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INEQUALITY DECOMPOSITIONS – A RECONCILIATION

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Inequality Decompositions – A Reconciliation

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Abstract

We show how classic source-decomposition and subgroup-decomposition methods can be reconciled with regression methodology used in the recent literature. We also highlight some pitfalls that arise from uncritical use of the regression approach. The LIS database is used to compare the approaches using an analysis of the changing contributions to inequality in the United States and Finland.

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1 Introduction

What is the point of decomposing income inequality and how should we do it? For some researchers the questions resolve essentially to a series of formal propositions that characterise a particular class of inequality measures. For others the issues are essentially pragmatic: in the same way as one attempts to understand the factors underlying, say, wage discrimination (Blinder 1973) one is also interested in the factors underlying income inequality and it might seem reasonable to use the same sort of applied econometric method of investigation. Clearly, although theorists and pragmatists are both talking about the components of inequality, they could be talking about very different things. We might wonder whether they are even on speaking terms.

In this paper we show how the two main strands of decomposition analysis that are often treated as entirely separate can be approached within a common analytical framework. We employ regression-based methods which are commonly used in empirical applications in various fields of economics.

The paper is organised as follows. Section 2 offers an overview of the decomposition literature. Our basic model is developed in section 3 and this is developed into a treatment of factor-source decomposition and subgroup decomposition in sections 4 and 5 respectively. Section 6 provides an empirical application, Section 7 discusses related literature and Section 8 concludes.

2 Approaches to decomposition

The two main strands of inequality-decomposition analysis that we mentioned in the introduction could be broadly labelled as "*a priori*" approaches and "explanatory models."

2.1 *A priori* approaches

Underlying this approach is the essential question "what is meant by inequality decomposition?" The answer to this question is established through an appropriate axiomatisation.

This way of characterising the problem is perhaps most familiar in terms of decomposition by subgroups. A coherent approach to subgroup decomposition essentially requires (1) the specification of a collection of admissible partitions – ways of dividing up the population into mutually exclusive and exhaustive subsets – and (2) a concept of representative income for each group. Requirement (1) usually involves taking as a valid partition any arbitrary grouping of population members, although other specifications also

make sense (Ebert 1988); requirement (2) is usually met by taking subgroup-mean income as being representative of the group, although other representative income concepts have been considered (Blackorby et al. 1981; Foster and Shneyerov 1999, 2000; Lasso de la Vega and Urrutia 2005, 2008). A minimal requirement for an inequality measure to be used for decomposition analysis is that it must satisfy a subgroup consistency or aggregability condition – if inequality in a component subgroup increases then this implies, ceteris paribus, that inequality overall goes up (Shorrocks 1984, 1988); the "ceteris paribus" clause involves a condition that the subgroup-representative incomes remain unchanged. This minimal property therefore allows one to rule out certain measures that do not satisfy the axioms from which the meaning is derived (Cowell 1988), but one can go further. By imposing more structure – i.e. further conditions – on the decomposition method one can derive particular inequality indices with convenient properties (Bourguignon 1979, Cowell 1980, Shorrocks 1980), a consistent procedure for accounting for inequality trends (Jenkins 1995) and an exact decomposition method that can be applied for example to regions (Yu et al. 2007) or to the world income distribution (Sala-i-Martin 2006). By using progressively finer partitions it is possible to apply the subgroup-decomposition approach to a method of "explaining" the contributory factors to inequality (Cowell and Jenkins 1995, Elbers et al. 2008).

The *a priori* approach is also applicable to the other principal type of decomposability – the break-down by factor-source (Paul 2004, Shorrocks 1982, 1983, Theil 1979). As we will see the formal requirements for factor-source decomposition are straightforward and the decomposition method in practice has a certain amount in common with decomposition by population subgroups. Furthermore the linear structure of the decomposition (given that income components sum to total income) means that the formal factor-source problem has elements in common with the regression-analysis approach that we review in Section 2.2.

Relatively few attempts have been made to construct a single framework for both principal types of decomposition - by subgroup and by factor source. A notable exception is the Shapley-value decomposition (Chantreuil and Trannoy 1999, Shorrocks 1999), which defines an inequality measure as an aggregation (ideally a sum) of a set of contributory factors, whose marginal effects are accounted eliminating each of them in sequence and computing the average of the marginal contributions in all possible elimination sequences. However, despite its internal consistency and attractive interpretation, the Shapley-value decomposition in empirical applications raises some dilemmas that cannot be solved on purely theoretical grounds. As argued by Sastre and Trannoy (2002), provided all ambiguities about different possible marginal-

istic interpretations of the Shapley rule are cleared up, this decomposition is dependent on the aggregation level of remaining income components and is highly nonrobust. Some refinements have been proposed to improve the Shapley inequality decomposition, including the Nested Shapley (Chantreuil and Trannoy 1999) and the Owen decomposition (Shorrocks 1999), based on defining a hierarchical structure of incomes. However, these solutions might face some difficulty in finding a sensible economic interpretation and some empirical solutions can only circumvent the problem without solving it (Sastre and Trannoy 2000, 2002).

2.2 Explanatory models

The second analytical strand of analysis that concerns us here derives from a mainstream econometric tradition in applied economics. Perhaps the richest method within this strand is the development of a structural model for inequality decomposition exemplified by Bourguignon et al. (2001, 2008), in the tradition of the DiNardo et al. (1996) approach to analysing the distribution of wages. This method is particularly attractive as an "explanatory model" in that it carefully specifies a counterfactual in order to examine the influence of each supposedly causal factor. However, its attractiveness comes at a price: a common criticism is that it is data hungry and, as such, it may be unsuitable in many empirical applications. Furthermore, the modelling procedure can be cumbersome and is likely to be sensitive to model specification.

A less ambitious version of the explanatory-model approach is the use of a simple regression model as in Fields (2003), Fields and Yoo (2000) and Morduch and Sicular (2002). As with the structural models just mentioned, regression models enjoy one special advantage over the methods reviewed in Section 2.1. Potential influences on inequality that might require separate modelling as decomposition by groups or by income components can usually be easily and uniformly incorporated within an econometric model by appropriate specification of the explanatory variables.

2.3 An integrated approach?

It is evident that, with some care in modelling and interpretation, the *a priori* method can be developed from an exercise in logic to an economic tool that can be used to address important questions that are relevant to policy making. One can use the subgroup-decomposition method to assign importance to personal, social or other characteristics that may be considered

to affect overall inequality. The essential step involves the way that between-group inequality is treated which, in turn, focuses on the types of partition that are considered relevant. One has to be careful: the fact that there is a higher between-group component for decomposition using partition A rather than partition B does not necessarily mean that A has more significance for policy rather than B (Kanbur 2006). However, despite this caveat, it is clear that there should be some connection between the between-group/within-group breakdown in the Section 2.1 approach and the explained/unexplained variation in the Section 2.2 approach.

We want to examine this connection using a fairly basic model.

3 Basic model

To make progress it is necessary focus on the bridge between formal analysis and the appropriate treatment of data. Hence we introduce the idea of data generating process (DGP), i.e. the joint probability distribution that is supposed to characterize the entire population from which the data set has been drawn.

Consider a set of random variables $\mathbf{H}$ with a given joint distribution $F(\mathbf{H})$, where $\mathbf{H}$ is partitioned into $[Y, \mathbf{X}]$ and $\mathbf{X} := (X_1, X_2, ..., X_K)$. Assume that we aim to explain Y as a function of explanatory variables $\mathbf{X}$ and a purely random disturbance variable U and that we can write the relation in an explicit form with Y as function of $(\mathbf{X}, U)$

$$Y = f(\mathbf{X}, U|\boldsymbol{\beta}) \tag{1}$$

where $\boldsymbol{\beta} := (\beta_1, ..., \beta_K)'$ is a vector of parameters. For example, we could think of Y as individual income, of $\mathbf{X}$ as a set of observable individual characteristics, such as age, sex, education, and of U as an unobservable random variable such as ability or luck.

Provided the functional form of f is known, and it is additively separable in $\mathbf{X}$ and U, we can write

$$Y = g(\mathbf{X}|\boldsymbol{\beta}) + U = E(Y|\mathbf{X}) + U \tag{2}$$

where $E(Y|\mathbf{X})$ is the regression function of Y on $\mathbf{X}$, which is used to estimate $\boldsymbol{\beta}$. For simplicity let us assume that the DGP represented by g takes a linear form:

$$Y = \beta_0 + \sum_{k=1}^{K} \beta_k X_k + U \tag{3}$$

Typically one observes a random sample of size n from $F(\mathbf{H})$,

$$\{(y_i, \mathbf{x}_i) = (y_i, x_{1i}, ...x_{ki}), i = 1, ...n\},$$

where the observations are independent over i. One then generates predictions of income for assigned values of individual characteristics using regression methods to compute a vector $\mathbf{b}$, as an estimate of $\boldsymbol{\beta}$. The true marginal distribution function of each random variable, which might be either continuous or discrete, is often unknown in economic applications, as data do not come from laboratory experiments, and one only knows the empirical distribution functions (EDF). The sample analogue of model (3) can be written as:

$$y = \beta_0 + \sum_{k=1}^{K} \beta_k x_k + \upsilon.$$

Provided that the functional form for g in (2) is correctly specified, and that standard assumptions such as exogenous covariates and spherical error variance hold, one could use OLS methods to estimate the income model obtaining

$$y = b_0 + \sum_{k=1}^{K} b_k x_k + u, \tag{4}$$

where b_k is the OLS estimate of β_k, $k = 0, ...K$, $u = y - E(y|x)$ is the OLS residual.

Using the upper case letter for denoting a random variable (whose distribution function is not known in typical survey settings) and the lower case letter for denoting a size-n random sample from the same distribution function, the mean and inequality function of Y are denoted by $\mu(Y)$ and $I(Y)$, the mean and the inequality statistics (i.e. functions of the data) with $\mu(y) = \mu(y_1, ..., y_n)$ and $I(y) = I(y_1, ..., y_n)$.

We can analyse the structure of the inequality of y (or of Y) in two different ways

- *Subgroup decomposition.* Suppose that a subset $T \subseteq \{1, ..., K\}$ of the observables consists of discrete variables such that x_k (X_k) can take the values ξ_{kj}, $j = 1, .., t_k$ where $k \in T$ and t_k is the number of values (categories) that can logically be taken by the kth discrete observable. Then in this case we could perform a decomposition by population subgroups, where the subgroups are determined by the t categories, where $t := \prod_{k \in T} t_k$. This decomposition could be informative – what you get from the within-group component is an aggregate of the amount of inequality that is attributable to the dispersion of the unobservable υ (U) and the remaining continuous observables x_k, $k \notin T$ (X_k, $k \notin T$). If all the observables were discrete the within-group component would be an aggregation of $I_{y|x}$ ($I_{Y|X}$) and the between-group component would

give the amount of inequality that would arise if there were no variation in $\upsilon\ (U)$.

- *Factor source decomposition.* We can also interpret (3) as the basis for inequality by factor source expressing $I\,(Y)$ in terms of component incomes $C_1, ..., C_{K+1}$, where

$$C_k := \beta_k X_k, k = 1, ..., K \tag{5}$$

$$C_{K+1} := U \tag{6}$$

– see section 4 below. In this case the term β_0 is irrelevant.

The application of these decomposition methods has been criticised on a number of grounds. Subgroup decomposition is criticised because it requires partitioning the population into discrete categories although some factors (for example, age) are clearly continuous variables. Moreover, handling more than very few subgroups at the same time can be cumbersome. The factor-source decomposition presented in the Shorrocks (1982) form presents the useful property of being invariant to the inequality measure adopted,[1] however it can be criticised as being limited to a natural decomposition rule where total income is the sum of different types of income (for example pension, employment income and capital income). The subgroup and factor source decomposition methods are sometimes criticised as being purely descriptive rather than analytical and as being irreconcilable one with another. Moreover they are tools which are often not well known in some fields of economics where the main focus is on the determinants of income or the market price of personal characteristics, which are estimated as the OLS coefficient in a Mincer-type wage regression.

The two decomposition methods – by population subgroup and by factor source – can be shown to be related to each other. This can be conveniently done using the model that we have just introduced.

4 Decomposition by factor source

Equation (3) is analogous to the case analysed by Shorrocks (1982) where income is the sum of income components (such as labour income, transfers

[1] Actually in some situations this might be regarded as a shortcoming, especially when the the change of inequality can have a different sign depending on the inequality measure adopted.

and so on). The inequality of total income, $I(Y)$, can be written using a natural decomposition rule such as:

$$I(Y) = \sum_{k=1}^{K+1} \Theta_k \tag{7}$$

where Θ_k depends on C_k and can be regarded as the contribution of factor k to overall income inequality. Define also the proportional contribution of factor k to inequality

$$\theta_k := \frac{\Theta_k}{I(Y)}.$$

Using (5) and (6) the results inShorrocks (1982) yield:

$$\theta_k = \frac{\sigma\left(C_k, Y\right)}{\sigma^2\left(Y\right)} = \frac{\sigma^2\left(C_k\right)}{\sigma^2\left(Y\right)} + \sum_{j \neq k}^{K+1} \rho(C_k, C_j) \frac{\sigma\left(C_k\right)\sigma(C_j)}{\sigma^2\left(Y\right)}, k = 1, ..., K+1$$

where $\sigma(X) := \sqrt{\text{var}(X)}$, $\sigma(X, Y) := \text{cov}(X, Y)$ and $\rho(C_i, C_j) := \text{corr}(C_i, C_j)$.

Since $\sigma\left(\beta_k X_k, Y\right) = \beta_k \sigma\left(X_k, Y\right)$ we have:

$$\theta_k = \beta_k^2 \frac{\sigma^2\left(X_k\right)}{\sigma^2\left(Y\right)} + \sum_{j \neq k}^{K} \beta_k \beta_j \frac{\sigma\left(X_k, X_j\right)}{\sigma^2\left(Y\right)} + \beta_k \frac{\sigma\left(X_k, U\right)}{\sigma^2\left(Y\right)} \tag{8}$$

from which we obtain

$$\theta_k = \beta_k^2 \frac{\sigma^2\left(X_k\right)}{\sigma^2\left(Y\right)} + \sum_{j \neq k}^{K} \beta_k \beta_j \rho(X_k, X_j) \frac{\sigma(X_j)\sigma(X_k)}{\sigma^2\left(Y\right)} + \beta_k \rho(X_k, U) \frac{\sigma\left(X_k\right)\sigma(U)}{\sigma^2\left(Y\right)}, \tag{9}$$

for $k = 1, ..., K$ and

$$\theta_{K+1} = \frac{\sigma^2\left(U\right)}{\sigma^2\left(Y\right)} + \sum_{k=1}^{K} \beta_k \rho(X_k, U) \frac{\sigma\left(X_k\right)\sigma(U)}{\sigma^2\left(Y\right)}. \tag{10}$$

Replacing β_k by its OLS estimate (b_k), and variances, covariances and correlation by their unbiased sample analogues, the estimate of θ_k, (z_k), can be obtained. A similar approach was followed by Fields (2003). Equations (9)-(10) provide a simple and intuitive interpretation and allow one to discuss the contribution of the value of characteristic $k, c_{k,}$ to inequality $I(y)$. If we impose more structure on the problem, by assuming that there is no multicollinearity among regressors and all regressors are non-endogenous $(\text{corr}(C_k, C_r) = 0, r \neq k)$, then (8) can be simplified to

$$\theta_k = \begin{cases} \beta_k^2 \frac{\sigma^2(X_k)}{\sigma^2(Y)}, k = 1, ..., K \\ \frac{\sigma^2(U)}{\sigma^2(Y)}, k = K+1 \end{cases} \tag{11}$$

and it can be estimated as

$$z_k = \begin{cases} b_k^2 \frac{\sigma^2(x_k)}{\sigma^2(y)}, k = 1, ..., K \\ \frac{\sigma^2(u)}{\sigma^2(y)}, k = K+1 \end{cases} \tag{12}$$

where $\sigma^2(x_k), \sigma^2(y), \sigma^2(u)$ stand for the unbiased sample variance of x_k, y, u, respectively. The sample analogue of the inequality decomposition as in (7) can be written as:

$$I(y) = \sum_{k=1}^{K+1} Z_k = \sum_{k=1}^{K+1} I(y) z_k = \sum_{k=1}^{K} I(y) b_k^2 \frac{\sigma^2(x_k)}{\sigma^2(y)} + I(y) \frac{\sigma^2(\upsilon)}{\sigma^2(y)}. \tag{13}$$

With some simplification, the right-hand-side of equation (13) might be interpreted as the sum of the effects of the K characteristics and of the error term, although one should consider it as the sum of the *total value* of the K characteristics, i.e. the product of each component's "price" as estimated in the income regression $(b_k, k = 1, ..., K)$ and its quantity $(x_k, k = 1, ..., K)$. One should also notice that the standard errors of (13) are not trivial to compute as they involve the ratio of variances of random variables coming from a joint distribution and the variance of inequality indices can be rather cumbersome to derive analytically (see for instance Cowell 1989). Bootstrap methods are suggested for derivation of standard errors of (13), although they are not presented for the empirical analysis which follows.

Equation (8) shows that θ_k $(k = 1, ..., K)$ can only be negative if

$$\beta_k(\textstyle\sum_{j \neq k} \beta_j \sigma(X_k, X_j) + \sigma(X_k, U)) < -\beta_k^2 \sigma^2(X_k), k = 1, ...K$$

for which a necessary condition is that there be either a nonzero correlation among RHS variables or at least one endogenous RHS variable.

It should be noted here that the decomposition (7) applies for natural decompositions only, i.e. if the LHS variable can be represented as a sum of factors. In the labour-economics literature it is customary to estimate a log-linear relation, such as

$$\log(y) = b_0 + \sum_{k=1}^{K} b_k x_k + u$$

based on arguments of better regression fit and error properties. In this case, the decomposition (7) can only be undertaken with $I(\log(y))$ on the LHS.

5 Decomposition by population subgroups

Assume now that X_1 is a discrete random variable that can take only the values $\{X_{1,j} : j = 1, ..., t_1\}$. In the general case, allowing for the possibility that $\text{corr}(X_{1,j}, X_{k,j}) \neq 0$ and that $\text{corr}(X_{1,j}, U) \neq 0$, equation (3) can be represented for each sub-group j as:

$$Y_j = \beta_{0,j} + \beta_{1,j} X_{1,j} + \sum_{k=2}^{K} \beta_{k,j} X_{k,j} + U_j. \tag{14}$$

Define $P_j = \Pr(X_1 = X_{1,j})$, the proportion of the population for which $X_1 = X_{1,j}$. Then within-group inequality can be written as

$$I_{\text{w}}(Y) = \sum_{j=1}^{t_1} W_j I(Y_j), \tag{15}$$

where t_1 is the number of groups considered, W_j is a weight that is a function of the P_j, and Y_j is given by (14). The decomposition by population subgroups allows one to write:

$$I(Y) = I_{\text{b}}(Y) + I_{\text{w}}(Y), \tag{16}$$

where I_{b} is between-group inequality, implicitly defined by (15) and (16) as

$$I_{\text{b}}(Y) := I(Y) - \sum_{j=1}^{t_1} W_j I(Y_j).$$

In the case of the Generalised Entropy (GE) indices we have, for any $\alpha \in (-\infty, \infty)$,

$$W_j = P_j \left[\frac{\mu(Y_j)}{\mu(Y)}\right]^{\alpha} = R_j^{\alpha} P_j^{1-\alpha}, \tag{17}$$

where $R_j := P_j \mu(Y_j)/\mu(Y)$ is the income share of group j, $\mu(Y_j)$ is mean income for subgroup j, $\mu(Y)$ is mean income for the whole population; we also have

$$I(Y) = \frac{1}{\alpha^2 - \alpha} \left[\int \left[\frac{Y}{\mu(Y)}\right]^{\alpha} dF(Y) - 1\right], \tag{18}$$

from which we obtain

$$I_{\text{w}}(Y) = \frac{1}{\alpha^2 - \alpha} \left[\sum_{j=1}^{t_1} P_j \left[\frac{\mu(Y_j)}{\mu(Y)}\right]^{\alpha} \int \left[\frac{Y_j}{\mu(Y_j)}\right]^{\alpha} dF(Y_j) - 1\right] \tag{19}$$

and

$$I_{\mathrm{b}}(Y) = \frac{1}{\alpha^2 - \alpha}\left[\sum_{j=1}^{t_1} P_j \left[\frac{\mu(Y_j)}{\mu(Y)}\right]^{\alpha} - 1\right]. \tag{20}$$

Let us now see how a decomposition by population subgroups could be adapted to an approach which uses the estimated DGP. Using a $n-$size random sample $\mathbf{y}, \mathbf{x}_1, .., \mathbf{x}_k$ from the joint distribution function $F(Y, X_1, .., X_k)$ one can estimate equation (14) by separate regressions for each different group obtaining:

$$y_j = b_{0,j} + \sum_{k=2}^{K} b_k x_{k,j} + u_j \tag{21}$$

where $b_{0,j}$ are OLS estimates of $\beta_{0,j} + \beta_{1,j}\mu(x_{1,j})$ in subsample j and u_j are the OLS residuals of each group.

Given the OLS assumptions, the unbiasedness property of OLS estimates allows one to write the mean of y_j in (21) as

$$\mu(y_j) = b_{0,j} + \sum_{k=2}^{K} b_{k,j}\mu(x_{k,j}).$$

The estimated between-group inequality I_b can then be written as:

$$I_{\mathrm{b}}(y) = \frac{1}{\alpha^2 - \alpha}\left[\sum_{j=1}^{t_1} p_j \left[\frac{b_{0,j} + \sum_{k=2}^{K} b_{k,j}\mu(x_{k,j})}{b_0 + \sum_{k=1}^{K} b_k \mu(x_k)}\right]^{\alpha} - 1\right] \tag{22}$$

where $p_j := n_j/n$ is the population share and n_j is the size of group j. Hence, the estimated within-group inequality is written as:

$$I_{\mathrm{w}}(y) = \sum_{j=1}^{t_1} w_j I(y_j)\left[\left(\sum_{k=2}^{K} b_{k,j}^2 \frac{\sigma^2(x_{k,j})}{\sigma^2(y_j)} + b_{k,j}\sum_{r\neq k} b_{r,j}\rho(x_{r,j}, x_k)\frac{\sigma(x_{r,j})\sigma(x_k)}{\sigma(y)} + b_{k,j}\rho(x_{k,j}, u_j)\frac{\sigma(x_{k,j})\sigma(u)}{\sigma(y)} + b_{k,j}\frac{\sigma\left(x_{k,j},u\right)}{\sigma^2(y)}\right) + \frac{\sigma^2(u)}{\sigma^2(y)}\right]. \tag{23}$$

where $w_j = (q_j)^{\alpha}(p_j)^{1-\alpha}$ and $q_j := p_j\mu(y_j)/\mu(y)$ is the income share of group j.

6 Empirical application

We applied the method outlined above to the Luxembourg Income Study (LIS) data set,[2] focusing on net disposable income for the United States

[2]Data are available from http://www.lisproject.org/. All empirical results can be replicated downloading relevant files as discussed in Appendix B. For a description of the

	equivalised disposable income inequality							
	United States			Finland			Finland/US	
	1986	**2004**	**change**	**1987**	**2004**	**change**	**1986-87**	**2004**
p90/p10	5.778	5.380	-7%	2.375	2.775	17%	-59%	-48%
p90/p50	2.076	2.080	0%	1.482	1.636	10%	-29%	-21%
p50/p10	2.786	2.584	-7%	1.603	1.698	6%	-42%	-34%
p75/p25	2.406	2.402	0%	1.557	1.687	8%	-35%	-30%
GE(0)	0.212	0.256	21%	0.066	0.101	54%	-69%	-60%
GE(1)	0.183	0.244	33%	0.063	0.124	96%	-65%	-49%
GE(2)	0.199	0.350	76%	0.070	0.315	347%	-65%	-10%
Gini	0.335	0.365	9%	0.193	0.240	24%	-42%	-34%

Table 1: Inequality statistics

and Finland in the mid 1980s and in 2004. We chose the United States and Finland as they are two relevant examples of countries belonging to the group of Anglo-Saxon and Scandinavian countries, the first being characterised by higher inequality of after-tax income and a light welfare state, the second being characterised by relatively lower inequality and a substantial welfare state – see for example Brandolini and Smeeding (2008a, 2008b). We focus on inequality computed for equivalised income, using the conventional square-root equivalence scale, so that each individual is given his family's income normalised by the square root of the family size.

We use these data also because they allow us to compare the distribution of an uniformly defined income variable at approximately the same periods. In fact, four data sets are considered: the United States in 1987 and 2004 and Finland in 1987 and 2004. As Table 1 shows equivalised income inequality in mid 1980s Finland was between 42% and 86% smaller than that in the US, according to inequality measures the GE and Gini indices, and between 29% and 59% smaller, using quantile ratios. Nearly twenty years later, inequality of equivalised income increased in both countries, especially for incomes in the upper tail of the income distribution, as GE(2) shows. Although equivalised-income inequality increased relatively more in Finland, it remained consistently lower in Finland with respect to the US.

We begin by examining the role of two important subgroups, those defined by sex and by education of the household head, where education is coded into four categories (less than high school, high school, college and Master/PhD). One way to investigate these issues is a decomposition by population subgroups of GE indices. Table 2 presents results by education and by sex subgroups: it first gives the measures of inequality computed in each subgroup and then shows the within- and between-subgroup decomposition of inequality for the three GE indices, for United States and then Finland

Luxembourg Income Study, see Gornick and Smeeding (2008).

in each period. Given the exact decomposability property of GE indices, the sum of the within and between components is equal to total inequality. One might conclude from Table 2 that, decomposing by educuation, both the inequality within educational subgroups and the inequality between groups increased in each country. In particular, between group inequality nearly doubled in both countries, while the trend of within-group inequality was more pronounced in Finland. By contrast, a decomposition by sex of the household head shows roughly the opposite pattern of within and between components: while the former clearly increased in both countries the latter was roughly stable in absolute value in Finland and clearly decreasing in the United States.[3]

From this analysis one cannot disentangle the changed contribution of a demographic characteristic of the population (e.g. education) while controlling for the other (e.g. sex). A possible solution would be to create a finer partition of the sample by interacting education and sex, as proposed in Cowell and Jenkins (1995). However, this method could become cumbersome if one wanted to control for some additional characteristics (e.g. ethnicity, area of residence), would need a discretisation of variables which might reasonably considered as continuous (e.g. age) and would reduce the sample size in each subgroup, hence the precision of the estimate.

6.1 Implementation of the basic model

What additional insights might a regression-based approach yield? To answer this question we estimated a model of equivalised disposable income as (3) where Y is household equivalised income and as covariates we used, for both countries in both periods, family variables (number of earners, number of children under age 18, whether the family rents or owns its own dwelling) and variables referring to the household head only (age, age squared, sex and four category dummies for education).[4]

In Table 3 we present results first for the United States and then for Finland.[5] The first two columns under each year and country presents the OLS

[3] A careful analysis of these inequality statistics should also assess the magnitude of the sampling error (Cowell 1989), however in this paper we use the empirical application as an illustration of the methodologies presented in the previous sections. Further discussions about confidence intervals estimation of inequality measures and its decompositions will be presented in Section 7.

[4] This is a clearly simplified model of equivalised income generation, however available data would not allow the develpment of a more complex structural model of household income. For further discussion of this issue, see Section 7.

[5] The sample sizes are quite different: in the US there were 32,452 observations in 1986 and 210,648 in 2004, in Finland the sample size decreased from 33,771 in 1987 to 29,112 in

Subgroups by education
United States

	1986			**2004**		
education	GE(0)	GE(1)	GE(2)	GE(0)	GE(1)	GE(2)
< high school	0.222	0.203	0.230	0.223	0.210	0.308
high school	0.177	0.150	0.156	0.210	0.192	0.262
college	0.135	0.127	0.144	0.185	0.182	0.248
Master/PhD	0.144	0.122	0.124	0.217	0.222	0.306
Within	0.179	0.150	0.165	0.206	0.195	0.298
Between	0.033	0.033	0.034	0.050	0.050	0.052

Finland

	1987			**2004**		
education	GE(0)	GE(1)	GE(2)	GE(0)	GE(1)	GE(2)
< high school	0.062	0.059	0.061	0.092	0.099	0.131
high school	0.058	0.055	0.061	0.075	0.082	0.193
college	0.051	0.051	0.063	0.102	0.144	0.424
Master/PhD	0.045	0.046	0.048	0.085	0.094	0.121
Within	0.059	0.056	0.062	0.088	0.110	0.300
Between	0.007	0.007	0.008	0.013	0.014	0.014

Subgroups by sex
United States

	1986			**2004**		
sex	GE(0)	GE(1)	GE(2)	GE(0)	GE(1)	GE(2)
male	0.183	0.162	0.176	0.226	0.225	0.323
female	0.270	0.246	0.290	0.283	0.263	0.377
Within	0.197	0.170	0.187	0.252	0.241	0.346
Between	0.015	0.013	0.012	0.004	0.003	0.003

Finland

	1987			**2004**		
sex	GE(0)	GE(1)	GE(2)	GE(0)	GE(1)	GE(2)
male	0.062	0.060	0.066	0.095	0.116	0.294
female	0.078	0.079	0.093	0.112	0.141	0.369
Within	0.063	0.061	0.068	0.100	0.122	0.313
Between	0.003	0.003	0.002	0.002	0.002	0.002

Table 2: Subgroup inequality decomposition by educational attainment and by sex of the householder.

coefficient estimates of an equivalised income regression with their p-values, as in equation (4). While number of earners in the family, age and high education of the household head are always positively associated with equivalised household income, number of children younger than 18, a rented dwelling and a female household head are consistently associated with lower equivalised household income in all the four samples considered. These controls are all individually and jointly statistically significant. Their contribution to total variability of the dependent variable in the specified model ranges from over 40% in the case of US 1986 to less than 11% in the case of Finland 2004.

Clearly this is not a structural model and its specification is unsuitable for a causal interpretation, however it is informative about the correlation of some key variables on equivalised household income. It should also be noted that the dependent variable (equivalised household income, y) was normalised to its mean in each sample to ensure scale consistency between different samples and the coefficients should be interpreted carefully. The constant captures the difference of the welfare state in the US and in Finland: equivalised income, an average twenty-year-old, uneducated, unemployed woman, living alone with no kids, in a rented house would have an income equal to 27% of the mean in US 1986 but only 9% of the mean in US 2004. The same person would have an income equal to 39% and 37% of the average income in Finland 1987 and 2004, respectively. In all the data sets considered, educational variables are highly relevant and their impact on income is important. Also the gender variable coefficient is relatively large and statistically significant in all samples.

Column (c), (d) and (e) in Table 3 present the results of the decomposition proposed in Section 4. Column (c) presents the decomposition estimates (z_k) as in (12), as a percentage of total inequality. However, the contribution of inequality of each factor depends on the magnitude of each factor relative to total income, i.e. on the factor share (c_k/y, with $c_k := b_k x_k$, $k = 1, ..., K$; $c_{k+1} := u$), which is reported in column (d). Hence, the ratio between the contribution of each income component, c_k, and the factor share provides an idea of how much an increase in the value of a factor might translate into a change of inequality. From Table 3 it emerges that, controlling for all the covariates jointly, in the US the number of earners in the household, number of children aged less than 18 and a rented dwelling accounted for about 22% of inequality in 1986 but less than 11% in 2004. Higher education (college and Master/PhD degrees) accounted for roughly 15% of inequality in both

2004, although according to the LIS documentation all four samples are representative of their respective population and this does not seem to have any relevant effect on statistical significance of each regressor included.

years considered, with Master/PhD consistently accounting for nearly 10%. In Finland in 1987, number of earners in the household, number of children aged less than 18 and a rented dwelling accounted for about 14% of total equivalised income inequality and in 2004 about 5%. Higher education is also important for inequality in Finland, accounting for over 11% and 4% in 1987 and 2004 respectively, although college education is between 3 and 10 times more important than a Master/PhD degree. High-school education always has an equalising effect. Female-headed households are associated with higher inequality, although it emerges that the contribution decreased by 90% in the US and by 75% in Finland. However, taking into account also the factor share of each right-hand-side variable it emerges clearly that the highest degree of education is consistently inequality-increasing in both countries, in other words, that euro for euro, a larger value of the highest degree of education has the largest inequality-increasing effect and a reduced penalty for rented housing almost always has a the largest redistributive impact.

However the proposed inequality decomposition is exact only if the contribution of the residual is not ignored. Indeed, Table 3 shows that after controlling for a set of individual and family characteristics, the residual still accounts for nearly 60% of inequality in US 1986 and nearly 90% of inequality in Finland 2004. However, the factor share of the residual is zero as it is, by construction, the OLS residual. It is worth recalling that this inequality decomposition enjoys the same properties as the factor source decomposition suggested in Shorrocks (1982), namely the fact that it is invariant to the inequality measure used.

Let us now assess the contribution of (the total value of) each right-hand-side variable to inequality applying a regression-based factor source decomposition as discussed in Section 4. Our subgroup decomposition allows us to assess whether one variable contributes uniformly to inequality in each subgroup or has a disproportionate effect across the subgroups. We estimate separate regressions for each subgroup as in (14) and present inequality decomposition estimates as in (23) for education subgroups in tables 4 and 5, and for gender subgroups in tables 6 and 7.[6] The decompositions by education subgroups show that in the US the contribution to inequality of the number of earners is larger for less educated subgroups and that the female penalty decreased uniformly across all education groups between 1986 and 2004. In Finland the results on the contribution to inequality of number of earners are similar, while the female penalty is larger for less educated house-

[6]Tables of results are presented omitting the OLS coefficient estimates and their significance, which could however be obtain from the author upon request.

holds. Decomposing the sample by sex of the householder, it emerges that the largest contribution to inequality relative to its share of total income, is due to the highest degree of education. In US 1986 the number of earners relative used to account for more inequality among females and the rented housing for more inequality among males, but these gender differences diminished by 2004. In Finland 1987 the contribution to inequality of number of earners is larger for females, while the contribution of the income penalty for young children at home is larger for males. By contrast educational gains make no gender-specific differential contribution to total inequality. In 2004 the contribution of different factors on total income is instead rather equally distributed between the genders.

6.2 Robustness checks

Ideally one would like to provide an analysis of the reliability of the estimates by producing standard errors of the computations.[7] Here we provide a simple robustness analysis of our results by testing whether they would change if different variables were included. Table 8 shows the decomposition with and without controls for age of the youngest child, number of people aged 65-74 and 75 or over, marital status, ethnicity (black and white for the US and Finnish or Swedish speaking for Finland) and a great number of regional dummies (when available in the data) and area dummies. It shows that although also these variables play a role in accounting for inequality (especially ethnicity in the US and age of youngest child in Finland), they do not greatly modify the conclusions outlined above. As our methodology is based on regression methods, it also allows us to interpret changes of contributions of different individual and household characteristics as an effect of omitted variable bias, which is relevant only where omitted and included explanatory variables are strongly correlated.

7 Discussion

Clearly any empirical methodology should come with a set of warnings about implementation: so too with the techniques illustrated in Section 6.

[7] A thorough treatmentis not a trivial task and it would involve the use of the bootstrap. We intend to discuss the correct bootstrap specification of our methodology in a separate paper.

United States

			1986					**2004**		
	(a) Coef.	(b) P>t	(c) deco.	(d) fac.sh.	(e) =(c/d)	(a) Coef.	(b) P>t	(c) deco.	(d) fac.sh.	(e) =(c/d)
Household charact.										
number of earners	0.117	0.000	7.164	23.265	0.308	0.148	0.000	4.400	27.238	0.162
num. children < 18	-0.118	0.000	11.166	-21.501	-0.519	-0.083	0.000	2.755	-14.361	-0.192
housing rented	-0.154	0.000	3.440	-4.658	-0.738	-0.240	0.000	3.201	-6.301	-0.508
Head charact.										
age	0.020	0.000	7.405	87.070	0.085	0.024	0.000	4.199	103.678	0.040
age squared	0.000	0.000	-4.924	-34.821	0.141	0.000	0.000	-2.723	-47.905	0.057
female	-0.200	0.000	2.805	-3.118	-0.900	-0.049	0.000	0.290	-2.274	-0.128
high school	0.206	0.000	-1.006	10.729	-0.094	0.196	0.000	-1.614	9.415	-0.171
college	0.443	0.000	4.581	6.290	0.728	0.497	0.000	4.451	13.086	0.340
master/PhD	0.685	0.000	9.619	7.115	1.352	0.964	0.000	9.254	8.803	1.051
constant	0.291	0.000	0.000	29.630		0.086	0.000	0.000	8.620	
residual			59.752	0.000				75.787	0.000	
Total			*100.000*	*100.000*				*100.000*	*100.000*	
obs.	32452					210648				
Prob > F	0.000					0.000				
R-squared	0.403					0.242				
Adj R-squared	0.402					0.242				

Finland

			1987					**2004**		
	(a) Coef.	(b) P>t	(c) deco.	(d) fac.sh.	(e) =(c/d)	(a) Coef.	(b) P>t	(c) deco.	(d) fac.sh.	(e) =(c/d)
Household charact.										
number of earners	0.094	0.000	9.446	21.471	0.440	0.110	0.000	2.814	23.058	0.122
num. children < 18	-0.058	0.000	3.677	-9.213	-0.399	-0.072	0.000	1.525	-12.619	-0.121
housing rented	-0.077	0.000	1.681	-1.806	-0.931	-0.122	0.000	1.060	-3.118	-0.340
Head charact.										
age	0.017	0.000	1.188	75.773	0.016	0.018	0.000	1.069	84.939	0.013
age squared	0.000	0.000	1.246	-35.707	-0.035	0.000	0.000	-0.264	-40.745	0.006
female	-0.133	0.000	2.001	-1.252	-1.599	-0.109	0.000	0.508	-3.403	-0.149
high school	0.076	0.000	-0.373	3.499	-0.106	0.030	0.010	-0.212	1.383	-0.153
college	0.359	0.000	10.214	4.677	2.184	0.279	0.000	3.255	10.272	0.317
master/PhD	0.458	0.000	1.342	0.441	3.046	0.676	0.000	0.972	0.921	1.055
constant	0.394	0.000	0.000	42.116		0.365	0.000	0.000	39.311	
residual			69.578	0.000				89.273	0.000	
Total			*100.000*	*100.000*				*100.000*	*100.000*	
obs.	33771					29112				
Prob > F	0.000					0.000				
R-squared	0.304					0.107				
Adj R-squared	0.304					0.107				

Notes: LHS is equivalised household income. Omitted variables are (characteristics of the household) housing owned, (characteristics of the householder) male, less than high school.

Table 3: OLS equivalised income regression and equivalised income decomposition by factor source as in eq. (12).

United States 1986

Education	**Less than high school**			**High school**			**College**			**Master/PhD**		
Popn. share	0.247			0.512			0.139			0.102		
Income share	0.169			0.492			0.181			0.158		
	(c)	(d)	(e)	(c)	(d)	(e)	(c)	(d)	(e)	(c)	(d)	(e)
	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)
Household charact.												
number of earners	14.189	31.754	0.447	8.844	26.294	0.336	1.201	7.406	0.162	3.539	12.932	0.274
num. children < 18	11.053	-22.457	-0.492	13.804	-23.139	-0.597	14.748	-24.348	-0.606	14.908	-22.019	-0.677
housing rented	4.277	-7.153	-0.598	4.127	-5.050	-0.817	1.530	-2.775	-0.551	3.717	-3.659	-1.016
Head charact.												
age	7.364	63.752	0.116	12.356	84.493	0.146	15.369	149.270	0.103	15.852	124.721	0.127
age squared	-4.461	-22.541	0.198	-8.718	-31.663	0.275	-13.267	-65.713	0.202	-12.578	-49.398	0.255
female	3.319	-4.535	-0.732	3.611	-3.606	-1.001	1.339	-2.000	-0.670	2.212	-1.962	-1.127
constant	0.000	61.181		0.000	52.671		0.000	38.160		0.000	39.386	
residual	64.259	0.000		65.977	0.000		79.080	0.000		72.350	0.000	

United States 2004

Education	**Less than high school**			**High school**			**College**			**Master/PhD**		
Popn. share	0.168			0.478			0.263			0.091		
Income share	0.096			0.420			0.325			0.159		
	(c)	(d)	(e)	(c)	(d)	(e)	(c)	(d)	(e)	(c)	(d)	(e)
	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)
Household charact.												
number of earners	17.912	45.844	0.391	8.294	33.677	0.246	2.132	17.879	0.119	0.770	10.171	0.076
num. children < 18	3.709	-18.932	-0.196	3.752	-14.973	-0.251	2.914	-14.880	-0.196	1.595	-13.042	-0.122
housing rented	2.712	-9.941	-0.273	3.915	-7.301	-0.536	2.550	-4.406	-0.579	1.498	-2.956	-0.507
Head charact.												
age	1.443	32.771	0.044	6.051	103.695	0.058	5.668	141.428	0.040	3.315	229.784	0.014
age squared	-0.791	-10.866	0.073	-4.191	-45.172	0.093	-4.299	-65.087	0.066	-1.727	-116.767	0.015
female	0.526	-2.842	-0.185	0.299	-2.199	-0.136	0.207	-2.122	-0.097	0.269	-2.776	-0.097
constant	0.000	63.965		0.000	32.274		0.000	27.187		0.000	-4.415	
residual	74.490	0.000		81.878	0.000		90.828	0.000		94.280	0.000	

Notes: LHS is equivalised household income. Omitted variables are (characteristics of the household) housing owned, (characteristics of the householder) male.

Table 4: Analysis of inequality decomposition by education subgroups.

Finland 1987

Education	**Less than high school**			**High school**			**College**			**Master/PhD**		
Popn. share	0.439			0.431			0.122			0.009		
Income share	0.404			0.424			0.159			0.013		
	(c)	(d)	(e)	(c)	(d)	(e)	(c)	(d)	(e)	(c)	(d)	(e)
	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)
num. Of earners	17.992	24.720	0.728	8.557	21.462	0.399	0.957	7.093	0.135	4.909	16.687	0.294
num. < 18	4.354	-7.774	-0.560	5.873	-10.635	-0.552	5.871	-12.110	-0.485	17.572	-18.997	-0.925
housing rented	0.713	-0.932	-0.765	3.008	-2.615	-1.151	2.132	-2.086	-1.022	0.446	-0.651	-0.686
age	-4.236	58.928	-0.072	10.671	72.644	0.147	14.348	107.473	0.134	8.858	33.968	0.261
age squared	7.354	-34.421	-0.214	-7.746	-28.463	0.272	-10.601	-40.185	0.264	-1.908	-3.459	0.551
female	2.420	-1.334	-1.815	2.267	-1.296	-1.749	1.511	-1.222	-1.236	0.251	-0.185	-1.358
constant		60.812			48.903			41.037			72.636	
residual	71.403			77.369			85.783			69.873		

Finland 2004

Education	**Less than high school**			**High school**			**College**			**Master/PhD**		
Popn. share	0.213			0.432			0.342			0.013		
Income share	0.179			0.391			0.410			0.021		
	(c)	(d)	(e)	(c)	(d)	(e)	(c)	(d)	(e)	(c)	(d)	(e)
	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)
num. Of earners	13.753	23.280	0.591	4.382	23.531	0.186	0.844	22.262	0.038	0.919	20.979	0.044
num. < 18	1.538	-8.875	-0.173	1.979	-11.645	-0.170	2.364	-17.213	-0.137	5.114	-11.164	-0.458
housing rented	3.194	-4.439	-0.720	1.907	-3.846	-0.496	0.393	-2.189	-0.179	1.534	-1.436	-1.068
age	-4.186	88.755	-0.047	4.725	96.671	0.049	0.889	42.467	0.021	-12.044	-95.515	0.126
age squared	7.737	-55.859	-0.139	-3.403	-43.002	0.079	-0.667	-16.288	0.041	19.319	72.085	0.268
female	1.595	-3.046	-0.524	1.206	-3.615	-0.334	0.286	-3.527	-0.081	0.254	2.401	0.106
constant		60.184			41.905			74.487			112.650	
residual	76.369			89.205			95.892			84.904		

Notes: LHS is equivalised household income. Omitted variables are (characteristics of the household) housing owned, (characteristics of the householder) male.

Table 5: Analysis of inequality decomposition by education subgroups.

United States 1986

sex		**Male**			**Female**	
Popn. share	0.848			0.152		
Income share	0.903			0.097		
	(c)	(d)	(e)	(c)	(d)	(e)
	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)
num. of earners	5.834	23.471	0.249	11.725	25.743	0.455
num. < 18	12.273	-22.040	-0.557	10.337	-19.856	-0.521
housing rented	2.971	-3.965	-0.749	3.405	-9.658	-0.353
age	8.937	96.528	0.093	2.800	28.245	0.099
age squared	-6.231	-39.116	0.159	-0.762	-4.202	0.181
high school	-1.426	11.150	-0.128	0.610	11.694	0.052
college	4.372	6.526	0.670	5.951	5.596	1.063
master/PhD	10.263	7.596	1.351	4.797	3.811	1.259
constant	0.000	19.852		0.000	58.626	
residual	63.008	0.000		61.138	0.000	

United States 2004

sex		**Male**			**Female**	
Popn. share	0.538			0.462		
Income share	0.579			0.421		
	(c)	(d)	(e)	(c)	(d)	(e)
	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)
num. of earners	3.136	24.547	0.128	6.019	30.854	0.195
num. < 18	2.739	-14.046	-0.195	2.671	-14.878	-0.180
housing rented	2.248	-4.547	-0.494	4.313	-8.882	-0.486
age	4.160	108.146	0.038	3.814	103.291	0.037
age squared	-2.810	-50.814	0.055	-2.250	-47.077	0.048
high school	-1.883	9.373	-0.201	-1.288	9.468	-0.136
college	3.960	13.056	0.303	5.104	13.118	0.389
master/PhD	10.659	10.393	1.026	7.045	6.707	1.050
constant	0.000	3.894		0.000	7.398	
residual	77.791	0.000		74.572	0.000	

Notes: LHS is equivalised household income. Omitted variables are (characteristics of the household) housing owned, (characteristics of the householder) male, less than high school.

Table 6: Analysis of inequality decomposition by sex subgroups.

Finland 1987

sex	**Male**			**Female**		
Popn. share	0.912			0.088		
Income share	0.932			0.068		
	(c)	(d)	(e)	(c)	(d)	(e)
	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)
num. of earners	8.005	21.624	0.370	14.199	18.786	0.756
num. < 18	5.050	-9.631	-0.524	-0.111	-2.621	0.042
housing rented	1.368	-1.632	-0.838	2.602	-4.147	-0.628
age	2.996	70.590	0.042	-7.151	127.644	-0.056
age squared	-1.183	-32.880	0.036	13.315	-65.313	-0.204
high school	-0.461	3.523	-0.131	-0.043	3.121	-0.014
college	10.458	4.717	2.217	9.694	4.272	2.269
master/PhD	1.418	0.468	3.028	0.253	0.078	3.238
constant	0.000	43.221		0.000	18.180	
residual	72.349	0.000		67.243	0.000	

Finland 2004

sex	**Male**			**Female**		
Popn. share	0.709			0.291		
Income share	0.737			0.263		
	(c)	(d)	(e)	(c)	(d)	(e)
	decomp.	fact. sh.	=(c/d)	decomp.	fact. sh.	=(c/d)
num. of earners	2.372	23.116	0.103	3.358	22.533	0.149
num. < 18	2.108	-13.554	-0.156	0.729	-10.249	-0.071
housing rented	1.195	-3.165	-0.378	0.622	-2.738	-0.227
age	2.466	105.720	0.023	-0.345	46.150	-0.007
age squared	-1.439	-49.200	0.029	0.845	-25.408	-0.033
high school	-0.283	1.863	-0.152	-0.011	0.062	-0.177
college	3.141	9.608	0.327	3.809	12.086	0.315
master/PhD	0.875	0.873	1.002	1.247	1.043	1.196
constant	0.000	24.740		0.000	56.522	
residual	89.566	0.000		89.745	0.000	

Notes: LHS is equivalised household income. Omitted variables are (characteristics of the household) housing owned, (characteristics of the householder) less than high school.

Table 7: Analysis of inequality decomposition by sex subgroups.

	United States				Finland			
	1986		2004		1987		2004	
								y
number of earners	7.164	7.538	4.400	4.343	9.446	9.447	2.814	2.755
num. children < 18	11.166	9.848	2.755	2.659	3.677	2.874	1.525	1.333
age of youngest child	no	-0.090	no	-0.028	no	0.696	no	-0.101
number aged 65-74	no	0.071	no	0.000	no	-0.110	no	0.084
number aged 75+	no	0.000	no	0.023	no	-0.166	no	-0.020
regional dummies	no	yes	no	yes	no	yes	no	yes
area dummies	no	yes	no	yes	no	no	no	yes
housing rented	3.440	3.811	3.201	3.227	1.681	2.082	1.060	1.374
age	7.405	9.727	4.199	4.164	1.188	1.267	1.069	1.272
age squared	-4.924	-6.948	-2.723	-2.701	1.246	1.419	-0.264	-0.299
female	2.805	2.545	0.290	0.294	2.001	1.429	0.508	0.476
married	no	0.054	no	0.453	no	0.855	no	0.163
ethnicity	no	3.020	no	0.818	no	0.011	no	0.281
high school	-1.006	-0.814	-1.614	-1.283	-0.373	-0.364	-0.212	-0.253
college	4.581	3.872	4.451	3.932	10.214	9.426	3.255	2.945
master/PhD	9.619	8.283	9.254	8.593	1.342	1.241	0.972	0.891
residual	59.752	55.105	75.787	73.772	69.578	66.361	89.273	87.845
Total	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000

Table 8: Equivalised income inequality decomposition with and without additional controls.

First, it is important to be clear whether inequality of income or inequality of predicted income is being considered: this follows from the point that decomposition is exact only if the residual is not ignored. To illustrate how important this may be Table 9 gives the decomposition of equivalised household income inequality $I(y)$ and the *predicted* equivalised household income inequality $I(\hat{y})$ for the same data sets considered in Section 6. For instance, from a first impression of inequality decomposition in Finland, one might conclude that college education contribution to inequality did not change substantially between 1987 and 2004, as its contribution to the decomposition of $I(\hat{y})$ decreased only from 33% to 30%. However, this is true only if the focus of the analysis is *predicted* income. Looking at the break-down of inequality of total income, in Finland one may conclude that the contribution of total value of college to equivalised income inequality decreased by over a third, from 10% to 3%, and most of the contribution now lies in the residual.

Second, although the computation of standard errors is sometimes treated as a trivial problem (as in Morduch and Sicular 2002), this is not so; the main reason for the complexity is that the inequality index computed from a random sample is itself a random variable and cannot be treated as deterministic in the calculation of standard errors (see Section 4); moreover, $I(y)$ often appears at the denominator of these decompositions making theoretical computation of standard errors cumbersome. A viable way to assess the reliability of calculation is to provide different specifications of the regression

	United States				Finland			
	1986		2004		1987		2004	
Decomposition of:	$I(y)$	$I(\hat{y})$	$I(y)$	$I(\hat{y})$	$I(y)$	$I(\hat{y})$	$I(y)$	$I(\hat{y})$
number of earners	7.164	17.798	4.400	18.173	9.446	31.049	2.814	26.230
num. children < 18	11.166	27.742	2.755	11.376	3.677	12.086	1.525	14.220
housing rented	3.440	8.546	3.201	13.218	1.681	5.527	1.060	9.881
age	7.405	18.399	4.199	17.340	1.188	3.905	1.069	9.967
age squared	-4.924	-12.234	-2.723	-11.245	1.246	4.094	-0.264	-2.462
female	2.805	6.969	0.290	1.199	2.001	6.579	0.508	4.735
high school	-1.006	-2.500	-1.614	-6.665	-0.373	-1.225	-0.212	-1.972
college	4.581	11.381	4.451	18.384	10.214	33.574	3.255	30.343
master/PhD	9.619	23.899	9.254	38.218	1.342	4.410	0.972	9.058
residual	59.752		75.787		69.578		89.273	
Total	100.000	40.249	100.000	24.213	100.000	30.422	100.000	10.727

Notes: LHS is equivalised household income. Omitted variables are: housing owned, male, less than high school. $y = b_0 + \sum_{k=1}^{k} b_k x_k + u$ and $\hat{y} = b_0 + \sum_{k=1}^{k} b_k x_k$.

Table 9: Equivalised income inequality decomposition of total and explained equivalised household incomes.

models, assessing the robustness of results to the inclusion or exclusions of some explanatory variables, as in Section 6.2, or even better by computing standard errors using the bootstrap.

Third, a single-equation model, such as that developed above, should only be interpreted as a descriptive model, showing correlations rather than causal relationships. Could we have done better by opting for a richer model such as the Bourguignon et al. (2001, 2008) simultaneous-equation extension of the Blinder-Oaxaca decomposition? Their interest is in the change across time of the full distribution of income and related statistics. The components of their model are an earnings equation for each household member (linking individual characteristics to their remuneration), a labour supply equation (explaining the decision of entering the labour force depending on individual and other household's members decisions) and a household income equation (aggregating the individuals' contributions to household income formation). The estimation of such an econometric model at two different dates allows one to disentangle: (i) a "price effect" (people with given characteristics and the same occupation get a different income because the remuneration structure has changed) (ii) a "participation" or "occupation effect" (individuals with given characteristics do not make the same choices as for entering the labour force because their household may have changed) and (iii) a "population effect" (individual and household incomes change because socio-demographic characteristics of population of households and individuals change). The main merit of such an approach is that it builds a comprehensive model of how decisions regarding income formation are taken, including the individual decision of entering the labour force and wage formation mechanism, into

a household-based decision process, extracting part of the information left in the residuals of single-equation linear models as the one used in this paper. Bourguignon et al. (2001) used this methodology to argue persuasively that the apparent stability of Taiwan's income inequality was just due to the offsetting of different forces. However, the rich structural model comes at the expense of increasing the complication of the estimation process and of introducing additional and perhaps questionable assumptions. Among the most important limitations of the Bourguignon et al. approach are: the robustness of the estimates of some coefficients, the problem of simultaneity between household members' labour-supply decisions, the issue of understanding what is left in the residuals of the labour supply equations and the counterfactual wage equations, the path-dependence problem (i.e. which counterfactual is computed first) is also a problem.[8] In sum, the full structural model approach for inequality analysis can be cumbersome and is likely to be sensitive to model specification.

8 Concluding comments

Our approach to reconciling the various strands of inequality-decomposition analysis is based on a single-equation regression, builds on the Shorrocks (1982) methodology and is aimed at providing a tool for understanding inequality, especially when the data are not sufficiently detailed to allow a structural model specification. It shares some features with the approach suggested by Fields (2003),[9] but improves on it by including in the analysis the decomposition by subgroups and in showing how this might also be useful to identify differences in determinants of inequality.

It is fairly robust, providing an improvement on other methods, but it provides results consistent with other decomposition methods. The simple specification enables one to distinguish clearly between "explanations" of inequality that rely solely on a breakdown of the factors that underlie predicted income and the breakdown of inequality of observed income.

[8] To get some idea of the magnitude of the path-dependence problem the authors computed all possible evaluations of price, participation and population effects, although the complex problem of computing proper confidence intervals for the structural model is not tackled. The problem has something in common with that of the Shapley-value method discussed in section 2.1.

[9] See also Fields and Yoo (2000), Morduch and Sicular (2002).

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A Appendix A: ancillary empirical results

In Table 10-13 the correlation matrix between RHS variables are presented.

United States 1986

	equiv. income	num. earners	num. <18	rented	age	age squared	female	high school	college	master/ PhD	residual
equiv. income	1.000										
num. earners	0.322	1.000									
	0.000										
num. <18	-0.363	0.022	1.000								
	0.000	*0.000*									
rented	-0.307	-0.182	0.132	1.000							
	0.000	*0.000*	*0.000*								
age	0.176	-0.006	-0.323	-0.292	1.000						
	0.000	*0.269*	*0.000*	*0.000*							
age squared	0.141	-0.069	-0.346	-0.250	0.985	1.000					
	0.000	*0.000*	*0.000*	*0.000*	*0.000*						
female	-0.244	-0.233	0.006	0.240	0.023	0.045	1.000				
	0.000	*0.000*	*0.299*	*0.000*	*0.000*	*0.000*					
high school	-0.062	0.012	-0.007	0.011	-0.170	-0.164	-0.003	1.000			
	0.000	*0.029*	*0.221*	*0.039*	*0.000*	*0.000*	*0.623*				
college	0.187	0.061	-0.070	-0.089	-0.025	-0.035	-0.068	-0.412	1.000		
	0.000	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*			
master/PhD	0.291	0.061	-0.029	-0.092	0.012	-0.010	-0.077	-0.345	-0.136	1.000	
	0.000	*0.000*	*0.000*	*0.000*	*0.028*	*0.073*	*0.000*	*0.000*	*0.000*		
residual	0.772	0.000	0.000	0.001	0.000	0.000	0.000	0.000	-0.001	0.000	1.000
	0.000	*0.938*	*0.953*	*0.931*	*0.998*	*0.987*	*0.971*	*0.982*	*0.929*	*0.993*	

Table 10: Pairwise correlation matrix of dependent, independent variables and residual . P-value in italics.

United States 2004

	equiv. income	num. earn.	num. <18	rented	age	age sq.	female	high sch.	college	mast/PhD	residual
equiv. income	1.000										
num. earn.	0.232	1.000									
	0.000										
num. <18	-0.182	0.028	1.000								
	0.000	*0.000*									
rented	-0.255	-0.119	0.070	1.000							
	0.000	*0.000*	*0.000*								
age	0.108	-0.132	-0.335	-0.284	1.000						
	0.000	*0.000*	*0.000*	*0.000*							
age sq.	0.074	-0.177	-0.360	-0.243	0.982	1.000					
	0.000	*0.000*	*0.000*	*0.000*	*0.000*						
female	-0.100	-0.090	0.060	0.122	-0.083	-0.070	1.000				
	0.000	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*					
high sch.	-0.138	0.000	-0.022	0.038	-0.038	-0.030	0.023	1.000			
	0.000	*0.953*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*				
college	0.171	0.045	-0.034	-0.140	-0.011	-0.032	-0.008	-0.571	1.000		
	0.000	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*				
mast/PhD	0.280	0.010	-0.046	-0.119	0.075	0.057	-0.071	-0.303	-0.188	1.000	
	0.000	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*		
residual	0.871	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
	0.000	*0.999*	*1.000*	*1.000*	*1.000*	*1.000*	*0.999*	*1.000*	*1.000*	*1.000*	

Table 11: Pairwise correlation matrix of dependent, independent variables and residual . P-value in italics.

Finland 1987

	equiv. income	num. earn.	num. <18	rented	age	age sq.	female	high sch.	college	mast/PhD	residual
equiv. income	1.000										
num. earn.	0.338	1.000									
	0.000										
num. <18	-0.143	0.158	1.000								
	0.000	*0.000*									
rented	-0.185	-0.169	-0.052	1.000							
	0.000	*0.000*	*0.000*								
age	0.020	-0.114	-0.279	-0.183	1.000						
	0.000	*0.000*	*0.000*	*0.000*							
age sq.	-0.021	-0.186	-0.314	-0.141	0.984	1.000					
	0.000	*0.000*	*0.000*	*0.000*	*0.000*						
female	-0.186	-0.257	-0.158	0.165	0.107	0.138	1.000				
	0.000	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*					
high sch.	-0.035	0.004	0.042	0.036	-0.330	-0.312	-0.018	1.000			
	0.000	*0.457*	*0.000*	*0.000*	*0.000*	*0.000*	*0.001*				
college	0.305	0.036	0.066	-0.052	-0.019	-0.034	-0.031	-0.324	1.000		
	0.000	*0.000*	*0.000*	*0.000*	*0.001*	*0.000*	*0.000*	*0.000*			
mast/PhD	0.109	0.017	0.031	-0.030	0.010	0.002	-0.025	-0.083	-0.035	1.000	
	0.000	*0.002*	*0.000*	*0.000*	*0.082*	*0.673*	*0.000*	*0.000*	*0.000*		
residual	0.834	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
	0.000	*0.998*	*0.986*	*0.996*	*0.993*	*0.993*	*1.000*	*1.000*	*0.981*	*0.988*	

Table 12: Pairwise correlation matrix of dependent, independent variables and residual . P-value in italics.

Finland 2004

	equiv. income	num. earn.	num. <18	rented	age	age sq.	female	high sch.	college	mast/PhD	residual
equiv. income	1.000										
num. earn.	0.170	1.000									
	0.000										
num. <18	-0.086	0.359	1.000								
	0.000	*0.000*									
rented	-0.151	-0.235	-0.052	1.000							
	0.000	*0.000*	*0.000*								
age	0.034	-0.157	-0.290	-0.245	1.000						
	0.000	*0.000*	*0.000*	*0.000*							
age sq.	0.009	-0.228	-0.325	-0.197	0.983	1.000					
	0.147	*0.000*	*0.000*	*0.000*	*0.000*						
female	-0.076	-0.138	-0.118	0.142	0.036	0.045	1.000				
	0.000	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*					
high sch.	-0.106	0.068	0.064	0.070	-0.196	-0.193	-0.049	1.000			
	0.000	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*				
college	0.181	0.095	0.044	-0.123	-0.057	-0.085	0.045	-0.629	1.000		
	0.000	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*				
mast/PhD	0.095	0.013	0.001	-0.049	0.022	0.014	-0.011	-0.099	-0.082	1.000	
	0.000	*0.023*	*0.912*	*0.000*	*0.000*	*0.020*	*0.054*	*0.000*	*0.000*		
residual	0.945	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
	0.000	*0.999*	*0.996*	*0.993*	*0.995*	*0.996*	*0.997*	*0.996*	*0.993*	*0.998*	

Table 13: Pairwise correlation matrix of dependent, independent variables and residual . P-value in italics.

B Appendix B: Files to replicate empirical results

All empirical results are computed using Stata (www.stata.com) on a remote machine, resident at LIS, and can be replicated using the relevant files from: http://www.economia.unimi.it/users/fiorio/ftp/projects/CowelFiorio_IneqDec/IneqDec.zip. The main results are obtained using a modification of the Stata routine ineqrbd (Fiorio and Jenkins 2007), which can also be downloaded from Stata typing "ssc install ineqrbd, replace" in the Stata command line.