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HOW MUCH WILL WE HAVE TO POSTPONE OUR RETIREMENT TO RECEIVE A DECENT PENSION? THE MECHANICS (AND GEOMETRY) OF NON-FINANCIAL DEFINED CONTRIBUTION PENSION SCHEMES

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How much will we have to postpone our retirement to receive a decent pension? The mechanics (and geometry) of Non-Financial Defined Contribution pension schemes

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Abstract
Non-financial Defined Contribution pension schemes have been proposed as an alternative to Defined Benefit pension schemes. The aim of the paper is the comparison of retirement incentives in a NDC and in a DB system. In particular, within a simplified but thorough analytical framework, the conditions under which the replacement ratio in a NDC system is equal to the replacement ratio in a DB system are worked out. Furthermore, the sensitivity of replacement ratios to postponement of retirement age is calculated. It is shown that, when life expectancy increases, legal retirement ages have to be adjusted in order to keep unchanged incentives and opportunities of NDC system.

1. Introduction

The Italian public pension reform in 1995 changed the computation scheme of pensions. The defined benefit scheme (DB) was substituted by a non-financial defined contribution scheme (NDC). Before the reform, the pension received by any retired worker was mainly based on the years of service and on the average wage received in the final part of working career. The 1995 reform based the computation of pensions on the life expectation of retired worker and on the present value of the pension contributions paid during her whole working career.

The paper identifies winners and losers in the new computation scheme. The analytical horizon is the long run. The aim is to cast some light on the choices made by the 1995 reform in order to have some suggestions on the design of future public pensions. That is why the rules of computation which are applied in the long transition period between the two regimes are not taken into account. At first, life expectancy is taken as given; later on the effects of increasing life expectancy are considered. Furthermore, workers are distinguished according to the rate of growth of their real wage.

The impact of the reform on pensions is computed by focusing on the rate of substitution, that is the ratio between first pension income and last wage income. The substitution rate is a very simple and intuitive indicator of pension adequacy: in the case of low income workers, a low rate of substitution points out a possible social sustainability problem. On the contrary, the substitution rate does not allow any comparison between the inter-temporal flow of contributions paid by each worker and the inter-temporal flow of pension income he receives. As far as the aim of the analysis is concerned, the substitution rate is claimed to be an useful indicator, which allows to indentify the rationale of the choice made by the 1995 reform. Furthermore, by controlling for the rate of growth of wages, the paper compares the substitution rates of workers who are supposed to have the same annual flow of wage income. This makes the inter-temporal comparison of contributions and pension income less relevant. Finally, the analytical framework developed in the paper shows that present and future substitution rates convey most of the information needed to a worker who has to decide when to retire.

The paper develops an analytical framework in order to compare the replacements rates in DB and NDC schemes and the incentives to retirement. The conditions under which the replacement ratios are equal in the two schemes are shown. Those conditions depend on the age of retirement and on the growth rate of wages, but they barely depend on the years of service. Therefore, we are able to identify the ages under which all the individuals with the same rate of growth of wage receive a smaller replacement ratio in the NDC scheme with respect to the DB scheme. Furthermore, the analytical expressions of the gain from postponing retirement in the two schemes are worked out. It is shown that, under not very binding conditions, the retirement age in NDC scheme is higher than the retirement age in DB scheme.

Finally, the effects of a higher life expectancy on the incentive to retirement are worked out. In particular, the attention is focused on the presence of a legal age of retirement which is non indexed to life expectancy. It is shown that the welfare of many retirees can undertake relevant reductions in the NDC scheme with respect to the DB scheme if the legal age of retirement is not linked to life expectancy. Old retirees and retirees which experienced a low rate of growth of wage experience the greater losses of welfare.

The last part of the paper carries out the empirical comparison of replacement rates. The institutional features of the Italian pension systems are considered in order to identify losers and winners in the NDC scheme with respect to DB scheme.

2. Related literature

3. The model

It is needed a model that can be used to predict the effects on retirement of different pension plans. The goal is to compare the effects of a DB scheme and a NDC scheme. The starting point is the option value model developed by Stock, Wise (1990). Each employee has to decide the year *r* when he retires. At the beginning of each year he computes the present value of future utility coming from two sources: first, wage income for all the years *s* when he continues to work, w_s, where $s < r$; second, social security pension received from year *r* to year *L*, when he dies with probability one. Given that the amount of pension depends on years of services and on the age of the retiree at the moment of retirement, the social security benefits will be written as $p_s(r)$, where *s* is greater or equal to *r*.

The following expression gives the indirect utility function evaluated at time *t*, discounted at the annual rate δ, and when he decides to retire at time *r*, where $r \geq t$:

$$V_t(r) = \sum_{s=t}^{r-1} \left(\frac{1}{1+\delta} \right)^{s-t} U_w(w_s) + \sum_{s=r}^{L} \left(\frac{1}{1+\delta} \right)^{s-t} U_r(p_s(r)).$$

The individual has to decide whether to work at *t* or to retire. In other words, he has to decide whether $r >$ t or whether $r = t$. The gain by postponing the retirement at time $r > t$ is given by

$$C_t(r) \equiv V_t(r) - V_t(t)$$

When $r = t+1$,

$$C_t(t+1) = U_w(w_t) - U_r(p_t(t)) + \sum_{s=t+1}^{L} \left(\frac{1}{1+\delta} \right)^{s-t} \left[U_r(p_s(t+1)) - U_r(p_s(t)) \right] \quad (1)$$

We adopt the following decision rule:

Decision rule

The employee continues to work at time t if there is a gain in postponing retirement, that is if $C_t(t+1) > 0$. On *the contrary, r = t when $C_t(t+1) \leq 0$*

In order to simplify our analysis we posit our indirect utility function to have the following specification:

$$V_t(r) = \sum_{s=t}^{r-1} \beta^{s-t} \ln(w_s) + \sum_{s=r}^{L} \left(\frac{1}{1+\delta} \right)^{s-t} [\ln(p_s(r)) + \gamma]$$

where γ represents the preference for leisure versus work.

The gain from one-year postponement of retirement becomes

$$C_t(t+1) = \ln(w_t) - \ln(p_t(t)) - \gamma + \sum_{s=t+1}^{L} \left(\frac{1}{1+\delta} \right)^{s-t} [\ln(p_s(t+1)) - \ln(p_s(t))]$$

It is useful to express the gain from retirement postponement in terms of the ratio between wage and pension in any single year, which we call the replacement ratio

$$\rho_t \equiv \frac{p_t(t)}{w_t}.$$

Furthermore, for simplification sake we assume no inflation[1], pensions fixed in nominal terms, that is $p_s(t) = p_t(t) \quad \forall \quad s \geq t$, and we posit

$$w_s \equiv w_{s-1}(1+\delta)(1+\lambda) = w_r \left(\frac{1}{(1+\delta)(1+\lambda)} \right)^{r-s} \qquad (2)$$

where λ is the differential rate of growth of wages with respect to δ[2].

Therefore, the new expression of the gain from retirement postponement is

$$C_t(t+1) = -\ln \rho_t - \gamma + \ln\left(\frac{\rho_{t+1}}{\rho_t}(1+\lambda)(1+\delta) \right) \sum_{s=t+1}^{L} \left(\frac{1}{1+\delta} \right)^{s-t}.$$

The previous expression makes it clear that the gain from one-year postponement of retirement is greater the smaller the replacement ratio, the smaller the preference for leisure versus work, the higher the present value of replacement ratio increase carried about by the retirement postponement. Furthermore, the expression shows that, in our model, the replacement ratios at time t and at time $t+1$ convey most of the information contained in the gain function.

The amount of pension in a NDC scheme

Assuming no inflation and pensions fixed in nominal terms, the amount of pension in a NDC scheme, $p_t^{NDC}(r)$, is given implicitly by the following expression:

$$\sum_{s=a}^{r-1} \tau w_s (1+\delta)^{r-s} = p_t^{NDC}(r) \sum_{s=r}^{L} \frac{1}{(1+\delta)^{s-r}} \qquad \forall \ t \geq r \qquad (3)$$

where a is the age at which the individual started working and L is the age of death of the individual.

The sum on the right hand side of the equality is called the annuitization factor

[1] All our results would be unaffected by assuming that wages and pensions are indexed to inflation.

[2] Strictly speaking the differential rate of growth is equal to $(1+\delta)\lambda$.

$$G(r,L) \equiv \sum_{s=r}^{L} \frac{1}{(1+\delta)^{s-r}} = \frac{1-\frac{1}{(1+\delta)^{L-r+1}}}{1-\frac{1}{1+\delta}} = \frac{1}{\delta}\,\frac{(1+\delta)^{L-r+1}-1}{(1+\delta)^{L-r}} \qquad (4)$$

where $L - r + 1$ are the years of life as a retiree. For values of δ close to zero, the full expression of annuitization factor can be approximated by the following:

$$G(r,L) \cong \frac{(L-r+1)}{1+(L-r)\delta} \ .$$

By substituting (2) and (4) in (3) it turns out that:

$$\tau\, w_r \sum_{s=a}^{r-1} \left[\frac{(1+\delta)}{(1+\delta)(1+\lambda)} \right]^{r-s} = p_t^{NDC}(r)\ \frac{\delta}{(1+\delta)} \left[1-(1+\delta)^{-(L-r+1)}\right]^{-1}$$

and

$$p_t^{NDC}(r) = w_r\, \frac{\tau\,\delta}{\lambda(1+\delta)}\ \frac{1-(1+\lambda)^{-(r-a)}}{1-(1+\delta)^{-(L-r+1)}}$$

where $r - a$ are the years of service, $L - r + 1$ are the years of life as a retiree, and, for $\lambda = 0$, the function is still continuous[3].

Therefore, the replacement ratio in NDC scheme is

$$\rho^{NDC}(r,L,\tau,\lambda) \equiv \frac{p_r^{NDC}(r)}{w_r} = \frac{\tau\,\delta}{\lambda(1+\delta)}\ \frac{1-(1+\lambda)^{-(r-a)}}{1-(1+\delta)^{-(L-r+1)}} . \qquad (5)$$

For values of λ and δ close to zero, the full expression of replacement ratio can be approximated by the following:

$$\rho^{NDC} = \tau \frac{1}{\lambda}\,\frac{(1+\lambda)^{r-a}-1}{(1+\lambda)^{r-a}}\,\delta\,\frac{(1+\delta)^{L-r}}{(1+\delta)^{L-r+1}-1} \cong \tau\,\frac{r-a}{L-r+1}\,\frac{1+(L-r)\delta}{1+(r-a)\lambda} \quad \forall\ \lambda > -\frac{1}{r-a},$$

where $r - a$ are the years of service and $L - r + 1$ are the years of life as a retiree.

In the Appendix A.1 it is shown that

$$\frac{\partial \rho^{NDC}}{\partial \lambda} < 0 .$$

Therefore, in a NDC scheme the replacement ratio is inversely connected to the growth rate of wages. The higher the wage growth, the smaller the ratio between first pension and last wage.

The amount of pension in a DB scheme

In a DB scheme, $\forall t \geq r$ and $Q \leq r-a$, where Q are the number of the final years of service that are considered in the computation of pension, the amount of pension is given by the following expression:

[3] In effect, $\lim_{\lambda \to 0} \frac{1}{\lambda} \left[1 - \left(\frac{1}{1+\lambda} \right)^{r-a} \right] = r-a$ and, when $\lambda = 0$, $p_t^{NDC}(r) = \tau\, w_r (r-a)\, G(r,L)^{-1}$.

$$p_t^{DB}(r) \equiv \alpha\,(r-a)\,\frac{1}{Q}\sum_{s=r-Q}^{r-1} w_s (1+\delta)^{r-s}$$

Therefore, by substitution of (2),

$$p_t^{DB}(r) = \alpha\,(r-a)\,\frac{1}{Q}\,w_r \sum_{s=r-Q}^{r-1}\left[\frac{(1+\delta)}{(1+\delta)(1+\lambda)}\right]^{r-s} = \alpha\frac{r-a}{Q}\,w_r\,\frac{1}{\lambda}\left[1-\left(\frac{1}{1+\lambda}\right)^{Q}\right]$$

where, for $\lambda = 0$, the function is still continuous[4].

Consequently,

$$\rho^{DB}(r,Q,\alpha,\lambda) \equiv \frac{p_r^{DB}(r)}{w_r} = \alpha\frac{r-a}{Q}\frac{1}{\lambda}\left[1-\left(\frac{1}{1+\lambda}\right)^{Q}\right].$$

For values of λ close to zero, the full expression of replacement ratio can be approximated by the following:

$$\rho^{DB}(r,Q,\alpha,\lambda) \cong \alpha(r-a)\frac{1}{1+Q\lambda} \quad \forall\ \lambda > -\frac{1}{Q}.$$

As in the case of NDC scheme, it is possible to show that

$$\frac{\partial \rho^{DB}}{\partial \lambda} < 0 \quad .$$

Therefore, as in the case of NDC scheme, higher wage growth implies smaller replacement ratios.

In order to make the replacement ratios in the two schemes comparable we introduce the following function:

$$I_Q(r) \equiv \frac{r-a}{Q}\frac{\sum_{s=r-Q}^{r-1}\left[\frac{1}{1+\lambda}\right]^{r-s}}{\sum_{s=r-a}^{r-1}\left[\frac{1}{1+\lambda}\right]^{r-s}}. \tag{6}$$

Both when $Q = r - a$, and when $\lambda = 0$, then $I_Q = 1$. Furthermore, it is possible to show that

Lemma 1
The function (6) is greater than one when $\lambda > 0$, and smaller than one when $\lambda < 0$.

Proof

See Appendix A.2

Making use of the new function (6), the replacement ratio in a DB scheme can be expressed as

$$\rho^{DB} = \alpha I_Q \frac{1}{\lambda}\left[1-\left(\frac{1}{1+\lambda}\right)^{r-a}\right]. \tag{7}$$

Therefore, when $\lambda > 0$, that is when the wage growth rate is greater than δ, the average present value wage computed on the whole working life is smaller than the average computed on the last Q years of service and

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[4] In effect, $\lim_{\lambda\to 0}\frac{1}{\lambda}\left[1-\left(\frac{1}{1+\lambda}\right)^{Q}\right] = Q$ and, when $\lambda = 0$, $p_t^{DB}(r) = \alpha(r-a)$.

$I_Q > 1$. On the contrary, when wage growth rate is smaller than δ, the average present value wage computed on the whole working life is greater than the average on the last Q years of service and $I_w < 1$. Finally, when $\lambda = 0$, then the value of the average present value wage does not depend on Q and it is equal to the average computed on the whole working life. Furthermore, when $Q = r - a$, then the replacement ratio does not depend on the wage growth.

Comparison between replacement rates

By comparing the expressions of the replacement ratios in the two schemes, it is easy to show that they can be equal under some conditions.

Proposition 1

The replacement ratio in a DB scheme, given by expression (7), and in a NDC scheme, given by expression (5), are equal under the following condition:

$$\alpha I_Q = \tau \ G(r, L)^{-1} \qquad \Rightarrow \qquad \frac{\alpha I_Q}{\tau} = \frac{\dfrac{\delta}{1+\delta}}{1 - \dfrac{1}{(1+\delta)^{L-r+1}}}$$

Therefore, there exists an r such that the replacement ratios in NDC and DB schemes are equal. Its analytical expression is the following:*

$$r^* = L + 1 + \frac{\ln\left(1 - \dfrac{\delta}{1+\delta}\dfrac{\tau}{\alpha I_Q}\right)}{\ln(1+\delta)} \qquad (8)$$

Therefore, r^* mainly depends on: the life length, L, the ratio between social contribution rate, τ, and the DB scheme parameter α, the discount rate δ, and the function I_Q, defined in (6). This means that when $Q = r - a$, and $I_Q = 1$, that is when, in the DB scheme, the average present value wage is computed on the whole working life, then the threshold age (8) is the same whatever the wage growth is and whatever the years of service are. That threshold is higher the longer the life is, and it is smaller the higher the ratio τ/α and the discount factor δ are. Furthermore, it is easy to show that when the actual retirement age is smaller than r^*, then the replacement ration in DB scheme is higher than in NDC scheme, otherwise it is smaller.

When $Q < S$, on the contrary, the threshold age (8) increases with the wage growth[5]. Therefore, the higher the wage growth, the higher the retirement age at which the replacement ratios in the two schemes are equal. The threshold age r^* depends on the years of service, via I_Q. However, a linear approximation of r^* shows[6] that, as in the case of $Q = r - a$, the threshold age is mostly determined by the life length, L, the ratio between social contribution rate, τ, and the DB scheme parameter α, the discount rate δ.

[5] See Appendix A.3.

[6] See Appendix A.4.

4. The incentive to postpone retirement in a NDC scheme

We now go back to the expression of the gain from one-year postponement of retirement and we compare the gain in a DB scheme and in a NDC scheme. We start from DB scheme, which is analytically more simple.

Postponing retirement in a DB scheme

We saw that the expression of the postponement gain (1) depends on the percentage increase of replacement ratio, ρ_{t+1}/ρ_t . Now, in a DB scheme and when $Q < r - a$, a one-year postponement of retirement always increases the replacement ratio by a fixed amount:

$$\Delta\rho^{DB} \equiv \rho^{DB}(r+1) - \rho^{DB}(r) \cong \frac{\alpha}{1+Q\lambda}.$$

However, the percentage increase of replacement ratio is smaller, the larger the retirement age, r, is:

$$\frac{\rho^{DB}(r+1)}{\rho^{DB}(r)} = \frac{r-a+1}{r-a} = 1 + \frac{1}{r-a}.$$

The full expression of the retirement postponement gain in a DB scheme is

$$C_t^{DB}(t+1) = -\ln\rho^{DB}(t) - \gamma + \ln\left(\frac{t+1-a}{t-a}(1+\lambda)(1+\delta)\right)\sum_{s=t+1}^{L}\left(\frac{1}{1+\delta}\right)^{s-t} =$$

$$= -\ln\left[\frac{\alpha}{\lambda}\frac{t-a}{Q}\left(1-\frac{1}{(1+\lambda)^{Q}}\right)\right] - \gamma + \frac{1}{\delta}\left(1-\frac{1}{(1+\delta)^{L-t}}\right)\ln\left[\left(1+\frac{1}{t-a}\right)(1+\lambda)(1+\delta)\right].$$

Therefore, in a DB scheme the gain from one-year postponement of retirement depends on seven parameters: years of service, $t - a$, the years of life as a retiree[7], $L - t$, the rate of growth of wages, λ, the preference for leisure versus work, γ, the time preference, δ, and the two parameters of pension computation, α and Q. As the year of retirement, t, increases the gain from the retirement postponement falls: on one hand, the replacement ratio increases, making more attractive the retirement, on the other hand, both the percentage increase of replacement ratio and the numbers of years during which the higher replacement ratio can be enjoyed, drop[8]. On the contrary, a higher wage growth creates the incentive for retirement postponement: it reduces the replacement ratio and increases the average wage which the pension computation is based on.

When, in a DB scheme, $Q = r - a$, the expression of the retirement postponement gain is slightly different. The percentage increase of replacement ratio becomes

$$\frac{\rho^{DB}(r+1)}{\rho^{DB}(r)} = \frac{1}{(1+\lambda)}\frac{(1+\lambda)^{r-a+1}-1}{(1+\lambda)^{r-a}-1}. \qquad (9)$$

However, as in the case with $Q < r - a$, the percentage increase of replacement ratio is greater than one and

[7] Actually, the years of life as a retiree are $L - t + 1$.

[8] $\frac{\mathrm{d}}{\mathrm{d}t}\left(1-\frac{1}{(1+\delta)^{L-t}}\right) = -(1+\delta)^{-(L-t)}\ln(1+\delta) < 0.$

decreases with the age of retirement[9].

By making use of the function $I_Q(r)$, defined in (6), the general expression of postponement gain becomes

$$C_t^{DB}(t+1) = -\ln\left[\frac{\alpha\, I_Q(t)}{\lambda}\left(1-\frac{1}{(1+\lambda)^{t-a}}\right)\right] - \gamma + \\ + \frac{1}{\delta}\left(1-\frac{1}{(1+\delta)^{L-t}}\right)\ln\left(\frac{I_Q(t+1)}{I_Q(t)}\frac{(1+\lambda)^{t-a+1}-1}{(1+\lambda)^{t-a}-1}(1+\delta)\right). \tag{10}$$

Postponing retirement in a NDC scheme

In a NDC scheme the analytical representation of the gain from retirement postponement is more complex. By substitution, it turns out that:

$$\frac{\rho^{NDC}(r+1)}{\rho^{NDC}(r)} = \left[\frac{1}{(1+\lambda)}\;\frac{(1+\lambda)^{r-a+1}-1}{(1+\lambda)^{r-a}-1}\right]\left[\frac{1}{(1+\delta)}\;\frac{(1+\delta)^{L-r+1}-1}{(1+\delta)^{L-r}-1}\right]. \tag{11}$$

The last expression makes it clear that, in a NDC scheme, the retirement postponement increases the replacement ratio by a greater amount[10] than it does in the case of a DB scheme with $Q = r - a$. In effect, in a NDC scheme the percentage increase of replacement ratio comprises two factors: the first factor depends on the years of service, $r - a$, and it is equal to the percentage increase of replacement ratio in a DB scheme with $Q = r - a$. The more the years of services, the higher the amount of contributions and the replacement ratio. The second factor, on the contrary, depends on the years of life as a retiree, $L - r + 1$, it is not present in the DB scheme, and it magnifies the first factor. The higher the retirement age, the smaller the years of life as a retiree and the higher the replacement ratio.

Furthermore, while the first factor is monotonically decreasing with age of retirement as in a DB scheme, the second factor is monotonically increasing with the retirement age[11].

The full expression of the retirement postponement gain in a NDC scheme becomes

$$C_t^{NDC}(t+1) = -\ln\left[\tau\frac{\delta}{\lambda(1+\delta)}\frac{1-(1+\lambda)^{-(t-a)}}{1-(1+\delta)^{-(L-t+1)}}\right] - \gamma + \\ + \frac{1}{\delta}\left(1-\frac{1}{(1+\delta)^{L-t}}\right)\ln\left(\frac{(1+\lambda)^{t-a+1}-1}{(1+\lambda)^{t-a}-1}\;\frac{(1+\delta)^{L-t+1}-1}{(1+\delta)^{L-t}-1}\right). \tag{12}$$

Therefore, the retirement postponement can produce a gain much higher that it does in the case of a DB scheme: primarily, when the $r < r^*$, the replacement ratio is smaller in a NDC scheme and this incentives the retirement postponement[12]; furthermore, the postponement originates a percentage increase of replacement ratio which is higher than in the case of DB scheme, as we have above pointed out.

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[9] See Appendix A.5 and A.6.

[10] Both the factors in the previous expression are greater than one. See Appendix A.6.

[11] See Appendix A.5.

[12] The postponement of retirement increases the replacement ration, since

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{\lambda}\left(1-\frac{1}{(1+\lambda)^{-(t-a)}}\right) = \frac{1}{\lambda}(1+\lambda)^{-(t-a)}\ln(1+\lambda) > 0.$$

Comparison of the gain from the retirement postponement in a DB and in a NDC scheme

We now want to compare the retirement decision in a DB and in a NDC scheme. By computing the difference between the gains from the retirement postponement at a given time in the two schemes it is possible to show that

Proposition 2

Assume that an employee follows the decision rule previously defined and that, in a DB scheme, she retires in the year r^{DB} and that $r^{DB} < r^*$, where r^* was identified in Proposition 1. We further assume that $C^{DB}_{r^{DB}}(r^{DB}+1)=0$. Then, in a NDC scheme, the same employee would retire at the year $r^{NDC} > r^{DB}$ under the following conditions:

a) always when $Q = r - a$, or when $\lambda \leq 0$;

b) under the condition $\frac{\rho^{NDC}(r^{DB}+1)}{\rho^{NDC}(r^{DB})} > 1 + \frac{1}{r^{DB}-a}$, when $Q < r - a$, and $\lambda > 0$.

Proof

See Appendix.

5. The life length and the optimal retirement age

6. An application to Italian pension reform

The comparison between substitution rates before and after the 1995 reform allows to point out a couple of interesting points (see Table 1). We distinguish three types of workers according to the rate of growth of their real wage. High growth workers receive a wage which increases by 2 per cent a year. The wage of low growth workers grows by 1 per cent a year. Intermediate growth wages grow at 1,5 per cent a year. First, after the reform young retirees receive always a pension smaller than the one they would have received before the reform. However, old retirees can receive a pension higher than before the reform. This is particularly true when the wage growth rate is low. Second, after the reform the higher the wage growth rate, the smaller is the substitution rate, given age and years of services. In particular, when the wage growth rate is equal to 2.0 per cent, substitution rates are always smaller after the reform, whatever the age and years of services are. In the end, in the 1995 reform the looser were young retirees and workers with a high wage growth rate.

The effects of the 1995 reform on the substitution rates can be graphically represented. In a plane with years of services on the horizontal axis and age on the vertical axis, isosubstitution curves give all the combinations of age and years of services that give the same substitution rate. Before the 1995 reform the isosubstitution curves are vertical. For instance, when the wage growth rate is one per cent per year, a rate of substitution of 70 per cent could be reached provided that the retiree had worked for 35.6 years, whatever the age of the retiree (see Figure 1, curve AA). A rate of substitution of 80 per cent requested 40.7 years of work (see Figure 1, curve CC). After the 1995 reform, the isosubstitution curves become downward sloped. Along an isosubstitution curve an increase of years of services has to be compensated by a fall of age. The more on the right the curve, the higher the rate of substitution. For instance, when the wage growth rate is one per cent per year, the combination of 35 years of service and 62.3 years of age is on the 70 per cent isosubstitution curve (see Figure 1, curve γγ). The same rate of substitution is obtained by a retiree with 36 years of service and 61.4 years of age. The slope of any isosubstitution curve is constant, even though any isosubstitution curve has a different slope: namely, the higher, the flatter[13].

Linearity of substitution rates is preserved also in the case of higher growth rate of wages (see Figure 2 and 3). However, the location of any isosubstitution curve is more on the right, the higher is the wage growth rate. For instance, given 1 per cent wage growth, the 70 per cent isosubstitution curve is represented by AA of Figure 1 in the system before the 1995 reform and by γγ of the same Figure in the system after the 1995 reform. Given higher wage growth, the 70 per cent isosubstitution curves move on the right (AA and δδ in Figure 2, AA and εε in Figure 3).

The two curves representing the same substitution rate after the 1995 reform and before it cross each other. In the figures we have highlighted the locus where the isosubstitution curves cross each other and we have called it indifference edge. The indifference edge identifies the retirees who benefit form the 1995 reform, and those who suffer because of it. All the workers who retire with an age-years of service combination higher then the indifference edge receive a greater pension than they do in the system before the 1995 reform. On the contrary, all the workers below the indifference edge are worse off in the system after the 1995 reform, and those on the edge are indifferent. The indifference edge identifies the threshold retirement age which we have called r^* in (8). Given a 1 per cent wage growth rate the indifference edge is linear, almost flat[14] and it ranges from 61.8 and 61.3 per cent (Figure 1). When the wage growth is 1.5 per cent, the indifference edge is nearly horizontal at 63.9 per cent level (Figure 2). Finally, when the wage growth is 2 per cent, the indifference edge lies above 65 years of age (Figure 3). Visual analysis makes it clear that most of retirees receive substitution rates smaller than those before the 1995 reform. However, retirees with low wage growth rates can be better off, provided that their age is higher than 62 years.

Visual analysis of isosubstitution curves highlights another crucial feature of the computation scheme introduced in 1995. Most of the retirees can reach the same substitution rate they benefitted before the reform by postponing the age of retirement. For instance, take the 57 years old retiree with a 1 per cent wage growth rate. With a 35.6 years of service that retiree received a rate of substitution of 70 per cent before the

[13] See Appendix A.9 for an explanation.
[14] See Appendix A.4 for an explanation.

reform. After the reform he received a substitution rate just above 60 per cent. However, he can reach the 70 per cent substitution rate by working about 2.5 years more. In fact, the aging path of that retiree (Figure 1) crosses the curve $\alpha\alpha$ at the age of about 59.2 years. The years of extra work needed to reach the same substitution rate than before the reform increase as the wage growth rate increases. For instance, when the wage growth rate is 2 per cent, a worker with 36.4 years of service and 57 years of age needs to postpone the retirement by little more than 5 years to get the 70 per cent substitution rate (Figure 3).

Notice that the linearity of the isosubstitution curves makes the extra years of work needed to reach the same substitution rate than before the reform to be mostly dependent on the age of the worker. In other words, years of service marginally impinge on the extra period of work. This is made clear by computing the extra years of work for any combination of age and years of service (Table 2). For instance, when the wage growth rate is 1 per cent and with 57 years of age, the extra years of work range from 2.5 with 35 years of services to 2.4 with 40 years of service. When the wage growth rate is 2 per cent, the range is slightly larger: from 5.3 to 5.8 years[15]. Obviously, the closer the combination age-service to the indifference edge, the smaller the extra years of work.

In conclusion, most of the retirees receive a substitution rate smaller than before the 1995 reform. However, most of them have the opportunity of reaching the same substitution rate they had before the reform by postponing the retirement age. Only a small group of workers do not have that opportunity. Namely, when the wage growth rate is 2 per cent, 64 years old workers with less than 41 years of service are not able to reach the substitution rate ante reform. This is so because they would need to postpone the retirement age by more than one year. But by doing so they would overtake the maximum retirement age of 65 years.

THIS SECTION HAS TO BE COMPETED BY AN ANALYSIS OF THE EFFECTS OF A HIGHER LIFE EXPECTANCY ON THE EXTRA YEARS OF WORK TO RECEIVE THE SAME SUBSTITUION RATE THAN IN THE PENSION SYSTEM BEFORE THE REFORM

[15] When years of service are higher than 40, the extra years of work fall because by law 80 per cent was the maximum substitution rate before the 1995 reform.

References

Bŏrsch-Supan, Axel H., 2006, "What Are NDC Systems? What Do They Bring to Reform Strategies?", in Robert Holzmann, Edward Palmer (eds.), *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*; Washington: The World Bank.

Chłoń-Domińczak, Agnieszka, Daniele Franco, Edward Palmer, 2009, The First Wave of NDC – Taking Stock Ten Years Plus Down the Road, presented at the Joint Swedish Social Insurance Agency – World Bank Conference on Non – Financial Defined Contribution (NDC) Pension Systems: Progress and New Frontiers in a Changing Pension World.

Gronchi, Sandro, Sergio Nisticò, 2006, "Implementing the NDC Theoretical Model: A Comparison of Italy and Sweden", in Robert Holzmann, Edward Palmer (eds.), *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*; Washington: The World Bank.

Gronchi, Sandro, Sergio Nisticò, 2008, "Theoretical foundations of Pay-As-You-Go Defined-Contribution Pension Schemes", *Metroeconomica* 59(2): 131-159.

Legros, Florence, 2006, "NDCs: A Comparison of the French and German Point Systems", in Robert Holzmann, Edward Palmer (eds.), *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*; Washington: The World Bank.

Lindeman, David, David Robalino, Michal Rutkowski, 2006, "NDC Pension Schemes in Middle- and Low-Income Countries", in Robert Holzmann, Edward Palmer (eds.), 2006, *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*; Washington: The World Bank.

Palmer, Edward, 2006, "What Is NDC?", in Robert Holzmann, Edward Palmer (eds.), *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*; Washington: The World Bank.

Pollastri, Corrado, 2008, Gli incentivi automatici nel sistema pensionistico italiano a contribuzione definita. Analisi microeconomica delle scelte di pensionamento e impatto sulla dinamica della spesa; XX Riunione scientifica SIEP, Pavia, 25-26 settembre.

Stock, S. H. , D. Wise, 1990, "Pensions, the option value of work, and retirement", *Econometrica* 58(5): 1151-1180.

Appendix

A.1 Derivative of replacement ratio with respect to wage growth

By making use of the Taylor series expansion of $(1+\lambda)^{r-a+1}$, it is easy to show that

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\frac{1}{\lambda}\left[1-(1+\lambda)^{-(r-a)}\right]=\frac{1+\lambda(r-a+1)-(1+\lambda)^{(r-a)+1}}{(1+\lambda)^{(r-a)+1}\lambda^2}<0,$$

and this proves that $\frac{\mathrm{d}\rho^{NDC}}{\mathrm{d}\lambda}<0$.

A.2 Proof of Lemma 1

The function I_Q can be represented as follows

$$I_Q=\frac{1}{\frac{Q}{r-a}+\frac{r-a-Q}{r-a}\frac{\frac{1}{r-a-Q}\sum_{s=a}^{r-Q-1}\left(\frac{1}{1+\lambda}\right)^{r-s}}{\frac{1}{Q}\sum_{s=r-Q}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}}}.$$

Therefore,

$$I_Q>1 \qquad \Rightarrow \qquad \frac{1}{r-a-Q}\sum_{s=a}^{r-Q-1}\left(\frac{1}{1+\lambda}\right)^{r-s}<\frac{1}{Q}\sum_{s=r-Q}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}.$$

Now, when $\lambda>0$, then

$$\left(\frac{1}{1+\lambda}\right)>\left(\frac{1}{1+\lambda}\right)^2>....>\left(\frac{1}{1+\lambda}\right)^{r-a}.$$

This implies

$$\sum_{s=r-Q}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}>Q\left(\frac{1}{1+\lambda}\right)^{Q} \quad \Rightarrow \quad \frac{1}{Q}\sum_{s=r-Q}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}>\left(\frac{1}{1+\lambda}\right)^{Q}$$

and

$$\sum_{s=a}^{r-Q-1}\left(\frac{1}{1+\lambda}\right)^{r-s}<(r-a-Q)\left(\frac{1}{1+\lambda}\right)^{Q+1} \quad \Rightarrow \quad \frac{1}{r-a-Q}\sum_{s=a}^{r-Q-1}\left(\frac{1}{1+\lambda}\right)^{r-s}<\left(\frac{1}{1+\lambda}\right)^{Q+1}.$$

Therefore,

$$\left(\frac{1}{1+\lambda}\right)^{Q+1}<\left(\frac{1}{1+\lambda}\right)^{Q} \quad \Rightarrow \quad \frac{1}{r-a-Q}\sum_{s=a}^{r-Q-1}\left(\frac{1}{1+\lambda}\right)^{r-s}<\frac{1}{Q}\sum_{s=r-Q}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s},$$

QED.

Similarly it is possible to show that when $\lambda<0$, then $I_Q<1$.

A.3 The derivative of I_Q with respect to λ

I_Q represents the ratio of the average addenda of the two series. The series at denominator is equal to the series at numerator plus some extra addenda.

$$I_Q = \frac{r-a}{Q} \frac{1-\left(\frac{1}{1+\lambda}\right)^Q}{\left[1-\left(\frac{1}{1+\lambda}\right)^Q\right]+\left(\frac{1}{1+\lambda}\right)^Q\left[1-\left(\frac{1}{1+\lambda}\right)^{r-a-Q}\right]}$$

When $\lambda > 0$, the extra addenda are all smaller than the addenda of the series at numerator. Therefore, the average addendum of the series at numerator is greater than the average addendum of the series at denominator and I_Q is greater than one. On the contrary, when $\lambda < 0$, the extra addenda are all greater than the addenda of the series at numerator. Therefore, the average addendum of the series at numerator is smaller than the average addendum of the series at denominator and I_w is smaller than one.

When the value of λ increases, the value of each extra addendum at denominator decreases, therefore I_w increases. In other words,

$$\frac{\mathrm{d}\, I_Q}{\mathrm{d}\,\lambda} > 0.$$

A.4 An approximation of *r in Proposition 1**

By making use of Taylor expansion, I_Q can be approximated by the following expression:

$$I_Q \equiv \frac{r-a}{Q} \frac{1-(1+\lambda)^{-Q}}{1-(1+\lambda)^{-(r-a)}} \cong \frac{r-a}{Q} \frac{1-[1-Q\lambda]}{1-[1-(r-a)\lambda]} = 1$$

Therefore, given that the retirement age which makes the replacement ratio in NDC scheme equal to the one in DB scheme is implicitly defined bye the following:

$$\alpha I_Q = \tau \frac{\frac{\delta}{1+\delta}}{1-(1+\delta)^{-(L-r+1)}},$$

and that, by using Taylor expansion once again, that expression can be approximated by

$$\alpha \cong \tau \frac{\frac{\delta}{1+\delta}}{(L-r+1)\delta},$$

the linear approximation of the threshold retirement age r^* is

$$r^* \cong L+1-\frac{\tau}{\alpha}\frac{1}{1+\delta},$$

which makes it clear that the approximation of r^* does not depend on the years of services.

A.5 Derivatives of the percentage increase of replacement ratio

The percentage increase of replacement ratio in the two schemes is given by (9) and (11). The expression in

(9) is equal to the first of the two factors in (11). The first factor in (11) decreases when the retirement year increases:

$$\frac{\mathrm{d}}{\mathrm{d}\,r}\frac{(1+\lambda)^{r-a+1}-1}{(1+\lambda)^{r-a+1}-(1+\lambda)}=\frac{-\lambda(1+\lambda)^{r-a+1}\ln(1+\lambda)}{\left((1+\lambda)^{r-a+1}-(1+\lambda)\right)^2}<0.$$

On the contrary, the second factor in (11) increases when the retirement year increases:

$$\frac{\mathrm{d}}{\mathrm{d}\,r}\frac{(1+\delta)^{r-a+1}-1}{(1+\delta)^{r-a+1}-(1+\delta)}=\frac{\delta\,(1+\delta)^{L-r+1}\ln(1+\delta)}{\left((1+\delta)^{r-a+1}-(1+\delta)\right)^2}>0.$$

A.6 Percentage increase of the replacement ratio in DB scheme

The percentage increase of replacement ratio is greater than one for any value of λ:

when $\lambda > 0$

$$(1+\lambda)>1 \quad \Rightarrow \quad \frac{\rho^{DB}(r+1)}{\rho^{DB}(r)}=\frac{1}{(1+\lambda)}\,\frac{(1+\lambda)^{r-a+1}-1}{(1+\lambda)^{r-a}-1}>1\,;$$

on the contrary, when $\lambda < 0$

$$(1+\lambda)<1 \quad \Rightarrow \quad (1+\lambda)^{r-a+1}-1 \;<\; (1+\lambda)^{r-a+1}-(1+\lambda) \;<0 \quad \Rightarrow$$

$$\Rightarrow \quad \frac{\rho^{DB}(r+1)}{\rho^{DB}(r)}=\frac{1}{(1+\lambda)}\,\frac{(1+\lambda)^{r-a+1}-1}{(1+\lambda)^{r-a}-1}>1,$$

A.7 Lemma 2

The function

$$\frac{I_Q(r+1)}{I_Q(r)}=\frac{r-a+1}{r-a}\,\frac{1-(1+\lambda)^{-(r-a)}}{1-(1+\lambda)^{-(r-a+1)}}=\frac{\dfrac{1}{r-a}\displaystyle\sum_{s=r-a}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}}{\dfrac{1}{r-a+1}\displaystyle\sum_{s=a}^{r}\left(\frac{1}{1+\lambda}\right)^{r-s+1}}$$

is greater than one when $\lambda > 0$*, and smaller than one when* $\lambda < 0$*.*

Proof

By making use of the transformation $j = s - 1$, the previous function can be represented as follows

$$\frac{I_Q(r+1)}{I_Q(r)}=\frac{\dfrac{1}{r-a}\displaystyle\sum_{s=r-a}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}}{\dfrac{1}{r-a+1}\left[\displaystyle\sum_{j=a}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-j}+\left(\frac{1}{1+\lambda}\right)^{r-a+1}\right]}=$$

$$=\frac{1}{\dfrac{r-a}{r-a+1}+\dfrac{1}{r-a+1}\,\dfrac{\left(\dfrac{1}{1+\lambda}\right)^{r-a+1}}{\dfrac{1}{r-a}\displaystyle\sum_{s=r-a}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}}}$$

Therefore,

$$\frac{I_Q(r+1)}{I_Q(r)}>1 \qquad \Rightarrow \qquad \left(\frac{1}{1+\lambda}\right)^{r-a+1}<\frac{1}{r-a}\sum_{s=r-a}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}.$$

Now, when $\lambda>0$, then

$$\left(\frac{1}{1+\lambda}\right)>\left(\frac{1}{1+\lambda}\right)^{2}>....>\left(\frac{1}{1+\lambda}\right)^{r-a+1}.$$

This implies

$$\sum_{s=r-a}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}>(r-a)\left(\frac{1}{1+\lambda}\right)^{r-a+1} \qquad \Rightarrow \qquad \frac{1}{r-a}\sum_{s=r-a}^{r-1}\left(\frac{1}{1+\lambda}\right)^{r-s}>\left(\frac{1}{1+\lambda}\right)^{r-a+1}$$

QED.

Similarly it is possible to show that when $\lambda<0$, then $I_Q(r+1)/I_Q(r)<1$.

A.8 Proof of Proposition 2

By assumption at time r^{DB} the gain function $C^{DB}_{r^{DB}}(r^{DB}+1)=0$. We now make the difference at time r^{DB} between the gain functions in the NDC scheme (12) and the same function in the DB scheme (10), and we will work out the conditions under which that difference is positive.

$$\begin{aligned}C^{NDC}_{r^{DB}}(r^{DB}+1)-C^{DB}_{r^{DB}}(r^{DB}+1)=&-\ln\left[\tau\frac{\delta}{\lambda(1+\delta)}\frac{1-(1+\lambda)^{-(r^{DB}-a)}}{1-(1+\delta)^{-(L-r^{DB}+1)}}\right]-\gamma+\\&+\frac{1}{\delta}\left(1-\frac{1}{(1+\delta)^{L-r^{DB}}}\right)\ln\left(\frac{(1+\lambda)^{r^{DB}-a+1}-1}{(1+\lambda)^{r^{DB}-a}-1}\,\frac{(1+\delta)^{L-r^{DB}+1}-1}{(1+\delta)^{L-r^{DB}}-1}\right)+\\&+\ln\left[\frac{\alpha\,I_Q(r^{DB})}{\lambda}\left(1-(1+\lambda)^{-(r^{DB}-a)}\right)\right]+\gamma-\\&-\frac{1}{\delta}\left(1-\frac{1}{(1+\delta)^{L-r^{DB}}}\right)\ln\left(\frac{I_Q(r^{DB}+1)}{I_Q(r^{DB})}\,\frac{(1+\lambda)^{r^{DB}-a+1}-1}{(1+\lambda)^{r^{DB}-a}-1}(1+\delta)\right).\end{aligned}$$

By simplification, the difference between the gain functions becomes

$$\begin{aligned}C^{NDC}_{r^{DB}}(r^{DB}+1)-C^{DB}_{r^{DB}}(r^{DB}+1)=&\ln\left(\frac{\alpha\,I_Q(r^{DB})}{\tau}\right)+\ln\left[\frac{(1+\delta)}{\delta}\left(1-\frac{1}{(1+\delta)^{L-r^{DB}+1}}\right)\right]+\\&+\frac{1}{\delta}\left(1-\frac{1}{(1+\delta)^{L-r^{DB}}}\right)\left[\ln\left(\frac{(1+\delta)^{L-r^{DB}+1}-1}{(1+\delta)^{L-r^{DB}}-1}\right)-\ln\left(\frac{I_Q(r^{DB}+1)}{I_Q(r^{DB})}(1+\delta)\right)\right].\end{aligned}$$

By assumption $r^{DB}<r^*$, and this implies that

$$\rho^{DB}(r^{DB})>\rho^{NDC}(r^{DB}).$$

In other words, when the retirement age is smaller than r^*, then the replacement ratio in a DB scheme at time r^{DB} is always greater than the replacement in a NDC scheme at the same time. But this implies that

$$\frac{\alpha\,I_Q(r^{DB})}{\tau}>\left[\frac{(1+\delta)}{\delta}\left(1-\frac{1}{(1+\delta)^{L-r^{DB}+1}}\right)\right]^{-1},$$

and that

$$\ln\left(\frac{\alpha\, I_Q(r^{DB})}{\tau}\right)+\ln\left[\frac{(1+\delta)}{\delta}\left(1-\frac{1}{(1+\delta)^{L-r^{DB}+1}}\right)\right]>0$$

Furthermore, given that

$$\frac{(1+\delta)^{L-r^{DB}+1}-1}{(1+\delta)^{L-r^{DB}}-1}>(1+\delta)\,,$$

we have that, when $Q = r - a$, or when $\lambda = 0$, and therefore $I_Q = 1$, then

$$\ln\left(\frac{(1+\delta)^{L-r^{DB}+1}-1}{(1+\delta)^{L-r^{DB}}-1}\right)-\ln\left(\frac{I_Q(r^{DB}+1)}{I_Q(r^{DB})}(1+\delta)\right)>0\,. \qquad \text{(A1)}$$

Therefore,

$$C^{NDC}_{r^{DB}}(r^{DB}+1)-C^{DB}_{r^{DB}}(r^{DB}+1)=C^{NDC}_{r^{DB}}(r^{DB}+1)>0$$

and, given the decision rule previously defined, this implies that $r^{NDC} > r^{DB}$.

We have proved[16] in Lemma 2 that, when $\lambda < 0$, $I_Q(t+1)/ I_Q(t) < 1$, and this implies that (A1) is true also in the case of $Q < r - a$ and $\lambda < 0$.

Finally, in the case of $\lambda > 0$, (A1) is true under the condition

$$\frac{(1+\delta)^{L-r^{DB}+1}-1}{(1+\delta)^{L-r^{DB}}-1}>\frac{I_Q(r^{DB}+1)}{I_Q(r^{DB})}(1+\delta)\,,$$

which can be written as

$$\frac{\rho^{NDC}(r^{DB}+1)}{\rho^{NDC}(r^{DB})}>1+\frac{1}{r^{DB}-a}$$

QED.

A.9 Iso-replacement curves

In the NDC scheme

$$\rho^{NDC}=\frac{\tau}{\lambda}\frac{\delta}{1+\delta}\left[1-(1+\lambda)^{-(r-a)}\right]\left[1-(1+\delta)^{-(L-r+1)}\right]^{-1}.$$

Therefore,

$$\begin{aligned}\mathrm{d}\,\rho^{NDC}=&\frac{\tau}{\lambda}\frac{\delta}{1+\delta}\left[1-(1+\delta)^{-(L-r+1)}\right]^{-1}(1+\lambda)^{-(r-a)}\ln(1+\lambda)\;\;\mathrm{d}(r-a)+\\ &-\frac{\tau}{\lambda}\frac{\delta}{1+\delta}\left[1-(1+\lambda)^{-(r-a)}\right]\left[1-(1+\delta)^{-(L-r+1)}\right]^{-2}(1+\delta)^{-(L-r+1)}\ln(1+\delta)\;\mathrm{d}(L-r+1).\end{aligned}$$

Along an iso-replacement curve

$$\frac{\mathrm{d}(L-r+1)}{\mathrm{d}(r-a)}=\frac{\ln(1+\lambda)}{\ln(1+\delta)}\,\frac{(1+\delta)^{L-r+1}-1}{(1+\lambda)^{r-a}-1}\cong\frac{L-r+1}{r-a}.$$

That is, the iso-replacement curves can be approximated by lines which start from the origin and have a constant slope.

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[16] See Appendix A.8.

Table 1 – Rates of substitution ante e post Dini's reform

wage rate of growth: 1.0% per year

				ante 1995 reform					
	years of service								
age	35	36	37	38	39	40	41	42	43
57-65	0.69	0.71	0.73	0.75	0.77	0.79	0.80	0.80	0.80
				post 1995 reform					
65	0.77	0.80	0.82	0.84	0.87	0.89	0.92	0.94	0.97
64	0.74	0.77	0.79	0.81	0.84	0.86	0.88	0.91	0.93
63	0.72	0.74	0.76	0.79	0.81	0.83	0.85	0.88	0.90
62	0.69	0.72	0.74	0.76	0.78	0.80	0.82	0.85	0.87
61	0.67	0.69	0.71	0.73	0.76	0.78	0.80	0.82	0.84
60	0.65	0.67	0.69	0.71	0.73	0.75	0.77	0.79	0.81
59	0.63	0.65	0.67	0.69	0.71	0.73	0.75	0.77	0.79
58	0.61	0.63	0.65	0.67	0.69	0.71	0.73	0.75	0.77
57	0.59	0.61	0.63	0.65	0.67	0.69	0.71	0.73	0.74

wage rate of growth: 1.5% per year

				ante 1995 reform					
	years of service								
age	35	36	37	38	39	40	41	42	43
57-65	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.80	0.80
				post 1995 reform					
65	0.71	0.73	0.75	0.77	0.79	0.81	0.83	0.85	0.87
64	0.68	0.70	0.72	0.74	0.76	0.78	0.80	0.82	0.84
63	0.66	0.68	0.70	0.72	0.73	0.75	0.77	0.79	0.81
62	0.64	0.66	0.67	0.69	0.71	0.73	0.75	0.76	0.78
61	0.62	0.63	0.65	0.67	0.69	0.70	0.72	0.74	0.76
60	0.60	0.61	0.63	0.65	0.66	0.68	0.70	0.72	0.73
59	0.58	0.59	0.61	0.63	0.64	0.66	0.68	0.69	0.71
58	0.56	0.58	0.59	0.61	0.63	0.64	0.66	0.67	0.69
57	0.55	0.56	0.58	0.59	0.61	0.62	0.64	0.65	0.67

wage rate of growth: 2.0% per year

				ante 1995 reform					
	years of service								
age	35	36	37	38	39	40	41	42	43
57-65	0.67	0.69	0.71	0.73	0.75	0.77	0.79	0.80	0.80
				post 1995 reform					
65	0.65	0.67	0.69	0.70	0.72	0.74	0.75	0.77	0.79
64	0.63	0.65	0.66	0.68	0.69	0.71	0.73	0.74	0.76
63	0.61	0.62	0.64	0.65	0.67	0.69	0.70	0.72	0.73
62	0.59	0.60	0.62	0.63	0.65	0.66	0.68	0.69	0.71
61	0.57	0.58	0.60	0.61	0.63	0.64	0.66	0.67	0.68
60	0.55	0.56	0.58	0.59	0.61	0.62	0.63	0.65	0.66
59	0.53	0.55	0.56	0.57	0.59	0.60	0.61	0.63	0.64
58	0.52	0.53	0.54	0.56	0.57	0.58	0.60	0.61	0.62
57	0.50	0.52	0.53	0.54	0.55	0.57	0.58	0.59	0.61

Table 2 - Extra years of work to receive the same substitution rate

wage rate of growth: 1.0% per year

age	years of service 35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
64																
63																
62																
61	0.4	0.3	0.3	0.3	0.2	0.2	0.04									
60	0.9	0.9	0.8	0.8	0.8	0.7	0.6	0.1								
59	1.4	1.4	1.4	1.3	1.3	1.3	1.1	0.7	0.2							
58	1.9	1.9	1.9	1.9	1.9	1.8	1.7	1.2	0.8	0.3						
57	2.5	2.4	2.4	2.4	2.4	2.4	2.2	1.8	1.3							

wage rate of growth: 1.5% per year

age	years of service 35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
64																
63	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.2								
62	1.1	1.1	1.1	1.1	1.1	1.1	1.1	0.8	0.4							
61	1.6	1.6	1.6	1.7	1.7	1.7	1.7	1.4	1.0	0.6	0.2					
60	2.2	2.2	2.2	2.2	2.3	2.3	2.3	2.0	1.6	1.2	0.8	0.6				
59	2.7	2.8	2.8	2.8	2.9	2.9	2.9	2.6	2.2	1.8	1.6					
58	3.3	3.3	3.4	3.4	3.4	3.5	3.5	3.2	2.8	2.6						
57	3.8	3.9	3.9	4.0	4.0	4.1	4.1	3.8	3.6							

wage rate of growth: 2.0% per year

age	years of service 35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
64	n.v.	n.v.	n.v.	n.v.	n.v.	n.v.	n.v.	n.v.	0.9	0.6	0.2					
63	1.7	1.7	1.8	1.8	1.9	2.0	n.v.	1.9	1.6	1.2	0.9	0.5	0.2			
62	2.3	2.3	2.4	2.5	2.5	2.6	2.7	2.6	2.2	1.9	1.5	1.2	0.8	0.5		
61	2.9	2.9	3.0	3.1	3.2	3.2	3.3	3.2	2.9	2.5	2.2	1.8	1.5			
60	3.5	3.5	3.6	3.7	3.8	3.9	4.0	3.9	3.5	3.2	2.8	2.5				
59	4.1	4.2	4.2	4.3	4.4	4.5	4.6	4.5	4.2	3.8	3.5					
58	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.2	4.8	4.5						
57	5.3	5.4	5.5	5.6	5.7	5.8	5.9	5.8	5.5							

Figure 1 - Curves of isosubstition
wage growht rate 1%

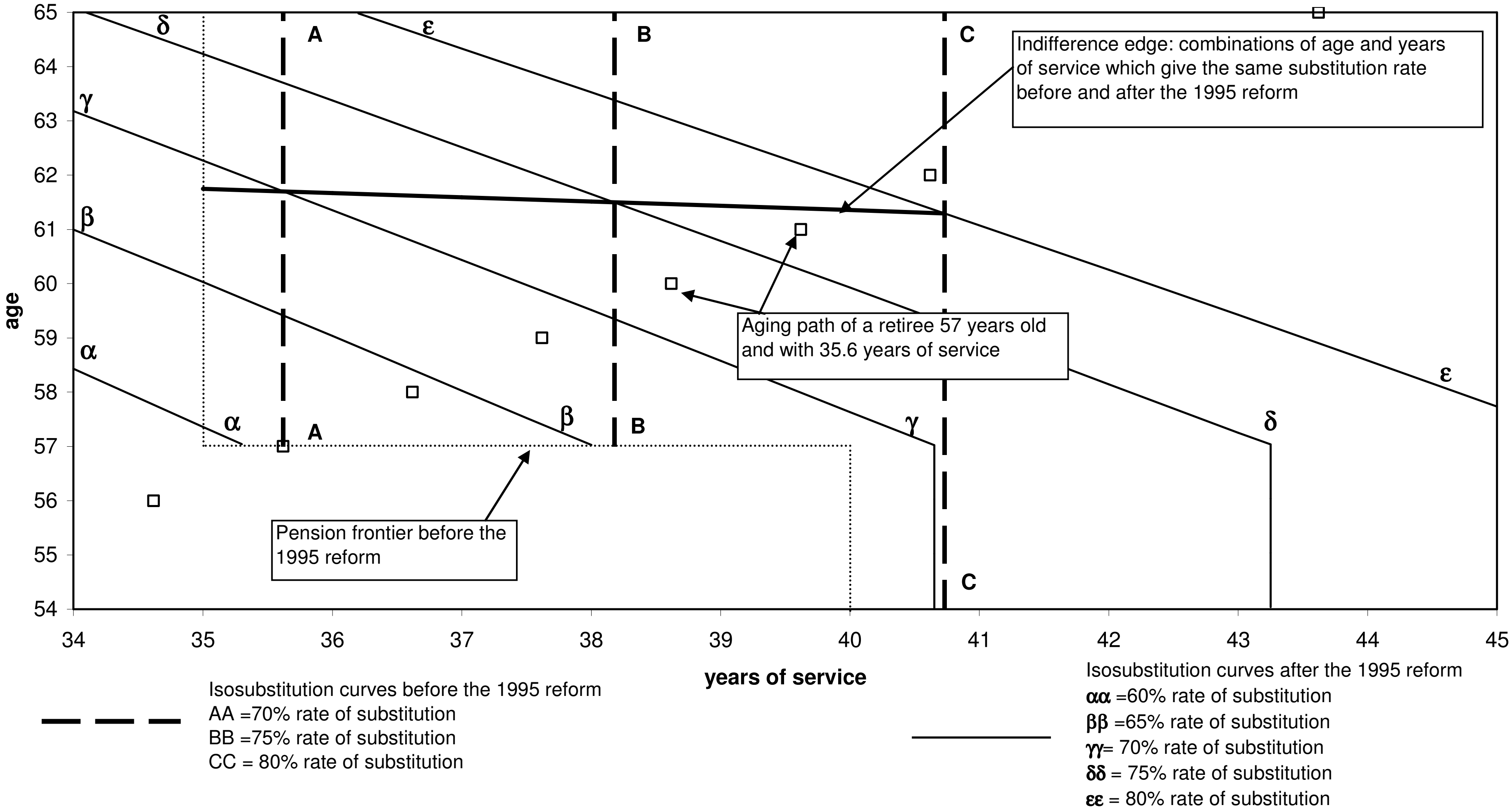


Figure 2 - Curves of isosubstition
wage growht rate 1.5%

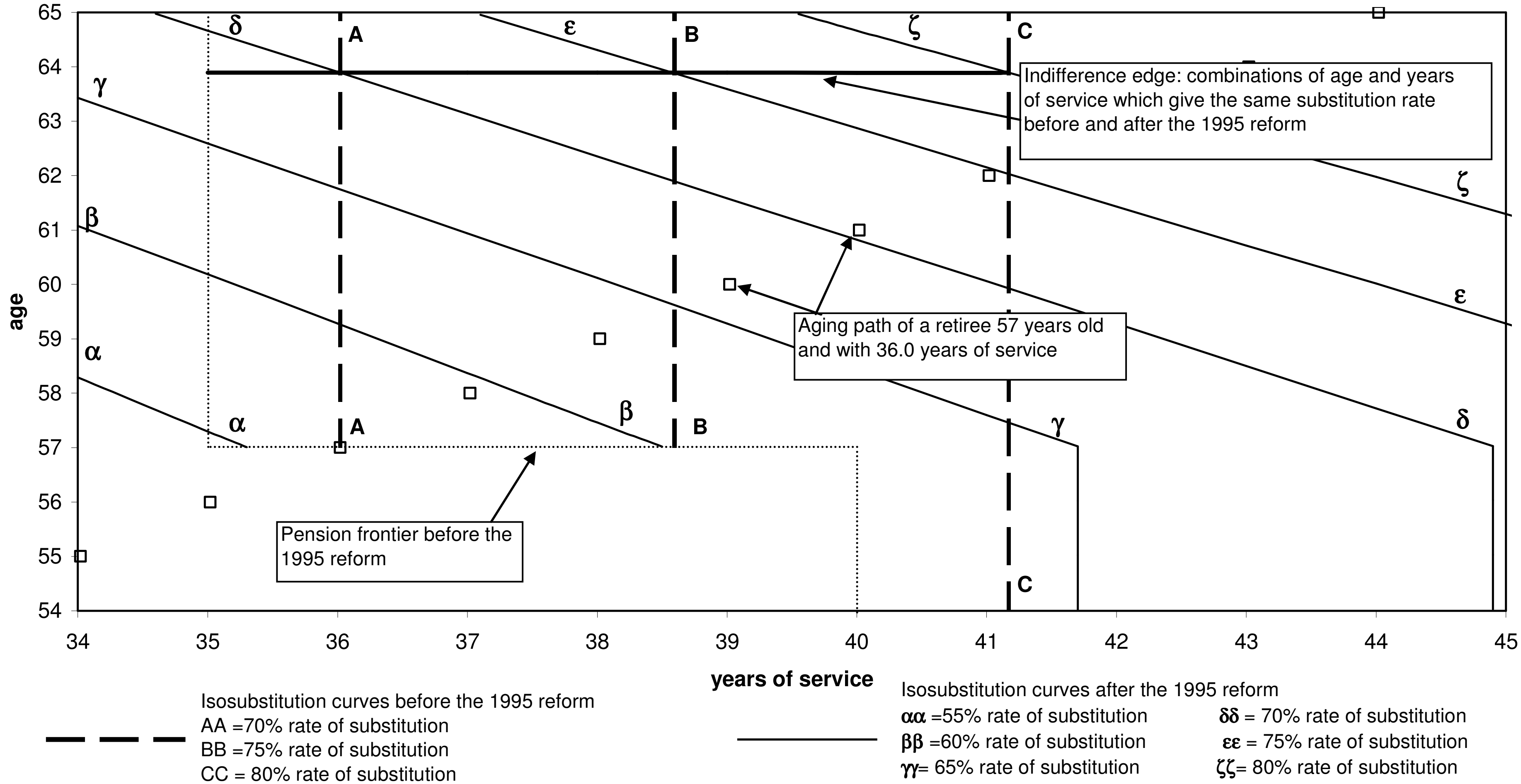
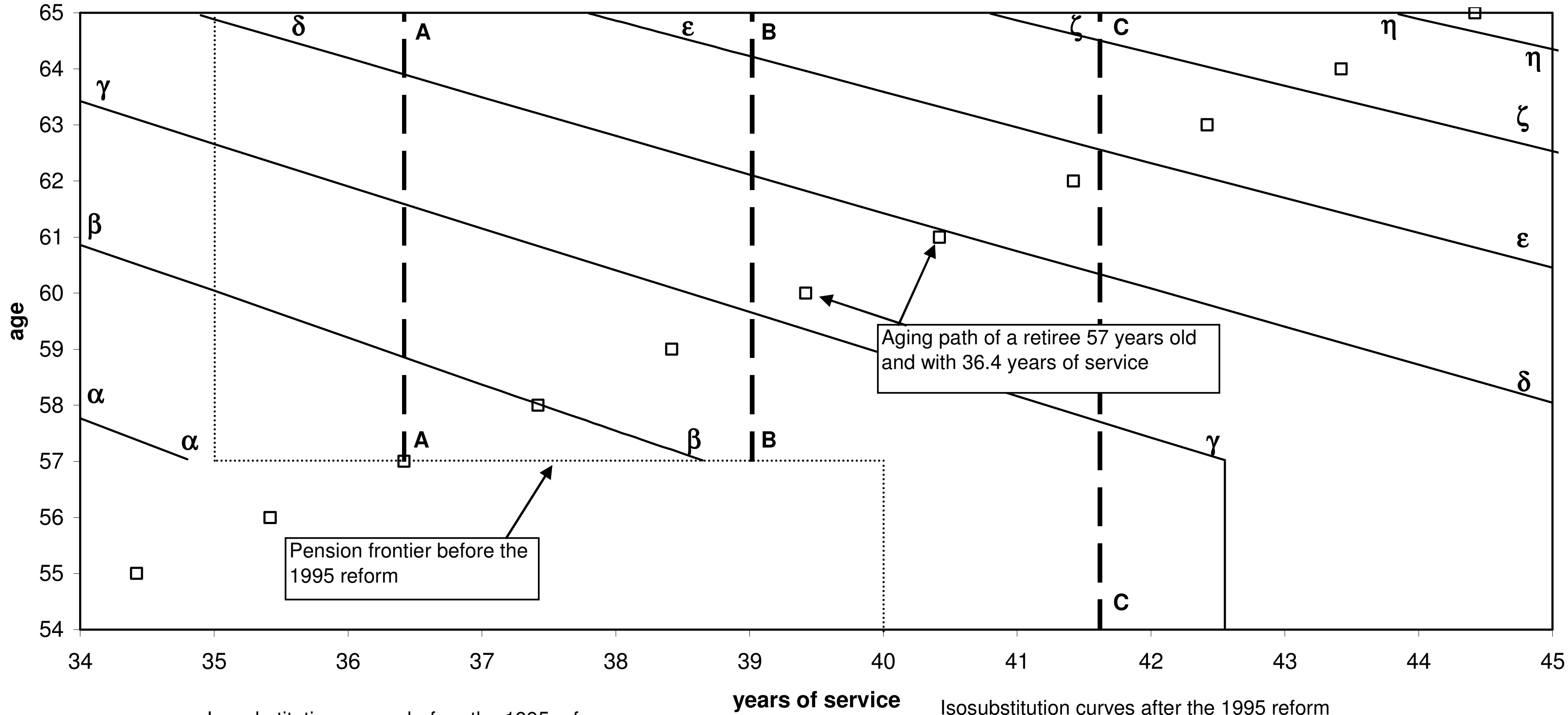


Figure 3 - Curves of isosubstition
wage growht rate 2%



Isosubstitution curves before the 1995 reform
AA =70% rate of substitution
BB =75% rate of substitution
CC = 80% rate of substitution

Isosubstitution curves after the 1995 reform
αα =50% rate of substitution
ββ =55% rate of substitution
γγ= 60% rate of substitution
δδ = 65% rate of substitution
εε = 70% rate of substitution
ζζ = 75% rate of substitution
ηη = 80% rate of substitution