta$ is equal to the university's average total cost,[22] assumed the same for all universities. The expected utility of a type-$i$ agent attending university ($i = a, s$) is then

$$\pi^i_{LF} (m_1, \theta) \equiv \max_{e,d} \int_x u_1 (m_1 - \theta) f dx - z^i(e, d) + \int_x \int_{m_2} u_2 (w_2 + m_2) g dm_2 f dx. \tag{11}$$

Recalling that it is not possible to borrow from the market for educational purposes, the agent will not enrol as a student if $m_1$ is lower than $\theta$. If he does not enrol, there is a social cost.

For each type $i$, we can define a threshold level of parental support, $\widetilde{m}^i_1$, defined by

$$\pi^i_{LF} \left(\widetilde{m}^i_1, \theta\right) = \pi_{LM}, \ i = a, s,$$

such that any agent of that type is indifferent between going to university, or straight into the labour market. An agent will then attend university if and

[21] Unless $\chi$ is conditional on degree result. In that case, since the final degree result will be known only at date 2, we must assume that $\chi$ is paid in installments, as partial results start to come in.

[22] Possibly inclusive of the cost of doing reasearch.

only if he receives parental support at least equal to the larger of the threshold, and the tuition fee.[23] In the Appendix, we prove the following.

**Proposition 1.** *In laissez faire,*

*i) all type-i students choose the same subject mix, $d^i_{LF}$, and effort level, $e^i_{LF}$;*

*ii) type-a students take fewer science subjects, and supply less effort, than type-s (the latter specialize completely in science),*

$$d^a_{LF} < d^s_{LF} = 1 \text{ and } e^a_{LF} < e^s_{LF};$$

*iii) the threshold level of parental support, below which an agent will go straight into the labour market, is higher for type a than for type s,*

$$\widetilde{m}^a_1 > \widetilde{m}^s_1.$$

The finding that an $s$ will take just science subjects, while an $a$ will not take just arts, is due to the lower expected earnings associated with arts subjects. The same may be said of the result that type $a$ will study less hard than type $s$, and that the threshold level of parental support is higher for the former than for the latter.

# 5 The principal

Let $j$ denote a generic agent, and $z^j$ the disutility of the effort supplied by this agent. If $j$ is of type $i$, $z^j \equiv z^i\left(e^j, d^j\right)$. The principal maximizes the Benthamite welfare function

$$W\left(y_1, y_2\right) = \sum_j \int_{x^j} \left( u_1\left(m^j_1 + y^j_1\right) - z^j + \int_{m^j_2} u_2\left(w_2 + m^j_2 - y^j_2\right) g dm^j_2 \right) f dx^j. \tag{12}$$

Given the i.i.d assumptions, net payments to or from $j$ can depend only on $j$'s own outcomes, $\left(x^j, m^j_2\right)$.[24] Since (12) is a sum of concave functions, the principal is averse to risk like his agents. Given the large number of agents, however, he does not face any risk regarding how much he will have to pay out in total to students at date 1, and how much he will get back in total from graduates at date 2. The principal's intertemporal budget constraint may then be written in expected-value terms as

$$\sum_j \left( \int_{x^j} \left( y^j_1 + p - \int_{m^j_2} y^j_2 g dm^j_2 \right) f dx^j \right) \leq S, \tag{13}$$

where $S$ is the amount by which the policy is subsidized out of general tax revenue. We assume that this subsidy is set equal to the monetary value of the higher-education externality.

[23] Subsistence consumption is implicitly normalized to zero.

[24] We can thus avoid writing the expected utilities of the different agents as functions of the joint density of the different outcomes; see Holmström (1982) and Mookherjee (1984).

Since individual study effort in not observable, each $j$ must be given the incentive to choose the $e^j$ assigned to him by the policy. In view of our assumptions regarding the distributions of $x^j$ and $m_2^j$, this incentive-compatibility constraint can be replaced by the first-order condition on $j$'s choice of $e^j$,

$$z_{e^j}^j = \int_{x^j} \left[ u_1 \left( m_1^j + y_1^j \right) + \int_{m_2^j} u_2 \left( w_2 + m_2^j - y_2^j \right) g dm_2^j \right] f_{e^j} dx^j. \tag{14}$$

There is no analogous constraint on $d^j$, because this is observable, and the principal can consequently force $j$ to choose any particular degree type.

There are then the university-participation constraints,

$$\pi_{PS}^j \left( m_1^j, y_1^j, y_2^j \right) \geq \pi_{LM}, \tag{15}$$

where

$$\begin{aligned} \pi_{PS}^j \left( m_1^j, y_1^j, y_2^j \right) \equiv \max_{e^j, d^j} \int_{x^j} u_1 \left( m_1^j + y_1^j \right) f dx^j - z^j \\ + \int_{x^j} \int_{m_2^j} u_2 \left( w_2 + m_2^j - y_2^j \right) g dm_2^j f dx^j. \end{aligned} \tag{16}$$

is the expected utility of going to university for agent $j$.

The first-order conditions for the maximization of (12) are

$$\left( u_1' \left( m_1^j + y_1^j \right) \left( 1 + \nu^j \right) - \lambda \right) f + \mu^j u_1' \left( m_1^j + y_1^j \right) f_{e^j} = 0, \tag{17}$$

$$\left( -u_2' \left( w_2 + m_2^j - y_2^j \right) \left( 1 + \nu^j \right) + \lambda \right) f - \mu^j u_2' \left( w_2 + m_2^j - y_2^j \right) f_{e^j} = 0, \tag{18}$$

$$\begin{aligned} \left( 1 + \nu^j \right) \left[ \int_{x^j} \int_{m_2^j} u_2 g_{d^j} dm_2^j f dx^j - z_{d^j}^j \right] + \\ \lambda \int_{x^j} \int_{m_2^j} y_2^j g_{d^j} dm_2^j f dx^j - \mu^j z_{e^j d^j}^j = 0, \end{aligned} \tag{19a}$$

and

$$\begin{aligned} \left( 1 + \nu^j \right) \left\{ \int_{x^j} \left[ u_1 + \int_{m_2^j} u_2 g dm_2^j \right] f_{e^j}^j dx^j - z_{e^j}^j \right\} \\ + \lambda \left( \int_{x^j} \left[ \int_{m_2^j} y_2^j g dm_2^j - y_1^j - p \right] f_{e^j}^j dx^j \right) + \\ \mu^j \left\{ \int_{x^j} \left[ u_1 + \int_{m_2^j} u_2 g dm_2^j \right] f_{e^j e^j} dx^j - z_{e^j e^j}^j \right\} = 0, \end{aligned} \tag{20}$$

where $\lambda$ is the Lagrange-multiplier associated with the principal's budget constraint (13), $\mu^j$ that associated with the incentive-compatibility constraint (14), and $\nu^j$ that associated with the university-participation constraint (15).[25] The term $\int_{x^j} \int_{m_2^j} u_2 g_{d^j} dm_2^j f_{e^j} dx^j$ in (19$a$) is equal to zero because an increase in study effort, and consequently in the expected degree result, does not affect the marginal effect of $d^j$ on date-2 expected utility (see Appendix A0, c)). Conditions (17) and (18) say that the principal must choose $\left(y_1^j, y_2^j\right)$ so as to equate the marginal benefit to the opportunity-cost at each date, and in each possible state of the world. The remaining conditions are slightly less straight-forward.

The first LHS term of (19$a$) is the expected private benefit, and the second the expected external benefit (*via* the government budget constraint), of inducing agent $j$ to take more science subjects. The third is the marginal effect of $d^j$ on the incentive-compatibility constraint, equal to the effect on the disutility of effort.

The first LHS term of condition (20) is equal to zero because of the incentive-compatibility constraint (14). The second term is the amount by which the principal's budget constraint is relaxed if $e^j$ increases a little. That is in turn the sum of two partial effects. One arises from the fact that, the harder $j$ studies, the more he is likely to earn, and thus to pay the principal, when he gets into the graduate labour market. The other arises from the fact that, the harder $j$ studies, the more the principal is likely to have to pay him while he is a student. The third term is proportional to the second-order condition for $j$'s maximization problem, hence negative.

In addition to (13) – (15), the principal may face the political (no-implicit-tax) constraint that tuition fees cannot exceed average total cost,

$$\theta^j(d^j) \leq p. \tag{21}$$

The logic of this restriction will be discussed in section 8, when we come to the interpretation of the optimal policy, but one thing is worth pointing out right away. Without (21), $\theta^j$ does not figure in the principal's optimization problem. All that matters is the *net* payment, $y_1^j = \chi^j - \theta^j$. Therefore, in the absence of further restrictions, $\chi^j$ and $\theta^j$ are left indeterminate. Further restrictions may arise if the policy is a loan scheme, and will be discussed in section 8.

# 6 First best

If study effort were observable by the principal, there would be no incentive-compatibility constraints. The maximum would then be a first best with the following properties (see Appendix for the proof).

**Proposition 2**. *In first best,*

[25] For brevity, we omit the arguments of $u_1$ and $u_2$, respectively $\left(m_1^j + y_1^j\right)$ and $\left(w_2 + m_2^j - y_2^j\right)$.

*i) consumption is equalized across agents, dates, and states of the world,* $c_1^j = c_2^j = c$*;*

*ii) students receive a net payment dependent only on parental support,* $y_1^j = y_{1FB}\left(m_1^j\right)$*, with* $y_{1FB}(.)$ *decreasing;*

*iii) graduates make a payment dependent only on the graduate premium,* $y_2^j = y_{2FB}\left(m_2^j\right)$*, with* $y_{2FB}(.)$ *increasing;*

*iv) all type-i students choose the same subject mix,* $d_{FB}^i$*, and effort level,* $e_{FB}^i$*;*

*v) type-a students take less science subjects, and supply less effort, than type-s (the latter specialize completely in science),* $d_{FB}^a < d_{FB}^s = 1$ *and* $e_{FB}^a < e_{FB}^s$*;*

*vi) the university participation constraints are not binding.*

Therefore, a first best is characterized by perfect equity, perfect consumption smoothing, and full insurance. Since agents of the same type differ only in the amount of support that they receive from their parents, this implies that all agents of the same type will behave the same. Although consumption is the same for both types of agent, utility may be different, because the two types have different disutility-of-effort functions, and we cannot tell whether $z^s(e_{FB}^s, d_{FB}^s)$ is higher or lower than $z^a(e_{FB}^a, d_{FB}^a)$. In standard principal-agent models, the full-insurance property descends from the assumption that the principal is less risk-averse than the agents. Here, by contrast, it is due to the fact that the principal does not face any budget uncertainty.[26] Notice that the optimal scheme redistributes not only from rich to poor students, but also from rich to poor graduates. It thus compensates students not only for any difference in the amount of support they receive from their parents, but also for any difference in the amount they will earn in the graduate labour market. That is indeed why the participation constraints are not binding.

The net payment $y_1^j$ due to $j$ at date 1 is a type-independent function of parental support, $m_1^j$, and thus of parental means. Depending on parameter values, it could be positive for all values of $m_1^j$, or positive for low, and negative for high ones. The payment $y_2^j$ due from $j$ at date 2 is a type-independent function of $j$'s graduate premium, $m_2^j$, and thus of his graduate earnings.

As in *laissez faire*, the study effort, and the science content of the degree, are higher for type-$s$, than for type-$a$ students. Intuitively, that is because the expected monetary return to investing in a university education is higher for the latter, than for the former. It thus makes sense to invest more in the education of scientifically talented students, than in that of artistic ones, as this will help relax the principal's budget constraint. As a consequence of this, and of stochastic dominance, type-$s$ graduates will earn more, on average, than type-$a$ graduates. At date 2, the former will thus pay the principal more, on average, than the latter (but a very successful arts graduate may well pay more than a not-so-successful science graduate). One might wonder whether $e^i$ and $d^i$ are higher in first best or in *laissez faire*. Recall that the policy enables students to discount their expected graduate earnings, and redistributes in favour of

[26] The same result emerges from Cigno *et al.* (2003)

students with low parental support. This implies that students with sufficiently low parental support will supply more effort and, if they are of the artistic type, take more science subjects, in first best than in laissez faire. The same is not necessarily true of richer students, who may be net contributors to the scheme.

Since the principal does not have to provide students with costly incentives to study hard, because effort is observable, it cannot be optimal to keep any agent (recall that "agents" are not all school leavers, but only those with the right personal characteristics) out of a university.

**Corollary 2** *In first best, all agents go to university.*

Since, in *laissez faire*, not necessarily all agents go to university, the first-best policy raises social welfare. Even though some very rich students may have a lower level of utility with, than without the policy, the implied redistribution in favour of very poor students will in fact enhance social welfare in view of the concavity of the utility functions.

# 7 Second best

We now turn to the more realistic situation where individual study effort is not observable, so that the principal faces also the incentive-compatibility constraints (14). The maximum is now a second best, with the following properties (see Appendix for the proof).

**Proposition 3.** *In second best,*

*i) all students receive a net payment* $y^j_{1SB} = y^i_{11}\left(m^j_1\right) + y^i_{12}\left(x^j\right)$, *where* $y^i_{11}(.)$ *is decreasing, and* $y^i_{12}(.)$ *increasing;*

*ii) all graduates make a payment* $y^j_{2SB} = y^i_{21}\left(m^j_2\right) + y^i_{22}\left(x^j\right)$ , *where* $y^i_{21}(.)$ *is increasing, and* $y^i_{22}(.)$ *decreasing;*

*iii) at* $t = 1, 2$, *all agents of the same type have the same expected consumption;*

*iv) all type-i students choose the same degree type,* $d^i_{SB}$, *and effort level,* $e^i_{SB}$, *with* $d^a_{SB} < d^s_{SB} = 1$ *and* $e^a_{SB} < e^s_{SB}$;

*v) the degree taken by artistically inclined students,* $d^a_{SB}$, *contains fewer science subjects, and the study effort delivered by either type of student,* $e^i_{SB}$, *will be lower, than would be efficient given* $y^i_{tSB}(.)$.

Therefore, as in first best, the policy redistributes in favour of students from poorer families. Now, however, equality of consumption is not achieved. Differences in parental support are fully compensated for students of the same type, because there would be no advantage in distorting the actions of students with the same disutility-of-effort function. Agents of the same type consequently enjoy the same expected level of consumption, and the same level of expected utility, at each date. But different types of student have different expected levels of consumption because they face different $y^i_{tSB}(.)$ schedules.

The net payment due to a student at date 1 is the sum of two type-specific functions, one decreasing in parental support, and the other increasing in degree result. The reason why this net payment must increase with academic performance is moral hazard. As individual effort is not observable, each student must in fact be given an incentive to study hard. Notice that it is optimal for the principal to give the same marginal incentive to all students of the same type. Incentives differ across types however, because different types have different disutility-of-effort functions. As the reward for a good degree result will be the same for all students of the same type, irrespective of how much each individual is getting from his parents, equality of consumption is achieved by redistributing in favour of relatively poor students. That is why the net payment due to a student has two components, one depending solely on "need" (insufficient parental support), the other solely on "merit" (academic performance).

The payment due by a graduate at date 2 is similarly the sum of two type-specific functions, one increasing in graduate earnings, the other decreasing in degree result. This payment must be lower for graduates with higher degree results because it reduces net earnings and, therefore, the incentive to study hard. Since $j$ does not know, at date 1, how much he will earn at date 2, the marginal disincentive effect of the payment is independent of the realization of $m_2^j$. The second-best date-2 payment may thus be viewed as the difference between an amount that increases with labour market performance, and a discount that increases with academic performance.

The second-best policy encourages students with a penchant for the arts to specialize in their favourite subjects more than would be efficient given the second-best payment schedules. The intuitive explanation is that, since effort is not observable, and providing an agent with the incentive to study hard is thus costly, the principal must make it easier for the student to get better degree results. For type-$a$ students, the marginal disutility of effort is increasing in the science content of the degree. As a consequence, in order to loosen their incentive constraint, their second-best subject mix will include less science. By contrast, since $z_{d^s}^s$ is negative, type-$s$ students will in fact take only science subjects as in first best ($d_{SB}^s = 1$). Therefore the subject mix chosen by students predisposed to the study of science will not be distorted. Regarding the choice of effort level, we can say that $e^i$ will always be inefficiently low, because the agents do not take into account the social benefit (*i.e.*, the effect on the principal's budget constraint) of individual effort. Notice that, given $d_{SB}^i$, effort is less costly for the scientific type, who is choosing his least-cost subject mix, than for the artistic one, who is not. The second-best choice of effort is consequently higher for the former than for the latter.

Can we be sure that, at date 2, type-$a$ will pay on average less than type-$s$ graduates as in first best? Since $x^a$ and $x^s$ have the same distribution (in other words, greater effort has the same effect on the probability of getting a good degree result for both types), and $e_{SB}^a$ is lower than $e_{SB}^s$, $x_{SB}^a$ will be on average lower than $x_{SB}^s$. This *may* modify the first-best conclusion that type-$s$ must pay more, on average, than type-$a$ graduates. In second best, $y_2^i(.)$ is in fact increasing in $m_2^j$, but decreasing in $x^j$. Although it is unlikely that the average type-$a$ graduate will pay more than the average type-$s$ graduate, this possibility cannot thus be ruled out.

Let us now consider the possibility that the university participation constraint (15) is binding for some agents. We know that this could not happen in first best. It could happen in second best, however, because the cost to the principal of providing an agent with the incentive to study hard may now outweigh the expected benefit. This has an important implication. By definition, a participation-constrained agent, $k$, enjoys the same level of expected utility irrespective of whether he goes to university, or straight into the labour market. If $k$ is expected to be a net contributor, he should be kept in the scheme (*i. e.*, offered the $y_1^k$ that satisfies (15) as an equation).[27] Otherwise, social welfare will be maximized by setting $y_1^k$ so low that $k$ will go into the labour market straight from school, and using the resources thus freed to raise the expected utility of unconstrained agents.

Recalling that it is optimal to equalize the starting points of all agents of the same type, it then follows that, if (15) is binding for some agent, it will be binding also for all other agents of the same type. If any of the university participation constraints is binding, only one type of agent will get public support. Agents of the other type will go to university only if they are rich. If any agent is denied support, he will then be of the artistic type. The principal will help only agents that are expected to earn a great deal of money in the graduate labour market.

Let us now relax the assumption that the random variables are i.i.d.. Suppose that the degree results of different agents are affiliated and dependent (Milgrom and Weber, 1982).[28] For example, competition for student numbers may have led to a general lowering of examination standards. In such a case, the net payment an agent receives at date 1, and the one he makes at date 2, should optimally depend not only on his own degree result, but also on those of all other students. In other words, $y_1^i$ and $y_2^i$ should be functions of the vector $x = \left(x^j, x^{-j}\right)$.[29] This happens because other people's degree results convey information about the agent's own study effort. It can be shown that the second-best $y_1^i(.)$ is increasing in $x^j$, and decreasing in each element of $x^{-j}$.[30] Similarly, the second-best $y_2^i(.)$ is decreasing in $x^j$, and increasing in each element of $x^{-j}$. By contrast, $y_2^i(.)$ does not depend on $m_2^{-j}$.

We can thus say something quite specific about the properties of the second-best payment schedules. Where the effects of $x^j$ and $x^{-j}$ are concerned, the

[27] Recall that $y_1^j$ can be negative for rich students. Such students will then be offered a gross transfer $\chi^j \geq 0$ that does not cover the tuition fee.

[28] Affiliation implies that the random variables tend to "move together". In other words, it is more likely that the realized values will be all high, or all low, than that some will be high, and others low. Since affiliated random variables have nonnegative covariance, affiliation includes, as a special case, independence.

[29] Consider the conditions determining the form of $y_1^i()$ and $y_2^i()$, i.e. (34) and (35) in the proof of proposition 3. If the random variable affecting $j$' degree result is stochastically dependent on the random variables that affect the degree results of others, the likelihood ratio $f_{e^j}(x|e)/f(x!e)$ is a function of the entire $x = \left(x^j, x^{-j}\right)$ vector (Holmström, 1982).

[30] That $y_1^i\left(m_1^j, x^j, x^{-j}\right)$ is increasing in $x^j$ when $\frac{f_{e^j}(x,!e)}{f(x!e)}$ is monotone in $x^j$ is a well known result. That $y_1^i\left(m_1^j, x^j, x^{-j}\right)$ is monotonically decreasing in $x^{-j}$ if and only if the random variables are affiliated is demonstrated in Luporini (2006).

implication is that the policy maker should look only at relative degree results, and not be fooled into making higher (lower) net payments to all students, or requiring lower (higher) payments from all graduates, if degree results drift upwards (downwards). The intuition is that, if all degree results move in the same direction, this may reflect the behaviour of the examiners, rather than the behaviour of the students. Where the effects of $m_2^j$ and $m_2^{-j}$ are concerned, the implication is rather that the policy maker should look only at a graduate's own earnings. That is because, for any given degree type and degree result, graduate earnings depend only on chance.[31]

# 8 Policy interpretation: scholarships or loans?

Both the first and the second best can always be implemented using a suitably designed scholarship scheme financed by a graduate tax. The transfers, $\chi_1^j \equiv y_1^j + \theta^j$ and $y_2^j$ , can in fact be interpreted as, respectively, a scholarship and a graduate tax. In this case, the only restriction, in addition to (13) – (15), is the no-implicit-tax constraint (21). As already noted, however, what matters is the difference between $\chi_1^j$ and $\theta^j$, not their absolute values. Therefore, if the policy is a scholarship scheme financed by a graduate tax, (21) is never binding. The principal can always set the tuition fee equal to total average cost for all students, and adjust the size of the scholarship until the net transfer reaches the desired level.[32]

Alternatively, we can interpret $\chi_1^j$ as a loan, and $y_2^j$ as a loan repayment. Compared with scholarships and graduate taxes, student loans face additional restrictions. One is that, since $j$ cannot be obliged to accept the loan, the policy must satisfy the credit-participation constraint

$$\pi_{PS}^j\left(m_1^j, y_1^j, y_2^j\right) \geq \pi_{LF}^j\left(m_1^j, \theta^j\right). \tag{22}$$

This is indeed the hallmark of a loan scheme. Another may be that the loan repayment cannot be larger than the loan capitalized at the market interest rate. Recalling that $y_2^j$ is discounted back to date 1 using the market rate of interest, we can write this "no usury" constraint as

$$y_2^j \leq y_1^j + \theta^j. \tag{23}$$

Unlike (22), this is not a necessary feature of loan schemes, but it can arise only in the context of such a scheme.

Can the policy maker implement the first, or the second-best, allocation using a loan scheme? We shall answer this question with regard to the first

[31] That would not be true if we allowed the probability distribution of graduate earnings to be conditional also on work effort or search intensity.

[32] Since $j$ cannot be obliged to accept a negative transfer, the gross payment must satisy

$$y_1^j + p \geq 0.$$

The analysis implicitly assumes that this constraint is never binding.

best, but the same kind of reasoning applies also to the second best. A first-best loan scheme offers students the loan schedules $\chi^j = y_{1FB}(.) + \theta^j(.)$, and faces graduates with the loan repayment schedule $y_{2FB}(.)$. Let us consider first the effect of the credit participation constraint, (22). Recall that, in first best, tuition fees are undetermined. As a consequence, the principal can choose $\theta^j(.)$ and $\chi^j$ to satisfy (22). Take a type-$i$ agent, $j$, rich enough to finance his education entirely out of $m_1^j$, and thus in a position to turn down the offer of a loan if the terms are not sufficiently favourable. Given that $j$ would be a net contributor,[33] the policy maker has an interest in inducing him to participate in the scheme. As $j$'s type is known, and the choice of subject mix is observable, the policy maker would then want to set $\theta^j(.)$ such that (a) $\theta^j\left(d_{FB}^i\right)$ is high enough to make it profitable for $j$ to accept the loan if he chooses $d_{FB}^i$, and (b) $\theta^j(d^j)$ is sufficiently higher than $\theta^j(d_{FB}^i)$, for any $d^j \neq d_{FB}^i$, to make it unprofitable for $j$ to turn down the offer, and choose $d^j$ instead of $d_{FB}^i$.[34] This is clearly not feasible if (21) prevents the policy maker from setting tuition fees higher than average total cost. If (21) is binding, (22) is binding too.

Let us now look at the effect of the no-usury constraint (23). Without (21), (23) would never be binding, because the principal could always offer $j$ a loan equal to $y_{1FB}^j\left(m_1^j\right) + \widetilde{\theta}^j(d_{FB}^i)$, and set the repayment equal to $y_{2FB}\left(m_2^j\right)$, conditional on $j$ choosing the subject mix $d_{FB}^j$, and supplying the effort level $e_{FB}^j$. Following the same line of argument used in the last paragraph, it can be easily shown that there exists a $\widetilde{\theta}^j(d_{FB}^i)$ such that (a) it is profitable for $j$ to accept the deal, and (b) (23) is not binding. For any $d^j \neq d_{FB}^i$, $\widetilde{\theta}^j(d^j)$ must be large enough to make it unprofitable for $j$ to turn down the loan, and choose $d^j$ instead of $d_{FB}^i$.[35] That is clearly not feasible in the presence of the no-implicit-tax constraint (21). In other words, the ceiling on the size of the loan repayment constitutes an additional constraint only if there is also a ceiling on the tuition fee.

This provides a natural justification for the existence of both (21) and (23). Although the former may exist independently of the presence of either scholarships or student loans, there is in fact a special logic in assuming that both these constraints will be in place if the government is allowed to guarantee student loans, but not to increase taxation (at all, or above the level justified by an education externality) for educational purposes. Suppose that the externality is "small". If the government wanted to subsidize poor students, it would then have to charge rich ones more than the average total cost. Similarly, if the government wanted to subsidize poor graduates, it would have to charge rich ones more than the market interest rate. These excess fees and interests would be

[33] He will be a net contributor if $\int_{x^j}\left(\int_{m_2^j} y_{2FB}^j g^j dm_2^j - y_{1FB}^j - p\right) f^j dx^j \geq 0$. Since students taking a loan will gain from consumption smoothing, net contributors are not necessarily credit constrained. For the same reason, however, credit-constrained agents are net contributors.

[34] Even if $m_1^j$ happens to be the highest possible.

[35] Even if $m_2^j$ happens to be the highest possible.

taxes in all but name. By imposing (21) and (23), we are in effect saying that, if the principal is not allowed to use a graduate tax, it is not allowed to charge excess fees and interests either.

Similar arguments apply to the second best. All we have to do is replace $y_{tFB}$ and $d^i_{FB}$ with $y_{tSB}$ and $d^i_{SB}$. The foregoing discussion can be summarized as follows.

**Proposition 4**. (i) *Both a first and a second best can always be implemented using a scholarship scheme financed by a graduate tax; (ii) they can be implemented using a loan scheme if and only if the policy maker is allowed to set tuition fees higher than average total cost*.

Let us now look at the implications of the various additional constraints we are considering, for the kind of student that would be excluded from a loan scheme. Ignore, for the moment, the ceiling on the size of the loan repayment, (23). Recall that, if the no-implicit-tax constraint (21) is binding for some agent, the credit-participation constraint (22) will be binding too. If these constraints are binding for anyone, that will be rich students who can finance their studies entirely out of family resources. The utility of these constrained students is the same whether they do, or do not, get a loan. The principal wants them in the scheme, because he expects them to make a net contribution. Since this contribution will be lower than it would be without (22), however, there will be less to redistribute than in either first or second best. That would have no implications for the number of agents attending university if effort were observable. Since it is not, however, part of the education budget will have to be used for providing costly incentives. The poorest agents may then be excluded from the loan scheme, and consequently from university.

The policy we have outlined may not be feasible if the loan scheme must satisfy also the no-usury constraint (23), because redistribution can then be carried out only by charging rich students higher tuition fees than poor students. But this can only go as far as (21) permits. As tuition fees cannot be higher than $p$, the only way poor students can be subsidized is then by drawing on the external subsidy, $S$. The same may be said of insurance. The principal will be able to offer some insurance only if the general tax payer is willing to pay for it.

Therefore, the subsidy $S$ is needed to cover the cost of setting

$$y_2^j < y_1^j + p$$

for some $j$, in some state of nature. Without this subsidy, poor students could not be charged less than average total cost, and unlucky graduates could not be allowed to pay back less than the capitalized value of the money they borrowed. If $S$ is not sufficiently large, some agents will not get a loan. Which ones? Recall that (21) has the effect of making the credit constraint (22) binding. Recall, also, that the principal wants rich students in the scheme only if they are expected to make a net contribution. But the constraint on the size of the loan repayment (23) prevents that. Furthermore, since loan repayments are conditional on graduate earnings ("income-contingent"), all students, including rich ones, must be subsidized in unfavourable states of nature. Therefore, the richest students will be excluded. If $S$ is sufficiently low, the poorest agents will be excluded too, because the principal will not have enough money to subsidize them. Unlike rich ones, these agents will be excluded not only from credit, but also from university. Since type-$a$ agents are expected to have lower graduate earnings, and thus to make smaller repayments, the poor agents excluded are likely to have a penchant for the arts.

If $S$ is zero,[36] loan repayments cannot be income contingent. In other words, we can only have mortgage-type loans, without redistribution or insurance. Such a scheme will exclude any agent who would not be able to pay back the full capitalized value of his loan in the worst possible state of the world. Since it will allow at least some agents to discount their expected graduate earnings, however, such a scheme will be nonetheless better than *laissez faire.*

# 9 Discussion

We started from the premise that, in the absence of full contingent markets, a number of school leavers who would otherwise choose to invest in a university education will not do so. There is thus scope for raising social welfare by allowing young persons with the right personal characteristics to discount the expected return from a university education (more if this education has positive external effects). The question is how. The language used in the public debate over scholarships and student loans seems deliberately designed to obscure the difference between the two. For example, the expression "income-contingent loan" is used to cover a variety of situations, ranging from conventional mortgage-type loans combined with some earnings insurance, to schemes where the so-called loan repayment bears little or no relation to the size of the so-called loan. In interpreting the analytical solution of the social optimization problem, we have adopted the convention of calling the policy a loan scheme if one can avoid having to make a payment as a graduate by turning down the offer of a payment as a student. Otherwise, we call the payment received by the student a scholarship, and the one made by the graduate a tax.

The straight answer to our central question is that a scholarship scheme financed by a tax on graduate earnings is always at least as good as a loan scheme (indeed, the former will implement the first best allocation if individual effort is observable, the second best if individual effort is not observable). The reason is that, in the case of a loan scheme, the terms of the credit contract must be such, that it will be in a person's interest to accept it. This constraint, absent in the case of a scholarship scheme financed by a graduate tax, is not by itself sufficient to make loan schemes inferior. For that to be the case, there must be another restriction, namely that tuition fees cannot be higher than average total cost (in other words, that universities may be allowed to practice discounts to deserving students, but not to overcharge the rest). This restriction makes no difference if the policy is a scholarship scheme financed by a graduate tax, because the policy maker can then use the tax to redistribute. If the policy is a

[36] In real life, all student loan schemes are subidized out of general tax revenue

loan scheme, however, the restriction on the size of the fee may have the effect of excluding some poor school leavers with the right intellectual qualities from the loan scheme, and thus from higher education.

If, in addition to the constraint on the size of tuition fees, there is also the restriction that a loan repayment cannot be greater than the amount borrowed, capitalized at market interest rates, this will have the effect of excluding not only some poor, but also some rich agents from the scheme. Unlike the poor, however, the rich will not be excluded from university. What this additional restriction effectively says is that, if the education authority is not allowed to use a graduate tax, it will not be allowed to surreptitiously introduce it under the guises of repayment terms more onerous for the rich. Otherwise, the policy would be, in all but name, a scholarship scheme financed by a graduate tax. A similar logic can be found for the constraint on the size of tuition fees. If the government is not allowed to raise taxes for educational purposes, it will not be allowed to impose an implicit tax on students from rich families in the form of excess fees. Garcia-Peñalosa and Wälde (2000) argue that an income-contingent loan scheme subsidized by general taxation is sub-optimal because it is regressive. We have shown that there are other reasons why it may be sub-optimal.

It is clear that, the greater is the education externality, and thus the extent to which the policy is subsidized out of general tax revenue, the higher will be the expected welfare gain associated with the policy. That is true in general, but more so if the policy is a loan scheme. If the education authority is not allowed to use differential tuition fees, or loan repayments, to cross-subsidize poor students, or unlucky graduates, redistribution and insurance will in fact be possible only if the policy is subsidized by the general tax payer. Without such a subsidy, loans could not be income-contingent. Any student who gets a loan would then have to pay it back in full, at market interest rates, as in a conventional mortgage.[37] The outcome would still be better than *laissez faire*, however, because the policy would at least help some of the talented young people, who could not have gone to university otherwise.

The second-best payment due to each student is the sum of two type-specific functions, one decreasing in parental support, the other increasing in degree result. The former may be regarded as a grant, dependent on "need", the latter as either a scholarship or a loan, dependent on "merit". The grant could be paid out at front. The scholarship or loan would have to be paid in installments, as exam results come in. Our finding contrasts with the one in Hanushek et al. (2004), that need-based grants perform better than merit-based ones. The difference is due to the fact those authors do not consider moral hazard. In our framework too, if individual effort were observable, the optimal grant would depend only on need.

The second-best payment due from a graduate is similarly the sum of two type-specific functions, one increasing in the graduate premium, the other de-

[37] Since human capital cannot be mortgaged, however, all unsecured loans are in effect income-contingent, because there is no way of recovering a credit from a poor debtor. Therefore, it is actually impossible for a non-usurarious loan scheme to be entirely self-financing.

creasing in degree result. The sum of these two functions may be interpreted as the difference between a surcharge on ordinary income tax, dependent on graduate earnings, and a discount for good academic performance. Since the policy maker maximizes a sum of expected utilities, rather than total tax revenue, the first-best policy does not oblige every student to take the subjects that promise a higher monetary reward, indeed the opposite. In second best, the "scientific" (short-hand for high expected earnings) type specializes completely in science subjects as in first best, but the "artistic" one is induced to take more of his favourite subjects. On the other hand, the policy may exclude poor agents of the artistic type. If it does not, they will be cross-subsidized by scientific agents.

If the random components of the degree results of the different students tend to move in the same direction, the second-best payment due to a student, and the second-best payment due from a graduate, take account not only of the person's own degree result, but also of those obtained by others. The implication is that the policy maker should not be fooled by grade drift into granting every student a higher scholarship or loan, and charging every graduate a lower tax or loan repayment. By contrast, even if the random components of graduate earnings tend to move together, the second-best tax or loan repayment due from a graduate will optimally depend only on this person's own earnings.

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# Appendix

**A0. Proof of concavity of the expected utility functions, and of the social welfare function**

The expected utility function of each agent is concave in $y_1$, and convex in $y_2$, for the assumption that $u_t(.)$ is concave. To show that the expected utility function is concave also in $d$, and in $e$, we write a student's expected time-2 utility conditional on $x$ as

$$\upsilon(x,d) \equiv \int_{m_2} u_2(w_2 + m_2 - y_2)\, g dm_2, \tag{24}$$

with $y_2 \equiv 0$ in *laissez faire*. The function $\upsilon(.)$ has the following properties:

a) $\upsilon_x(x,d) > 0$, because $G_x(m_2|x,d) \leq 0$, with the strict inequality sign holding for some value of $m_2$ at each $x$ (i.e., because of FOSD of $G(m_2|x,d)$ for higher values of $x$), given that $u_2()$ is increasing in $m_2$;

b) $\upsilon_d(x,d) > 0$, again because $G_d(m_2|x,d) \leq 0$, with the strict inequality sign holding for some value of $m_2$ at each $d$ (i.e., because of FOSD of $G(m_2|x,d)$ for higher values of $d$), given that $u_2()$ is increasing in $m_2$;

c) $\upsilon_{dx}(x,d) = 0$, because $G_{dx} = 0$ implies that an increase in $x$ does not affect the FOSD effect of an increase in $d$ with regard to the distribution of $m_2$. Note that this also implies that $\int_x \upsilon_d(x,d)\, f_e dx = \int_x \int_{m_2} u_2 g_d dm_2 f_e dx = 0$.

The agent's expected utility function can be written as

$$\int_x (u_1(m_1 + y_1) + \upsilon(x,d))\, f dx - z^i(e,d), \tag{25}$$

with $y_1 \equiv 0$ in *laissez faire*. To show that this function is concave in $e$, let us integrate the above expression by parts. Recalling that $x \in X = [\underline{x}, \overline{x}]$, we obtain

$$u_1[(m_1 + y_1(\overline{x})) + \upsilon(\overline{x},d)] - \int_x \left( u_1'(m_1 + y_1) \frac{\partial y_1}{\partial x} + \upsilon_x(x,d) \right) F(x|e) dx - z^i(e,d)$$

The first term is a constant. The term in brackets under the integral sign is positive, because $u_1' > 0$, $\partial y_1 / \partial x > 0$ from the MLR assumption (see the proof of proposition 3 below), and $\upsilon_x(x,d) > 0$. Considering that $F(x|e)$ is convex in $e$ for the CDF assumption, the expression under the integral sign is also convex in $e$. Since $z^i(e,d)$ is also in $e$, it follows that the expected utility function is concave in $e$.

Similarly, to prove that expected utility is concave in $d$, let us integrate the RHS of (24) by parts with respect to $m_2$. Recalling that $m_2 \in M_2 = [0, \overline{m_2}]$, we obtain

$$u_2(w_2, \overline{m_2}) - \int_{m_2} u_2'(w_2 + m_2)\, G(m_2|e,d) dm_2.$$

Again, the first term is a constant, and the expression under the integral sign is convex in $d$, because $u_2'$ is constant with respect to $d$, and $G(.|e,d)$ is convex because of the CDF assumption. Since $z^i(e,.)$ is also convex, it follows that the expected utility function is concave in $d$.

Since the expected utility function of each agent is concave, the social welfare function, which is the sum of individual utility functions, is concave too.

**A1. Proof of Proposition 1**

**A1.1. Proof of parts (i) and (ii)**

In the case of an interior solution, the first-order conditions for (11), are

$$z_d^i(e,d) = \int_x \int_{m_2} u_2(w_2+m_2)\, g_d dm_2 f dx \tag{26}$$

and

$$z_e^i(e,d) = \int_x \left[ u_1(m_1-\theta) + \int_{m_2} u_2(w_2+m_2)\, g dm_2 \right] f_e dx, \tag{27}$$

which simplifies to

$$z_e^i(e,d) = \int_x \int_{m_2} u_2(w_2+m_2)\, g dm_2 f_e dx \equiv \int_x \upsilon(x,d)\, f_e dx, \tag{28}$$

because $u_1$ is constant with respect to $x$, and $\int_x f_e dx = 0$.

In view of (26) and (28), at an interior solution, $(d_{LF}, e_{LF})$ depends only on the expected date-2 gain. Since this expectation is the same for all the students of the same type, all type-$i$ students will choose the same subject mix and effort level, $\left(d_{LF}^i, e_{LF}^i\right)$. Where type $a$ is concerned, the choice of subject mix is always interior, because $z_{d^a}^a > 0$. Therefore, $0 < d_{LF}^a < 1$. Where type $s$ is concerned, the choice of subject mix is always at a corner, because $z_{d^s}^s < 0$. Therefore, $d_{LF}^s = 1$.

The effort level $e_{LF}^i$ is determined by (28) for $i = a, s$. To prove that $e_{LF}^s > e_{LF}^a$, suppose that the $s$ type chooses $e^s = e_{LF}^a$. Given the assumptions 2, 3 and 6 on the form of $z^i(e,d)$, this implies that the LHS of equation (28) is lower for the $s$ than for the $a$ type. On the other hand, the RHS is higher for the $s$ than for the $a$ type, because of b) and c) in **A0**. Since the $a$ type is choosing optimally, we know that (28) holds for $i = a$. As a consequence, for the $s$ type, the RHS would be higher than the LHS, implying that the $e^s$ must be raised above $e_{LF}^a$ for (28) to hold. As $e^s$ rises, $z_e^s$ will in fact increase because of assumption 3, while $\int_x \int_{m_2} u_2(w_2+m_2)\, g dm_2 f_e dx$ will decrease because $\int_x \int_{m_2} u_2(w_2+m_2)\, g dm_2 f dx$ is concave in $e$.□

**A1.2. Proof of part (iii)**

Consider (11). For any given $m_1$, it must be the case that

$$\int_x \int_{m_2} u_2(w_2+m_2)\, g(m_2|e_{LF}^s, 1) dm_2 f(x|e_{LF}^s) dx - z^s(e_{LF}^s, 1) >$$
$$\int_x \int_{m_2} u_2(w_2+m_2)\, g(m_2|e_{LF}^a, d_{LF}^a) dm_2 f(x|e_{LF}^a) dx - z^a(e_{LF}^a, d_{LF}^a)$$

Otherwise a type-$s$ student could increase his utility by choosing $e^s = e_{LF}^a$, and $e_{LF}^s$ would not be optimal. It then follows immediately that $\widetilde{m}_1^a > \widetilde{m}_1^s$.□

**A2. Proof of Proposition 2**

Let us first look at the characteristics of the solution when participation constraints (15) are not binding ($\nu^j = 0$). We will then show that (15) cannot be binding for any agent.

**A2.1. Proof of parts (i), (ii) and (iii)**

In first best, $\mu^j = 0$ for all $j$. Therefore, given that $\nu^j = 0$, (17) reduces to

$$u_1'\left(m_1^j + y_1^j\right) = \lambda, \tag{29}$$

and (18) to

$$u_2'\left(w_2 + m_2^j - y_2^j\right) = \lambda, \tag{30}$$

implying that

$$c_1^j = c_2^j = c \tag{31}$$

irrespective of $m_1^j$, and of the realizations of $x^j$ and $m_2^j$. On the other hand, (29) implies part (ii), and (30) part (iii) of the proposition.□

**A2.2. Proof of parts (iv) and (v)**

Using $\mu^j = \nu^j = 0$, (29) and

$$\int_{m_2^j} g_{d^j}^j dm_2^j = \int_{x^j} f_{e^j}^j dx^j = 0,$$

the conditions on $d^j$ and $e^j$, (19$a$) and (20), may be written as

$$z_{d^j}^j = \lambda \int_{x^j} \int_{m_2^j} y_2\left(m_2^j\right) g_{d^j}^j dm_2^j f^j dx^j \tag{32}$$

and

$$z_{e^j}^j = \lambda \int_{x^j} \int_{m_2^j} y_2\left(m_2^j\right) g dm_2 f_{e^j} dx^j \tag{33}$$

In view of (32) and (33), at an interior solution, $\left(d_{FB}^j, e_{FB}^j\right)$ depend only on the expected date-2 gain. Since this expectation is the same for all the students of the same type, all type-$i$ students will choose the same subject mix and effort level, $\left(d_{FB}^i, e_{FB}^i\right)$. Where type $a$ is concerned, the choice of subject mix is always interior, because $z_{d^a}^a > 0$. Therefore, $0 < d_{FB}^a < 1$. Where type $s$ is concerned, the choice of subject mix is always at a corner, because $z_{d^s}^s < 0$. Therefore, $d_{FB}^s = 1$.

The level of effort $e_{FB}^i$ is determined by (33) for $i = a, s$. Note, first of all, that $y_2(.)$ is increasing in $m_2^j$ like $u_2(.)$, and $\int_{x^j}\int_{m_2^j} y_2\left(m_2^j\right) gdm_2^j f dx^j$ concave in $e^j$ like $\int_{x^j}\int_{m_2^j} u_2\left(w_2 + m_2^j\right) gdm_2^j f dx^j$. As a consequence we can follow the same line of reasoning as in the proof of b) and c) of A0, to show that b') $\int_{m_2^j} y_2\left(m_2^j\right) g_{d^j}^j dm_2^j > 0$ and c') $\int_{x^j}\int_{m_2^j} y_2\left(m_2^j\right) g_{d^j} dm_2 f_{e^j} dx^j = 0$. To prove that $e_{FB}^s > e_{FB}^a$, suppose that the $s$ type chooses $e^s = e_{FB}^a$. Given assumptions 2, 3 and 6 on $z^i$, this implies that the LHS of (33) is lower for the $s$ than for the

$a$ type. On the other hand, given b') and c'), the RHS is higher for the $s$ than for the $a$ type. Since type-$a$ students are choosing optimally, we know that (33) holds for $i = a$. As a consequence, for the $s$ type, the RHS would be higher than the LHS, implying that $e^s$ must be raised above $e^a_{FB}$ for (33) to hold. As $e^s$ rises, $z^s_{e^j}$ increases because of assumption 3, while $\int_{x^j} \int_{m^j_2} y_2 \left(m^j_2\right) gdm^j_2 f_{e^j} dx^j$ decreases for concavity of $\int_{x^j} \int_{m^j_2} y_2 \left(m^j_2\right) gdm^j_2 f dx^j$.□

**A2.3. Proof of part (vi)**

Recall that, by definition, an agent is an individual that would profit from higher education if he were able to trade in a complete market system. Provided he were able to get credit and to insure against low academic results and low earnings, an agent would have a higher expected utility level from going to university than from going straight to the labour market. Consider the poorest among the agents, i.e. those individuals with $m^j_1 = \underline{m}^j_1$. The scheme provides them precisely with insurance and credit. Moreover it redistributes in their favour. As a consequence they will go to university, and the same will be true for agents with $m^j_1 > \underline{m}^j_1$.□

**A3. Proof of Proposition 3**

**A3.1. Proof of parts (i)-(iii)**

To check that $\mu^j$ is positive, we re-write the first-order condition on $y^j_1$, (17), as

$$\frac{\lambda}{u'_1\left(m^j_1 + y^j_1\right)} = 1 + \nu^j + \mu^j \frac{f_{e^j}(x^j|e^j)}{f(x^j|e^j)}, \tag{34}$$

where $\nu^j$ is equal to zero if the participation constraint (15) is not binding. Given that $\frac{f_{e^j}}{f^j}$ is increasing in $x^j$ for the MLR assumption, if $\mu^j \leq 0$, the agent would always choose the lowest possible level of effort. With $\mu^j < 0$ $(= 0)$, the RHS of (35) is decreasing (constant) in $x^j$, while the LHS is increasing in $c^j_1$, itself an increasing function of $y^j_1$. Hence, $y^j_1$ should be a decreasing function of $x^j$, but this implies that the agent will always choose the lowest possible level of effort. Hence, $\mu^j$ is positive.

Given $\mu^j > 0$, $y^j_1$ depends on $x^j$, as well as $m^j_1$. Since $f_{e^j}/f$ and, therefore, the RHS of (34) is increasing in $x^j$, while the LHS is increasing in $c^j_1$, $y^j_1$ is decreasing in $m^j_1$, and increasing in $x^j$.

The first-order condition on $y^j_2$ , (18), may similarly be re-written as

$$\frac{\lambda}{u'_2\left(w_2 + m^j_2 \ - y^j_2\ \right)} = 1 + \nu^j + \mu^j \frac{f_{e^j}(x^j|e^j)}{f(x^j|e^j)}. \tag{35}$$

Since the LHS of (35) is increasing in $c^j_2$, $y^j_2$ is again increasing in $m^j_2$. Since the RHS of (35) is increasing in $x^j$, and $u'_2$ in $y^i_2$, the latter is decreasing in $x^j$.

The separable form of the payment function derives from the fact that $m^j_1$ does not enter the RHS of (34), and $m^j_2$ that of (35). Note that, given (34) and (35), $u'_t$ varies $x^j$, but not with $m^j_t$, $t = 1, 2$. This can be achieved by compensating for differences in $m_t^j$, and giving all agents of the same type the same marginal incentive. All agents of the same type have then the same expected consumption at both dates. Since different types of student have different costs, $\mu^i$ will vary across types, implying that there is a different $y_t^i(.)$ for each $i$.□

**A3.2. Proof of parts (iv)-(v)**

Given that agents are compensated for differences in $m_t^j$, all type-$i$ students make the same choice of $d^i$ and $e^i$, $i = a, s$. Given $z_{d^s}^s < 0$, it follows immediately that $d_{SB}^s = 1$. The level of $d_{SB}^a$ is determined by $(19a)$. Given that the last LHS term of $(19a)$, $\mu^j z_{e^j d^j}^j$ is positive, $d_{SB}^a$ is lower than would be optimal given the $y_t^a(.)$ schedules $(t = 1, 2)$.

Given that, for $d_{SB}^s = 1$ and $0 < d_{SB}^a < 1$, effort is less costly for an $s$ than for an $a$ type. Moreover $e^s$ has a higher positive effect on the agent's expected income, and hence on the principal's budget constraint, than $e^a$. It then follows that $e_{SB}^s > e_{SB}^a$. $e_{SB}^i$ is determined by the incentive compatibility constraint (14).That $e_{SB}^i$ is inefficiently low given the $y_t^i(.)$ schedules $(t = 1, 2)$, follows from the fact that (14) ignores the marginal effect of $e_{SB}^i$ on the principal's budget constraint, measured by the second LHS term of (20). Such effect is positive because $\lambda > 0$, the first LHS term of (20) is equal to zero, and the last LHS term is negative.□