# COMPETITION BETWEEN STATE UNIVERSITIES 

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# Competition between State Universities 

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#### Abstract

We analyse how state university competition to collect resources may affect both the quality of teaching and research. By considering a set-up where two state universities behave strategically, we model their interaction with potential students as a sequential noncooperative game. We show that different types of equilibrium may arise, depending on the mix of research and teaching activity supplied by each university, and the mix of low and high ability students attending each university.


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JEL Classification: H52; I22; I23.

[^0]
## 1 Introduction

Notwithstanding researchers are part of it, the economic literature on education has traditionally ignored the competition between public universities for students and public funding (Boroah (1994), De Fraja and Iossa (2002), Johnes (2007)). Instead, there exist several theoretical and empirical works on competition between private and public schools and universities (Epple and Romano (1998, 2008), Bailey et al. (2004), Bertola and Checchi (2003), Oliveira (2006)).

This paper aims to analyse how state university competition to collect resources may affect both the quality of teaching and research. In this respect, two main remarks are in order. First, as it was suggested by Rothschild and White (1995), universities compete for students because universities adopt a customer-input technology, i.e. students are at once inputs and customers of the educational process. More precisely, students are inputs needed by universities to produce education, and they also provide funds to universities both by paying tuition fees, and allowing universities to receive transfers from the government. Second, Cohn and Cooper (2004) stress the fact that universities can be seen as multi-products institutions which supply three types of output: teaching, research, and public services. Teaching has the aim to deliver knowledge both at undergraduate and postgraduate level. Research has, instead, the aim to create knowledge with externalities for all society, and it may be considered as complementary to teaching, in case of postgraduate courses, while it is probably substitutable, in case of undergraduate courses. Finally, universities produce a third output which can be thought of as a public service: for example, in Italy, as well as in many other countries, university diploma have a legal value.

To tackle such an issue, we consider a second best set-up where two state autonomous universities behave strategically: ${ }^{1}$ their interaction with the potential students is thus modelled as a sequential noncooperative game. Given a public funding mechanism, at the first stage, the two universities choose their tuition fees and investments in teaching and research, which will determine their quality on the basis of different production functions; at the second stage, students choose which university to attend depending on a benefit-cost comparison. Under the assumption of perfect mobility of students, the cost of attending one university or the other depends on tuition fees (for simplicity, other costs are assumed equal). The benefit derived by attending one university or the other, instead, depends not only on the quality of teaching, but also on each student ability and medium ability of students attending each university, i.e. a peer group effect (Epple and Romano (1998)).

By solving the model, we can show that different types of equilibrium may arise, depending on the mix of research and teaching activity supplied by each university, and the mix of low and high ability students attending each university. More precisely, each equilibrium is characterized by two points of view. On the one side, universities may choose to specialize only in research or teaching, or instead to supply both of them. On the other side, students with different ability allocate between universities in different ways. Possible equilibria are the following: 1) an equilibrium where there is

[^1]complete segregation, i.e. all high ability students attend one university, and all low ability students attend the other university; 2) an equilibrium where all high ability students attend one university, and low ability students attend both universities; 3) an equilibrium where all students attend one university, and the other only produces research.

Our paper is related to two strands of economic literature which we try to combine in order to gather some new hints on universities' incentives. More specifically, we refer to both the literature on public universities competition, and the literature on capital tax competition with household mobility.

As we stressed above, the economic literature has devoted limited attention to public universities competition, even if some recent papers have tried to shed some light on such an issue. Del Rey (2001) uses a spatial competition model to analyse a game between two universities which provide both research and teaching, and use admission standards to control average ability of enrolled students. Depending on preferences, technologies, and public policies, different types of symmetric equilibrium may arise: both universities admit only some of the applicants and provide research; both universities satisfy all students' demand and provide research; both are "research only" universities. In a related paper, De Fraja and Iossa (2002) focus the attention on how students' mobility costs may affect the equilibrium configuration. In particular, if such a mobility cost is high, as in Del Rey (2001), the equilibrium is symmetric, and both universities admit the same number of students, and research investments are also the same. However, if mobility costs are sufficiently low, the resulting equilibrium, when it exists, is asymmetric, i.e. one university admit the best students, and provides more research than the other ("élite institution"). ${ }^{2}$ More recently, Kemnitz (2007) examines how different public funding schemes may affect universities' competition, and thus the quality of their teaching and research activities.

The literature on capital tax competition is instead quite large (Wellish (2000), Hindriks and Myles (2006)). In this respect, a familiar result is the one that shows that, when households are perfectly immobile, tax competition for perfectly mobile capital results in an underprovision of local public goods. However, such a result does not hold anymore when households are allowed to be perfectly mobile. This is due to the fact that fiscal externalities which are at the basis of the result on local public good underprovision disappear when households are mobile: each region/country internalizes the effects of its own policies on the welfare of nonresidents by taking the migration equilibrium into account. Accordingly, introducing mobility of households in the standard capital tax competition model mitigates the downward pressure on local public goods provision.

The aim of this paper is thus to combine these two strands of literature in order to analyse how students' mobility affect universities' competition on both tuition fees, and expenditure in research and teaching activities. To perform such a task, contrary to most of the existing literature on state university competition, we do not use a spatial competition model, but we use the methodological tools offered by the literature on capital tax competition. Further, in our paper, universities do not

[^2]set admission standards, thus students are free to attend which university they prefer on the basis of a cost-benefit analysis. This scenario fits better the European set-up than the U.S. one, and is probably more suitable to describe undergraduate degrees.

The plan of the paper is as follows. Section 2 describes the model. Section 3 analyses students' university choice and characterizes the different type of stable equilibria which may arise. Section 4 examines how universities compete with respect to their choice of tuition fees and research and teaching expenditure. Finally, section 5 contains some concluding remarks.

## 2 The model

Consider two universities $j, j=A, B$, operating in the same district, and differing with respect to quality of teaching, $q_{j}$, and quality of research, $r_{j}$. Students have to choose which university to attend. Students differ with respect to their ability, $e^{i}$, which can be high, $e^{h}$, or low, $e^{l}$, i.e. $e^{h}>e^{l}$. The preferences of each student, are represented by the following utility function

$$
\begin{equation*}
U_{j}^{i}=U^{i}\left(q_{j}\right)-b_{j}, \quad i=h, l ; \quad j=A, B, \tag{1}
\end{equation*}
$$

where $b_{j}$ denotes the per-student tuition fee paid to university $j$. We assume that high ability students derive a higher level of utility from any given level of $q_{j}$, i.e. $U^{h}\left(q_{j}\right)>U^{l}\left(q_{j}\right)$, and that university quality positively affects students' utility, $\frac{\partial U^{i}}{\partial q_{j}}>0$, with $\frac{\partial U^{h}}{\partial q_{j}}>\frac{\partial U^{l}}{\partial q_{j}}$. The exogenous total number of students is $\bar{N}=\sum_{i=h, l} \bar{N}^{i}$, where $\bar{N}^{h}$ is the total number of high ability students, and $\bar{N}^{l}$ the total number of low ability students. We assume that all students attend one of the two universities and thus $\bar{N}=n_{A}+n_{B}$, where $n_{j}$ denotes the total number of students attending university $j, j=A, B$. Further, $n_{j}^{i}, i=h, l$, denotes the total number of students belonging to each type and attending each university so that $n_{j}=\sum_{i=h, l} n_{j}^{i}, j=A, B$, and $\bar{N}^{i}=\sum_{j=A, B} n_{j}^{i}, i=h, l$. Let us denote $\bar{e}_{j}$ the average ability of students attending university $j$. Accordingly, the average ability of students attending university $j, j=A, B$, obtains as

$$
\begin{equation*}
\bar{e}_{j}=\frac{\sum_{i=h, l} n_{j}^{i} e^{i}}{n_{j}}=\lambda_{j}^{h} \Delta+e^{l}, \tag{2}
\end{equation*}
$$

with $\lambda_{j}^{h} \equiv \frac{n_{j}^{h}}{n_{j}}$, and $\Delta \equiv e^{h}-e^{l}$.
Each university may receive two types of transfer from the government. Let $t_{j} \geq 0$ denote a per-student transfer to university $j$, and $\tau_{j} \geq 0$ denote a lump-sum transfer, $j=A, B$. Accordingly, the budget constraint of each university $j, j=A, B$, obtains as

$$
\begin{equation*}
\left(t_{j}+b_{j}\right) n_{j}+\tau_{j}=T_{j}+R_{j}, \quad j=A, B \tag{3}
\end{equation*}
$$

where $T_{j} \geq 0$ and $R_{j} \geq 0$ represent expenditure on teaching and research by university $j, j=A, B$, respectively. Notice that universities are not constrained in the use of the transfers. The sums thus received can be used either to finance teaching or research.

Further, we assume that each university produces teaching activity according to the following production function ${ }^{3}$

$$
\begin{equation*}
q_{j}=\alpha \bar{e}_{j}+\beta \frac{T_{j}}{n_{j}}, \quad j=A, B, \text { with } q_{j}=0 \text { when } n_{j}=0 \tag{4}
\end{equation*}
$$

and produces research according to the following production function with decreasing returns

$$
\begin{equation*}
r_{j}=R_{j}^{\gamma_{j}}, \quad j=A, B, 0<\gamma_{j}<1 \tag{5}
\end{equation*}
$$

Thus, in this set-up, each university can improve the quality of its teaching by augmenting its teaching expenditure, for example by increasing the teacher/students ratio, and it can improve the quality of its research by augmenting its expenditure on research activity, for example, by recruiting better researchers and by purchasing more sophisticated equipments (De Fraja and Iossa (2002)).

Finally, each university cares about both teaching and research and thus we assume the following objective functions ${ }^{4}$

$$
\begin{equation*}
W_{j}=\sum_{i=h, l} n_{j}^{i} q_{j}+r_{j}, \quad j=A, B . \tag{6}
\end{equation*}
$$

The game is solved by backward induction. We first examine the students' decision on which university to attend and then the universities' decisions on tuition fees, on research and teaching expenditure.

## 3 Students' university choice and characterization of stable equilibria

If both universities enrol students of a given type, at equilibrium, those students must be indifferent with respect to which university to attend. This implies that the following arbitrage condition has to hold ${ }^{5}$

$$
\begin{equation*}
U^{i}\left(q_{A}\right)-b_{A}=U^{i}\left(q_{B}\right)-b_{B}, \quad i=h, l . \tag{7}
\end{equation*}
$$

The quality of teaching depends on per-student expenditure and on average students' ability. It is consequently affected both by the number of students and by the proportion of high ability individuals. By using (4), and (3) into (1), the effect of the number of students on individual utility obtains as

$$
\begin{equation*}
\frac{d U^{i}}{d n_{j}^{i}}=\frac{\partial U^{i}}{\partial q_{j}} \frac{\partial q_{j}}{\partial n_{j}^{i}}, \quad i=h, l ; \quad j=A, B . \tag{8}
\end{equation*}
$$

[^3]Accordingly, $\operatorname{sign} \frac{d U^{i}}{d n_{j}^{i}}=\operatorname{sign} \frac{\partial q_{j}}{\partial n_{j}^{i}}$, because $\frac{\partial U^{i}}{\partial q_{j}}>0$ by assumption. Further, by using (2) and (3) into (4), the effect of the number of students on teaching quality obtains as

$$
\begin{equation*}
\frac{\partial q_{j}}{\partial n_{j}^{i}}=\alpha \frac{\partial \bar{e}_{j}}{\partial n_{j}^{i}}+\beta \frac{\partial\left(T_{j} / n_{j}\right)}{\partial n_{j}^{i}}, \quad i=h, l, \quad j=A, B, \tag{9}
\end{equation*}
$$

when $q_{j}>0$. More specifically, for high ability students, $i=h$, equation (9) rewrites as

$$
\begin{equation*}
\frac{\partial q_{j}}{\partial n_{j}^{h}}=\frac{1}{n_{j}^{2}}\left[\alpha \Delta n_{j}^{l}+\beta\left(R_{j}-\tau_{j}\right)\right], \quad j=A, B, \tag{10}
\end{equation*}
$$

and for low ability students, $i=l$, equation (9) rewrites as

$$
\begin{equation*}
\frac{\partial q_{j}}{\partial n_{j}^{l}}=\frac{1}{n_{j}^{2}}\left[-\alpha \Delta n_{j}^{h}+\beta\left(R_{j}-\tau_{j}\right)\right], \quad j=A, B \tag{11}
\end{equation*}
$$

Notice that the effect of $n^{i}$ on quality depends on two terms. The first one represents the direct effect of an additional student on average quality and is positive (negative) for high (low) ability students. The second one represents the indirect effect that an additional student has on perstudent teaching expenditure and is positive (negative) if research expenditure is higher (lower) than the lump-sum transfer. The reason is that the excess of research expenditure over the lump sum transfer is financed by the fees paid by a higher (lower) number of students.

We are now in a position to determine the sign of $\frac{\partial q_{j}}{\partial n_{j}^{2}}, i=h, l$.
Lemma 1: i) $\frac{\partial q_{j}}{\partial n_{j}^{h}}>0$ if $\tau_{j}-R_{j}<\frac{\alpha}{\beta} \Delta n_{j}^{l}$, with $n_{j}^{l} \geq 0 ; \quad \frac{\partial q_{j}}{\partial n_{j}^{l}}>0$ if $R_{j}-\tau_{j}>\frac{\alpha}{\beta} \Delta n_{j}^{h}$, with $n_{j}^{h} \geq 0 ;$ ii) $\frac{\partial q_{j}}{\partial n_{j}^{h}}<0$ if $\tau_{j}-R_{j}>\frac{\alpha}{\beta} \Delta n_{j}^{l}$, with $n_{j}^{l} \geq 0 ; \quad \frac{\partial q_{j}}{\partial n_{j}^{j}}<0$ if $R_{j}-\tau_{j}<\frac{\alpha}{\beta} \Delta n_{j}^{h}$, with $n_{j}^{h} \geq 0$.

Notice that for $q_{j}=0$ it is

$$
\left.\frac{\partial q_{j}}{\partial n_{j}^{i}}\right|_{q_{j}=0}=\alpha e_{j}^{i}+\beta\left(t_{j}+b_{j}+\tau_{j}-R_{j}\right)>0, \quad j=A, B,
$$

because $t_{j}+b_{j}+\tau_{j}-R_{j}=T_{j} \geq 0$.
The sign of $\frac{\partial q_{j}}{\partial n_{j}^{i}}, i=h, l, j=A, B$, is crucial in determining the type of locally stable equilibrium which occurs at the students' subgame. In this respect, we can state the following

Proposition 1 There does not exist a stable equilibrium where $h$ students attend both university $A$ and $B$.

Proof. We divide the proof in two cases, showing that there cannot exist: i) a stable equilibrium where all $l$ students attend university $A$ and $h$ students attend both university $A$ and $B$; ii) a stable equilibrium where both $l$ and $h$ students attend both university $A$ and $B$. i) Suppose, contrary to proposition 1 , that there exists a stable equilibrium where all $l$ students choose university $A$ and $h$ students attend both university $A$ and $B$. Stability requires that $\frac{\partial q_{j}}{\partial n_{j}^{h}}<0, j=A, B$, and $\frac{\partial q_{A}}{\partial n_{A}^{L}}>0$. Let us consider university $A$. In order to have $\frac{\partial q_{A}}{\partial n_{A}^{h}}<0$, it must be that $\tau_{A}-R_{A}>\frac{\alpha}{\beta} \Delta \bar{N}^{l}>0$,
from Lemma 1. But, in order to have $\frac{\partial q_{A}}{\partial n_{A}^{\prime}}>0$, Lemma 1 prescribes that $R_{A}-\tau_{A}>\frac{\alpha}{\beta} \Delta n_{A}^{h}>0$, which contradicts the previous condition. ii) In order to have a stable equilibrium where both $l$ and $h$ students attend both university $A$ and $B$ the following conditions should be satisfied:

$$
\begin{aligned}
& U^{h}\left(\alpha\left(e^{l}+\frac{\Delta n_{A}^{h}}{n_{A}^{h}+n_{A}^{l}}\right)+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{n_{A}^{h}+n_{A}^{l}}\right)\right)-b_{A}= \\
& =U^{h}\left(\alpha\left(e^{l}+\frac{\Delta n_{B}^{h}}{n_{B}^{h}+n_{B}^{l}}\right)+\beta\left(t_{B}+b_{B}+\frac{\tau_{B}-R_{B}}{n_{B}^{h}+n_{B}^{B}}\right)\right)-b_{B} \\
& U^{l}\left(\alpha\left(e^{l}+\frac{\Delta n_{A}^{h}}{n_{A}^{h}+n_{A}^{l}}\right)+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{n_{A}^{h}+n_{A}^{A}}\right)\right)-b_{A}= \\
& =U^{l}\left(\alpha\left(e^{l}+\frac{\Delta n_{B}^{h}}{n_{B}^{h}+n_{B}^{l}}\right)+\beta\left(t_{B}+b_{B}+\frac{\tau_{B}-R_{B}}{n_{B}^{B}+n_{B}^{B}}\right)\right)-b_{B}
\end{aligned}
$$

But, given the assumption that $\frac{\partial U^{h}}{\partial q_{j}}>\frac{\partial U^{l}}{\partial q_{j}}$, these equations cannot be simultaneously satisfied.
The reason why a situation where $h$ students are found in both universities cannot represent a stable equilibrium is that an additional $h$ student tends to improve the quality of the university he enrols in. Consequently, it is profitable for $h$ students to concentrate in the same university. ${ }^{6}$ We are then left with the following stable equilibria: ${ }^{7}$,

Equilibrium I: all $h$ students go to university $A$ and all $l$ students go to university $B$.
Equilibrium II: all $h$ students go to university $A$ and $l$ students attend both university $A$ and $B$.

Equilibrium III: all students go to university $A$. University $B$ only produces research.

## Equilibrium I

For all $h$ students to choose university $A$ and all $l$ students to choose university $B$, the following conditions must be satisfied: ${ }^{8}$

$$
\begin{equation*}
U^{h}\left(\alpha e^{h}+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{\bar{N}^{h}}\right)\right)-b_{A} \geq U^{h}\left(\alpha e^{l}+\beta\left(t_{B}+b_{B}+\frac{\tau_{B}-R_{B}}{\bar{N}^{l}}\right)\right)-b_{B} \tag{12}
\end{equation*}
$$

and

$$
\begin{equation*}
U^{l}\left(\alpha e^{l}+\beta\left(t_{B}+b_{B}+\frac{\tau_{B}-R_{B}}{\bar{N}^{l}}\right)\right)-b_{B} \geq U^{l}\left(\alpha e^{h}+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{\bar{N}^{h}}\right)\right)-b_{A} . \tag{13}
\end{equation*}
$$

In order for this equilibrium to be stable it must be the case that $\frac{\partial q_{j}}{\partial n_{j}^{i}}>0, j=A, B, i=l, h$. This means that the effect of the number of students on quality is positive, i.e. that for low ability students the indirect effect through teaching expenditure is higher than the direct effect through

[^4]the level of average ability. By Lemma 1 , this equilibrium arises if and only if $R_{j}>\tau_{j}, j=A, B$, i.e. if the lump-sum transfer is not high enough to cover research expenditure. In this case an increase in the number of students raises per-student teaching expenditure as it reduces the per-student amount of resources substracted from teaching activity. Consequently the quality of its teaching increases.

## Equilibrium II

For all $h$ students to choose university $A$ and $l$ students attend both university $A$ and $B$, the following conditions must be satisfied:

$$
\begin{gathered}
U^{h}\left(\alpha\left(e^{l}+\frac{\Delta \bar{N}^{h}}{\bar{N}^{h}+n_{A}^{l}}\right)+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{\bar{N}^{h}+n_{A}^{l}}\right)\right)-b_{A}> \\
U^{h}\left(\alpha e^{l}+\beta\left(t_{B}+b_{B}+\frac{\tau_{B}-R_{B}}{n_{B}^{l}}\right)\right)-b_{B}
\end{gathered}
$$

and

$$
\begin{gathered}
U^{l}\left(\alpha e^{l}+\beta\left(t_{B}+b_{B}+\frac{\tau_{B}-R_{B}}{n_{B}^{l}}\right)\right)-b_{B}= \\
=U^{l}\left(\alpha\left(e^{l}+\frac{\Delta \bar{N}^{h}}{\bar{N}^{h}+n_{A}^{l}}\right)+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{\bar{N}^{h}+n_{A}^{l}}\right)\right)-b_{A} .
\end{gathered}
$$

In order for this equilibrium to be stable it must be the case that $\frac{\partial q_{j}}{\partial n_{j}^{h}}>0, j=A, B$, and $\frac{\partial q_{j}}{\partial n_{j}^{l}}<0, j=A, B$. This means that quality increases with high ability students and decreases with low ability ones for both universities. By Lemma 1 this implies $-\frac{\alpha}{\beta} \Delta \bar{N}^{h}<\tau_{A}-R_{A}<\frac{\alpha}{\beta} \Delta n_{A}^{l}$, and $0<\tau_{B}-R_{B}<\frac{\alpha}{\beta} \Delta n_{B}^{l}$. For university $B$ the lump sum transfer $\tau_{B}$ must exceed research expenditure. Funds in excess can thus be used to improve teaching quality. As a consequence university $B$ has no need to attract too many students. For university $A, \tau_{A}$ may exceed or be lower than $R_{A}$. In both cases, however, there is an incentive to attract students in order to finance teaching.

Further, for low ability students, we can state the following
Lemma 2. At equilibrium II, for low ability students it is $\frac{d n_{j}^{l}}{d b_{j}} \leq 0, \frac{d n_{j}^{l}}{d R_{j}}<0, \frac{d n_{j}^{l}}{d t_{j}}>0$, and $\frac{d n_{j}^{l}}{d \tau_{j}}>0$.
Proof. By totally differentiating (7), the following equation obtains

$$
\begin{gather*}
\frac{\partial U_{A}^{i}}{\partial q_{A}} \sum_{i=h, l} \frac{\partial q_{A}}{\partial n_{A}^{i}} d n_{A}^{i}+\frac{\partial U_{A}^{i}}{\partial q_{A}} \frac{\partial q_{A}}{\partial R_{A}} d R_{A}+\frac{\partial U_{A}^{i}}{\partial q_{A}} \frac{\partial q_{A}}{\partial A_{A}} d b_{A}-d b_{A}+ \\
-\frac{\partial U_{B}^{i}}{\partial q_{B}} \sum_{i=h, l} \frac{\partial q_{B}^{B}}{\partial n_{B}^{i}} d n_{B}^{i}-\frac{\partial U_{B}^{i}}{\partial q_{B}} \frac{\partial q_{B}}{\partial R_{B}} d R_{B}-\frac{\partial U_{B}^{i}}{\partial q_{B}} \frac{\partial q_{B}}{\partial b_{B}} d b_{B}+d b_{B}=0 . \tag{14}
\end{gather*}
$$

By using the market clearing condition, $d n_{B}^{i}=d n_{A}^{i}, i=h, l$, into (14), for low ability students, $i=l$, it follows that

$$
\begin{equation*}
\frac{d n_{j}^{l}}{\partial b_{j}}=-\frac{\beta \frac{\partial U_{j}^{l}}{\partial q_{j}}-1}{J^{l}}, \quad j=A, B \tag{15}
\end{equation*}
$$

$$
\begin{align*}
& \frac{d n_{j}^{l}}{\partial R_{j}}=\frac{\frac{\beta}{n_{j}} \frac{\partial U_{j}^{l}}{\partial q_{j}}}{J^{l}}, \quad j=A, B,  \tag{16}\\
& \frac{d n_{j}^{l}}{\partial t_{j}}=-\frac{\beta \frac{\partial U_{j}^{l}}{\partial q_{j}}}{J^{l}}, \quad j=A, B, \tag{17}
\end{align*}
$$

and

$$
\begin{equation*}
\frac{d n_{j}^{l}}{\partial \tau_{j}}=-\frac{\frac{\beta}{n_{j}} \frac{\partial U_{j}^{l}}{\partial q_{j}}}{J^{l}}, \quad j=A, B, \tag{18}
\end{equation*}
$$

where

$$
\begin{equation*}
J^{l}=\sum_{j=A, B} \frac{\partial U_{j}^{l}}{\partial q_{j}} \frac{\partial q_{j}}{\partial n_{j}^{l}}, \quad j=A, B \tag{19}
\end{equation*}
$$

Given that $\frac{\partial q_{j}}{\partial n_{j}^{l}}<0$, in (19) $J^{l}<0$ since $\frac{\partial U_{j}^{l}}{\partial q_{j}}>0$, by assumption. Then $\frac{d n_{j}^{l}}{d R_{j}}<0, \frac{d n_{j}^{l}}{d t_{j}}>0$, and $\frac{d n_{j}^{l}}{d \tau_{j}}>0$, follow immediately from (16), (17), and (18), respectively. Moreover, it follows from (15) that $\frac{\partial U_{j}^{l}}{\partial q_{j}} \gtreqless \frac{1}{\beta} \Longleftrightarrow \frac{d n_{j}^{l}}{d b_{j}} \gtreqless 0$. Note however that it cannot be $\frac{\partial U_{j}^{l}}{\partial q_{j}}>\frac{1}{\beta}$ at equilibrium because this would imply that students' utility could be increased by increasing $b_{j}$ (which would obviously improve also universities' welfare). Hence $\frac{d n_{j}^{l}}{d b_{j}} \leq 0$.

We can see that with a low $\beta$, i.e. a low impact of per-student teaching expenditure on quality, it is quite likely that a large number of low skill students decide to move away from the university with a higher tuition fee. On the contrary, the location choice of high ability students is not affected by marginal changes in the policy variables, $b_{j}, t_{j}, \tau_{j}$, and $R_{j}$, because the corresponding locally stable equilibrium is a corner one.

## Equilibrium III

For all students to choose university $A$, so that university $B$ only produces research, the following conditions must be satisfied:

$$
\begin{equation*}
U^{h}\left(\alpha\left(e^{l}+\frac{\Delta \bar{N}^{h}}{\bar{N}}\right)+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{\bar{N}}\right)\right)-b_{A} \geq U^{h}(0)=0 \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
U^{l}\left(\alpha\left(e^{l}+\frac{\Delta \bar{N}^{h}}{\bar{N}}\right)+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{\bar{N}}\right)\right)-b_{A} \geq U^{l}(0)=0 \tag{21}
\end{equation*}
$$

where $b_{B}=0$. In order for this equilibrium to be stable it must be the case that $\frac{\partial q_{A}}{\partial n_{A}^{2}}>0, i=h, l$. By Lemma 1, this implies that $R_{A}-\tau_{A}>\frac{\alpha}{\beta} \Delta \bar{N}^{h}$. Notice that for $n_{B}=0, \frac{\partial q_{B}}{\partial n_{B}^{i}}>0$ and $R_{B}=\tau_{B}$. In words, this means that equilibrium III arises if university $A$ 's investment in research, $R_{A}$, is greater than the transfer received by the government to finance research, $\tau_{A}$, and the effect of an increase in the number of low ability students on university $A$ 's investment in teaching is greater than the effect on university $A$ 's average ability of students. University $B$ only produces research, and thus the government only provides a lump-sum transfer which is entirely spent on research.

Further, at equilibrium III, the location choice of both high and low ability students is not affected by marginal changes in the policy variables, i.e. $b_{j}, t_{j}, \tau_{j}$, and $R_{j}$.

## 4 Universities' competition: Research expenditure and tuition fees

At the first stage of the game, each university solves its maximisation problem in accordance with the type of equilibrium arising at the second stage of the game. In particular each university behaves à la Nash with respect to its competitor but is a Stackelberg leader with respect to students. This means that each university decides tuition fees $b_{j}$, and research expenditure $R_{j}$ by taking into account the reaction of students, i.e. their location decisions. Starting from each equilibrium of the second stage, we solve the first stage considering that the objective function (6) must incorporate the corresponding equilibrium.

### 4.1 Equilibrium I

At equilibrium I of the second stage, where the students' location decisions are such that $\overline{N^{h}}=$ $n_{A}, \overline{N^{l}}=n_{B}$, the universities' objective functions are as follows

$$
W_{A}=\overline{N^{h}} \alpha e^{h}+\beta\left[\left(t_{A}+b_{A}\right) \overline{N^{h}}+\left(\tau_{A}-R_{A}\right)\right]+R_{A}^{\gamma_{A}},
$$

and

$$
W_{B}=\overline{N^{l}} \alpha e^{l}+\beta\left[\left(t_{B}+b_{B}\right) \overline{N^{l}}+\left(\tau_{B}-R_{B}\right)\right]+R_{B}^{\gamma_{B}} .
$$

Accordingly, the f.o.c. w.r.t. $R_{j}, j=A, B$, are as follows

$$
\begin{equation*}
\partial W_{j} / \partial R_{j}=\gamma_{j} R_{j}^{\gamma_{j-1}}-\beta=0, \quad j=A, B \tag{22}
\end{equation*}
$$

As far as the tuition fees, are concerned, we have that both universites pay-offs are increasing monotonic functions of $b_{j}, j=A, B$ :

$$
\begin{align*}
\partial W_{A} / \partial b_{A} & =\beta \overline{N^{h}}>0  \tag{23}\\
\partial W_{B} / \partial b_{B} & =\beta \overline{N^{l}}>0
\end{align*}
$$

### 4.1.1 Optimal research expenditure

From (22), the optimal level of research, $R_{j}^{I}$, obtains as

$$
\begin{equation*}
R_{j}^{I}=\left(\frac{\beta}{\gamma_{j}}\right)^{\frac{1}{\gamma_{j-1}}}, \quad j=A, B . \tag{24}
\end{equation*}
$$

Thus the optimal level of research is given by technological elements capturing, respectively, the impact of per-student teaching expenditure on the quality of teaching (efficacy of teaching expenditure), $\beta$, and the coefficient transforming expenditure on effective research activity (efficacy
of research expenditure), $\gamma$. Interesting enough a high efficacy of teaching expenditure implies a high level of optimal research expenditure. The explanation derives from the potential divertion of resources on financing research when one $€$ of teaching expenditure is highly efficient. On the contrary, the higher the efficacy of research expenditure the lower is the sum that is optimally allocated to research activities.

Remark 1. At equilibrium $I, \frac{\partial R_{j}^{I}}{\partial \tau_{j}}=0$.
The expenditure on research is, somewhat surprisingly, independent of the lump sum transfer by the central government. Recall that the latter is not sufficient to cover research expenditure, $R_{j}^{I}$, as this Equilibrium requires $R_{j}>\tau_{j}, j=A, B$.

### 4.1.2 Optimal per student tuition fee

Given (23), each university will choose the highest possible value of $b_{j}, j=A, B$. Such values will then result from the solution to the system formed by (12) and (13) when they hold as equalities.

Proposition 2 In Equilibrium $I, b_{A}^{I}=b_{B}^{I}=b^{I}$.
Proof. Given that the values of $b_{j}^{I}, j=A, B$ results from the solution to the system formed by conditions (12) and (13) holding as equalities, the following must hold:

$$
\begin{aligned}
& U^{h}\left(\alpha e^{h}+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{\bar{N}^{h}}\right)\right)-U^{h}\left(\alpha e^{l}+\beta\left(t_{B}+b_{B}+\frac{\tau_{B}-R_{B}}{\bar{N}^{l}}\right)\right)=b_{A}^{I}-b_{B}^{I} \\
& U^{l}\left(\alpha e^{h}+\beta\left(t_{A}+b_{A}+\frac{\tau_{A}-R_{A}}{\bar{N}^{h}}\right)\right)-U^{l}\left(\alpha e^{l}+\beta\left(t_{B}+b_{B}+\frac{\tau_{B}-R_{B}}{\bar{N}^{l}}\right)\right)=b_{A}^{I}-b_{B}^{I}
\end{aligned}
$$

Given the assumption that $\frac{\partial U^{h}}{\partial q_{j}}>\frac{\partial U^{l}}{\partial q_{j}}$ the above system of equations has either no solution or the unique solution $b_{A}^{I}=b_{B}^{I}=b^{I}$. $\square$

The following Corollary immediately follows from Proposition 2.
Corollary 1: For equilibrium I to exist, $t_{B}-t_{A}+\frac{R_{A}-\tau_{A}}{\bar{N}^{h}}-\frac{R_{B}-\tau_{B}}{\bar{N}^{l}}=\frac{\alpha}{\beta} \Delta>0$, where $R_{j}-\tau_{j}>0$, $j=A, B$.

For equilibrium I to exist, the government must give a relatively higher per-student and lump sum transfers to university $B$. Notice that the difference in the transfers to universities $B$ and $A$ is positively related to the difference in students' ability. University $B$ must be compensated for the lower quality of its students.

Corollary 1 implies that if both $t_{B}$ and $t_{A}$ increase (decrease) by the same amount, $b_{A}^{I}$ and $b_{B}^{I}$ must decrease (increase), remaining however always equal. A variation in $t_{B}$ and/or $t_{A}$ can be also compensated by changes in $\tau_{A}$ and/or $\tau_{B}$. In any case $b^{I}$ will vary in the opposite direction.

Notice that Proposition 2 implies that the equilibrium values of the arguments of $U^{l}($.$) and$ $U^{l}($.$) are the same. Moreover, it imposes b_{A}^{I}=b_{B}^{I}=b^{I}$ but it does not impose any constraint on
the level of the fee. As a consequence, considering that for any $q, U^{h}(q)>U^{l}(q)$ by assumption, the value of $b^{I}$ is found from the solution to ${ }^{9}$

$$
\begin{equation*}
U^{l}\left(\alpha e^{l}+\beta\left(t_{B}+b^{I}+\frac{\tau_{B}-R_{B}}{\bar{N}^{l}}\right)\right)-b^{I}=0 \tag{25}
\end{equation*}
$$

This implies that $t_{B}$ and $b^{I}$ are complements. A higher level of $t_{B}$ in fact enables the universities to raise $b^{I}$ and, consequently, to raise teaching quality.

### 4.2 Equilibrium II

At equilibrium II of the second stage, where the students' location decisions are such that $n_{A}=$ $\overline{N^{h}}+n_{A}^{l}$ and $n_{B}=n_{B}^{l}$, university $A$ solves the following maximisation problem

$$
\begin{array}{cc}
\max _{b_{A}, R_{A}} & \left.W_{A}=\overline{\left(N^{h}\right.}+n_{A}^{l}\right) q_{A}+r_{A} \\
\text { s.t. } & q_{A}=\alpha \bar{e}_{A}+\beta \frac{T_{A}}{n_{A}}, \\
& r_{A}=R_{A}^{\gamma_{A}}, \\
& \left.\left(t_{A}+b_{A}\right) \overline{\left(N^{h}\right.}+n_{A}^{l}\right)+\tau_{A}=T_{A}+R_{A}
\end{array}
$$

and University $B$ solves

$$
\begin{array}{cc}
\max _{b_{B}, R_{B}} & W_{B}=n_{B}^{l} q_{B}+r_{B} \\
\text { s.t. } & q_{B}=\alpha \bar{e}_{B}+\beta \frac{T_{B}}{n_{B}^{l}}, \\
& r_{B}=R_{B}^{\gamma_{B}} \\
& \left(t_{B}+b_{B}\right) n_{B}^{l}+\tau_{B}=T_{B}+R_{B}
\end{array}
$$

Accordingly, the f.o.c. of these two problems are

$$
\begin{equation*}
R_{A}: \quad \alpha \frac{\partial n_{A}^{l}}{\partial R_{A}} e^{l}+\beta\left[\left(t_{A}+b_{A}\right) \frac{\partial n_{A}^{l}}{\partial R_{A}}-1\right]+\gamma_{A} R_{A}^{\gamma_{A}-1}=0 \tag{26}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.b_{A}: \quad \alpha \frac{\partial n_{A}^{l}}{\partial b_{A}} e^{l}+\beta\left[\left(t_{A}+b_{A}\right) \frac{\partial n_{A}^{l}}{\partial b_{A}}+\overline{\left(N^{h}\right.}+n_{A}^{l}\right)\right]=0 \tag{27}
\end{equation*}
$$

for University $A$, and

$$
\begin{equation*}
R_{B}: \quad \alpha \frac{\partial n_{B}^{l}}{\partial R_{B}} e^{l}+\beta\left[\left(t_{B}+b_{B}\right) \frac{\partial n_{B}^{l}}{\partial R_{B}}-1\right]+\gamma_{B} R_{B}^{\gamma_{B}-1}=0 \tag{28}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.b_{B}: \quad \alpha \frac{\partial n_{B}^{l}}{\partial b_{B}} e^{l}+\beta\left[\left(t_{B}+b_{B}\right) \frac{\partial n_{B}^{l}}{\partial b_{B}}+n_{B}^{l}\right)\right]=0 \tag{29}
\end{equation*}
$$

for university $B$.

[^5]
### 4.2.1 Optimal research expenditure

Substituting (27) in (26) and (29) in (28), the optimal level of research $R_{j}^{I I}, j=A, B$, is the solution to

$$
\alpha\left(\frac{\partial n_{j}^{l}}{\partial R_{j}} e^{l}-\frac{\frac{\partial n_{j}^{l}}{\partial R_{j}}}{\frac{\partial n_{j}^{l}}{\partial b_{j}}} \frac{\partial n_{j}^{l}}{\partial b_{j}} e^{l}\right)-\beta\left(1+n_{j} \frac{\frac{\partial n_{j}^{l}}{\frac{\partial R_{j}}{\partial n_{j}^{l}}} \frac{\partial b_{j}}{\partial b_{j}}}{l}\right)+\gamma_{j} R_{j}^{\gamma_{j}-1}=0,
$$

or

$$
\gamma_{j} R_{j}^{\gamma_{j}-1}=\beta\left(1-\frac{\frac{\partial n_{j}^{l}}{\partial R_{j}}}{D_{j}}\right),
$$

and then

$$
\begin{equation*}
R_{j}^{I I}=\left[\frac{\beta}{\gamma_{j}}\left(1-\frac{\frac{\partial n_{j}^{l}}{\partial R_{j}}}{D_{j}}\right)\right]^{\frac{1}{\gamma_{j}-1}}=\left[\frac{\beta}{\gamma_{j}}\left(1+\Omega_{j}\right)\right]^{\frac{1}{\gamma_{j}-1}}>0 \tag{30}
\end{equation*}
$$

where we define $D_{j} \equiv-\frac{\sum_{i=h, l} \frac{\partial n_{j}^{i}}{n_{j}}}{n_{j}}=-\frac{\frac{\partial n_{j}^{l}}{\partial b_{j}}}{n_{j}}$ and $\Omega_{j} \equiv-\frac{\frac{\partial n_{j}^{l}}{\partial R_{j}}}{D_{j}}$. The first is an index of tuition fee competition, given that it measures the semi-elasticity of students w.r.t the fee, i.e. the percentage of unskilled students outflight due to an increse of the fee. The second is an index of the unskilled students outflight due to an increase in expenditure on research, $\frac{\partial n_{j}^{l}}{\partial R_{j}}<0$, relatively to the tuition fee competition $D_{j}$. Notice $D_{j}>0$ since $\frac{\partial n_{j}^{l}}{\partial b_{j}}<0$ from Lemma 2. Recall also that it is $\gamma_{j}<1$.

Then we have that optimal research expenditure is given by

$$
R_{j}^{I I}=\left(\frac{\beta}{\gamma_{j}}\right)^{\frac{1}{\gamma_{j}-1}}\left(1+\Omega_{j}\right)^{\frac{1}{\gamma_{j}-1}}>0 .
$$

While in equilibrium I $R_{j}^{I}$ was entierly determined by technological parameters, now $R_{j}^{I I}$ results from the product of the "technological component" $\left[\frac{\beta}{\gamma_{j}}\right]^{\frac{1}{\gamma_{j}-1}}$ by the "students' response component" $\left[\left(1+\Omega_{j}\right)\right]^{\frac{1}{\gamma_{j}-1}}$.

Notice that it is $\Omega_{j}>0$, because $\frac{\partial n_{j}^{l}}{\partial R_{j}}>0$ and $\frac{\partial n_{j}^{l}}{\partial b_{j}}<0$ from Lemma 2. Further, if $\Omega_{j}$ tends to be low, $R_{j}^{I I}$ tends to be given only by the technological parameters as in Equilibrium I. $R_{j}^{I I}$ tends, instead, to increase if $\Omega_{j}$ increases, and this is going to happen if $\frac{\partial n_{j}^{l}}{\partial R_{j}}$ is high with respect to $\left|\frac{\partial n_{j}^{l}}{\partial b_{j}}\right|$.

Moreover, $\Omega_{j}$ can be re-written as

$$
\begin{equation*}
\Omega_{j}=\frac{\frac{n_{j}^{l}}{n_{n}} \frac{\partial U_{j}^{l}}{\partial q_{j}}}{\frac{\partial U_{j}^{l}}{\partial q_{j}}-1 / \beta}, \tag{31}
\end{equation*}
$$

with $\frac{n_{A}^{l}}{n_{A}}<1=\frac{n_{B}^{l}}{n_{B}}$ and $\frac{\partial U_{j}^{l}}{\partial q_{j}}<1 / \beta$ from Lemma 2. Given that $\frac{n_{A}^{l}}{n_{A}}<\frac{n_{B}^{l}}{n_{B}}$, the relation between $\Omega_{A}$ and $\Omega_{B}$ depends on the relative quality of teaching. Since $U($.$) is concave, \Omega_{j}$ is certainly lower for university $A$ if $q_{A}>q_{B}$.

We are now in a position to prove the following proposition.

Proposition 3 At equilibrium II, the optimal research expenditure for university $j, R_{j}^{I I}$, is independent of $\tau_{j}$.

Proof. Follows from

$$
\frac{\partial R_{j}^{I I}}{\partial \tau_{j}}=\left[\frac{\beta}{\gamma_{j}}\right]^{\frac{1}{\gamma_{j}-1}} \frac{1}{\gamma_{j-1}}\left[\left(1+\Omega_{j}\right)\right]^{\left(\frac{1}{\gamma_{j}^{-1}}-1\right)} \frac{\partial \Omega_{j}}{\partial \tau_{j}},
$$

where, substituting for $\frac{\partial n_{j}^{l}}{\partial R_{j}}$ from (16), $\Omega_{j}=-\frac{\frac{\partial n_{j}^{l}}{\partial R_{j}}}{D_{j}}$ can be rewritten as

$$
\Omega_{j}=\frac{1}{1-\frac{1}{\beta \frac{\partial U_{j}^{\top}}{\partial q_{j}}}} .
$$

Hence $\frac{\partial \Omega_{j}}{\partial \tau_{j}}=0$, because $\frac{\partial U_{j}^{l}}{\partial q_{j}}$ is independent of $\tau_{j}$.
Thus $R_{j}^{I I}$ is independent from $\tau_{j}$ also in this equilibrium. Actually $\tau_{j}$ acts as a lump sum non matching grant.

### 4.2.2 Optimal per student tuition fee

As far as the tuition fee is concerned, by solving (27) and (29) we obtain

$$
\begin{equation*}
b_{A}=-\frac{1}{\frac{\partial n_{A}^{l}}{\partial b_{A}}}\left(\frac{\alpha}{\beta} \frac{\partial n_{A}^{l}}{\partial b_{A}} e^{l}+\left(\bar{N}^{h}+n_{A}^{l}\right)\right)-t_{A} \tag{32}
\end{equation*}
$$

which can be written as

$$
\begin{aligned}
b_{A} & =-\frac{\bar{N}^{h}+n_{A}^{l}}{\frac{\partial n_{A}^{l}}{\partial b_{A}}}\left(\frac{\alpha}{\beta} \frac{\frac{\partial n_{A}^{l}}{\partial b_{A}} \lambda_{A}^{l} e^{l}}{n_{A}^{l}}+1\right)-t_{A} \\
& =-\frac{\alpha}{\beta} \frac{\bar{N}^{h}+n_{A}^{l}}{n_{A}^{l}} e^{l} \frac{n_{A}^{l}}{\bar{N}^{h}+n_{A}^{l}}-\frac{\bar{N}^{h}+n_{A}^{l}}{\frac{\partial n_{A}^{l}}{\partial b_{A}}}-t_{A}
\end{aligned}
$$

and

$$
\begin{equation*}
b_{B}=-\frac{1}{\frac{\partial n_{B}^{l}}{\partial b_{B}}}\left(\frac{\alpha}{\beta} \frac{\partial n_{B}^{l}}{\partial b_{B}} e^{l}+n_{B}^{l}\right)-t_{B} . \tag{33}
\end{equation*}
$$

Thus, considering that $D_{A} \equiv-\frac{\frac{\partial n_{A}^{l}}{\partial b_{A}}}{\bar{N}^{h}+n_{A}^{l}}$, the optimal fee for university $A$ becomes

$$
b_{A}^{I I}=-\frac{\alpha}{\beta} e^{l}-\frac{\bar{N}^{h}+n_{A}^{l}}{\frac{\partial n_{A}^{l}}{\partial b_{A}}}-t_{A}=-\frac{\alpha}{\beta} e^{l}+\frac{1}{D_{A}}-t_{A},
$$

and taking into account that $D_{B} \equiv-\frac{\frac{\partial n_{B}^{l}}{\partial b_{B}}}{n_{B}^{I}}$ the optimal fee for University $B$ is

$$
\begin{equation*}
b_{B}^{I I}=-\frac{\alpha}{\beta} e^{l}-\frac{n_{B}^{l}}{\frac{\partial n_{B}^{l}}{\partial b_{B}}}-t_{B}=-\frac{\alpha}{\beta} e^{l}+\frac{1}{D_{B}}-t_{B} . \tag{34}
\end{equation*}
$$

Therefore $b_{j}^{I I}, j=A, B$, in general decreases with $\alpha / \beta$ and $e^{l}$, and decreases with $D_{j}$, the fee competition.

However, $b_{j}^{I I}$ could be positive, as well as negative: university $j$ can tax or subsidize its students according to the following relation

$$
b_{j}^{I I} \gtreqless 0 \quad \text { iff } \quad 1 / D_{j} \gtreqless \frac{\alpha}{\beta} e^{l}+t_{j} \quad \text { or iff } \quad t_{j} \lesseqgtr \theta_{j} \equiv 1 / D_{j}-\frac{\alpha}{\beta} e^{l} .
$$

Moreover, we have that $b_{j}^{I I}$ and $t_{j}$ are perfect substitute since the first two terms in (34) are independent of $t_{j}$. We thus have

Remark 2. In equilibrium II, $\frac{\partial b_{j}^{I I}}{\partial t_{j}}=-1, \quad j=A, B$.
Let us consider the case when the public transfer is sufficiently low, i.e. $t_{j}<\theta_{j}$. Then both universities will fix a positive tuition fee. University $B$ (attended only by low ability students) has an incentive to fix a positive tuition fee to cover its expenditure on teaching, and to avoid to be attended by all low ability students (which would be the case covered by equilibrium I).

### 4.3 Equilibrium III

At equilibrium III of the second stage, where $n_{A}=\bar{N}$ and $n_{B}=0$, universities' objective functions are as follows

$$
W_{A}=\bar{N} \alpha\left(\frac{\overline{N^{h}}}{\bar{N}} \Delta+e^{l}\right)+\beta\left[\left(t_{A}+b_{A}\right) \bar{N}+\left(\tau_{A}-R_{A}\right)\right]+R_{A}^{\gamma_{A}}
$$

and

$$
W_{B}=R_{B}^{\gamma_{B}}
$$

Accordingly, the foc for university $A$ with respect to research expenditure is

$$
\partial W_{A} / \partial R_{A}=\gamma_{A} R_{A}^{\gamma_{A}{ }^{-1}}-\beta=0,
$$

while with respect to tuition fee we have

$$
\begin{equation*}
\partial W_{A} / \partial b_{A}=\beta \bar{N}>0 \tag{35}
\end{equation*}
$$

so that $A$ 's pay-off is monotonically increasing with $b_{A}$.
For university $B$, we obviously have that the pay-off is increasing in research expenditure as

$$
\partial W_{B} / \partial R_{B}=\gamma_{B} R_{B}^{\gamma_{B}-1}>0
$$

### 4.3.1 Optimal research expenditure

At this Equilibrium $R_{j}^{I I I}$ obtains as

$$
\begin{equation*}
R_{A}^{I I I}=\left[\frac{\beta}{\gamma_{A}}\right]^{\frac{1}{\gamma_{A}-1}} \tag{36}
\end{equation*}
$$

for university $A$, and

$$
R_{B}^{I I I}=\tau_{B} .
$$

for university $B$.
Remark 3. At equilibrium III, $\frac{\partial R_{A}^{I I I}}{\partial \tau_{A}}=0$, and $\frac{\partial R_{B}^{I I I}}{\partial \tau_{B}}=1$.
As in Equilibrium I, research expenditure $R_{A}^{I I I}$ depends only on technological parameters, while $R_{B}^{I I I}$ is exactly equal to the lump sum transfer because there is no teaching activity.

### 4.3.2 Optimal per student tuition fee

At equilibrium III, the government does not finance teaching at university $B$, i.e. $t_{B}=0$. Given (35) and given that $U^{h}(q)>U^{l}(q) \forall q$, university $A$ will choose the value of $b_{A}^{I I I}$ by solving the following equation:

$$
\begin{equation*}
U^{l}\left(\alpha\left(e^{l}+\frac{\Delta \bar{N}^{h}}{\bar{N}}\right)+\beta\left(t_{A}+b_{A}^{I I I}+\frac{\tau_{A}-R_{A}^{I I I}}{\bar{N}}\right)\right)-b_{A}=0 . \tag{37}
\end{equation*}
$$

Notice that this implies that

$$
\begin{equation*}
U^{h}\left(\alpha\left(e^{l}+\frac{\Delta \bar{N}^{h}}{\bar{N}}\right)+\beta\left(t_{A}+b_{A}^{I I I}+\frac{\tau_{A}-R_{A}^{I I I}}{\bar{N}}\right)\right)-b_{A}^{I I I}>0 \tag{38}
\end{equation*}
$$

high ability students enjoy a higher level of utility than low ability ones. ${ }^{10}$
Analogously to equilibrium I, $t_{A}$ and $b_{A}^{I I I}$ are complements. A higher level of $t_{A}$ in fact enables university $A$ to raise $b_{A}^{I I I}$ and, consequently, to raise teaching quality.

## 5 Concluding remarks

In this paper we have analysed the topic of the impact of student mobility on the characteristics of two competing state universities. Assuming two types of students ("high ability" and "low ability"), the composition of student population impacts on the quality of teaching. The latter is an argument of the individual utility function ("peer effect") as well as of the universities' objective functions. The level of research (which is linked to research expenditure by efficiency parameters) is the other argument of the universities' objective functions. Each university decides the level of its tuition fees and of its research expenditure. The government contributes to financing the universities by a lump sum transfer and a matching grant per student.

By selecting locally stable equilibria, the analysis has ruled out some institutional settings in favour of some others. One of the main results is that high ability students always concentrate in the same university. Due to the existence of a positive peer effect there cannot exist a stable equilibrium where high ability students divide between different universities. We have three types of equilibria. In Equilibrium I, an élite institution is created with only high ability individuals

[^6](university A) while low ability students are segregated in a different institution (university B). In Equilibrium II, all high ability and part of the low ones attend one university (university A) while the rest of the low ones attends the other university (university B). In Equiliibrium III, all students are concentrated in one university (university A), while the other institution becomes a research centre.

Equilibrium II is most likely to occur because it is conditional on less restrictive requirements. As far as university B is concerned, for Equilibrium II to realize, the lump sum transfer from the government must be greater than research expenditure. The residual part of these funds are devoted to finance teaching. In this case a relatively low number of students is sufficient because research is self sustaining. Part of the low skilled students go to university A where the lump sum transfer may be lower than research expenditure. As shown in Proposition 2, the lump sum transfer does not influence the level of research expenditure. Moreover, as emphasized in Remark 2, the tuition fee and the per-student transfer are perfect substitutes.

The other two equilibria are strictly dependent on precise government behavioural parameters and then policies. In order to have equilibrium I, the lump sum transfer must be lower than research expenditure. So part of the latter must be financed by tuition fee revenue. The level of research expenditure is entirely explained by technological parameters of the research production function. Thus, efficiency is crucial in defining the level of public expenditure. As far as tuition fees are concerned, a somewhat surprising result is that tuition fees must be equal in both universities. However, the government tends to compensate the effect of the low skilled students in university B because, as shown in Corollary 1, the per student transfer in this university must be higher than in university A.

In the peculiar equilibrium III, the government intends to separate teaching from research. A reasearch institution is created, totally financed with a lump sum transfer. In the university, attended by all students, both high and low skilled, the lump sum transfer is lower than research expenditure. Here research expenditure is significatively financed by tuition fee revenue.

The role played by the governement in characterizing the types of equilibrium is crucial in our analysis, however government decision making has not been modelled so far. Our future research will be devoted to this topic with the intent to extend and qualify the results of the present paper.

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[^1]:    ${ }^{1}$ See Aghion et al. (2008) for an empiriacal analysis of the link between universities' autonomy, competition, and research performance. See also Veugelers and Van Der Ploeg (2008).

[^2]:    ${ }^{2}$ Optimal research and teaching decisions are also analysed by De Fraja and Valbonesi (2009) who however consider that in each local education market there is a single university which acts as a monopoly.

[^3]:    ${ }^{3}$ This is a common form for the teaching production function, see e.g. Del Rey (2001).
    ${ }^{4}$ The same type of objective function is also used by Del Rey (2000) while de Fraja and Iossa (2002) assume that universities are interested in maximising their prestige which is formalized as a function of the number of students, the average ability of the student body, and research expenditure. More recently, De Fraja and Valbonesi (2008) suppose that universities are only interested in maximising their amount of research, so that teaching is not an end in itself, but a mean to fund research.
    ${ }^{5}$ This condition is quite familiare in the literature dealing with tax competition with household mobility. See for instance Wellish (2000, p.111).

[^4]:    ${ }^{6}$ More precisely, quality is increased unless the university is attended only by $h$ and the lump sum transfer exceeds research expenditure. This could be the case for one university but not for the other.
    ${ }^{7}$ More precisely, there are three types of equilibria. For each type, there actually exist two symmetric equilibria. The second one can be obtained by simply exchanging the subscript $A$ for $B$ and viceversa.
    ${ }^{8}$ We assume that universities fix tuition fees without taking into account the marginal effect of a student movement on teaching quality. Given that $\bar{N}$ is large, such effect is negligible.

[^5]:    ${ }^{9}$ We are implicitly assuming that $\partial U^{h}\left(\alpha e^{h}+\beta\left(t_{A}+b^{I}+\frac{\tau_{A}-R_{A}}{\bar{N}^{h}}\right)\right) / \partial b \leq 1 / \beta$ at the equilibrium.

[^6]:    ${ }^{10}$ By only looking at equilibria where all potential students go to university we are implicitly assuming that the increase in university $A$ 's payoff from raising the tuition fee up to the level that would equate to zero the utility of high ability students is lower than the loss due to the fact that low ability students would not enrol.

