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ENVIRONMENT AND SUSTAINABLE GROWTH IN A TWO-SECTOR MODEL WITH VARIABLE LABOR GROWTH RATE

MASSIMILIANO FERRARA, LUCA GUERRINI

Environment and sustainable growth in a two-sector model with variable labor growth rate

Massimiliano Ferrara Luca Guerrini

Abstract

We investigate a closed economy in which the production depends essentially on physical capital, natural capital and labor. Contrary to the standard literature, we assume that the labor growth rate is non constant but variable over time. We show that the economy is sustainable in the long run if human activities have a net zero or positive effect on the environment. Moreover, for any given tax rate (or MPC, i.e. marginal propensity to consume), we derive the set of sustainable MPCs (or tax rates).

1. Introduction

The works by Solow (1956) and Swan (1956) are an important milestone in the theory of economic growth. Their contributions have sparked substantial interest in a class of growth models, known as neoclassical growth models, in which capital and labor can be continuously substituted for each other. Most popular intermediate macroeconomic textbooks almost uniformly start with some variants of the Solow-Swan model (see, for example, Hall and Taylor, 1997; Mankiw, 1997; Romer, 1996), but, as noticed by Dasgupta (1996), in those books there is no mention of environmental resources. The implicit assumption, clearly undesirable, is that natural resources are neither scarce now nor scarce in the future. An effort to address this omission by treating natural capital as an essential factor of production was done by Tran-Nam (2001). By modeling the natural capital stock as a renewable resource, in the sense that damages done to the environment production and consumption externalities are reversible, and can be corrected by collective maintenance actions, he showed that the economy is sustainable in the long run if human activities have a positive effect on the environment. A natural question to be asked in this model is what the impact of changes in the population growth rate would be. For this purpose, we turn to an examination of the consequences of relaxing the assumption of constant population growth rate in Tran-Nam's paper (2001). Following Guerrini (2006), we assume that the labor growth rate is variable over time and controllable subject to be between prescribed upper and lower limits. In this framework, we find that the economy is sustainable in the long run if human activities have a net zero or positive effect on the environment. Moreover, for any given tax rate (or marginal propensity to consume, say MPC), we derive the set of sustainable MPCs (or tax rates). Finally, note that our paper focuses on economic theory, so all practical problems associated with measuring natural capital are assumed away.

2. The model

We consider a closed economy in continuous time equipped with the aggregate production function

$$Y = F(K, E, L) = K^{\eta} E^{\theta} L^{1-\eta-\theta}, \ \ \eta, \theta \in (0, 1),$$

where Y is the flow of output, K is the physical capital stock, E is the natural capital stock, and L is the employed population. Time index is omitted to ease the burden of notations. In addition, we assume that $0 < \eta + \theta < 1$, so that this production function exhibits constant returns to scale, it has positive and diminishing returns, and it satisfies the Inada conditions. Output is a homogenous good that can be consumed, saved or spent to maintain or improve the environment. Since the economy is closed, households cannot buy foreign goods or assets, as well as sell home goods or assets abroad. Therefore, output equals income, and the amount invested I equals the amount saved S. The national accounting can be written as

$$Y = C + S + T \tag{2.1}$$

where C and T represent consumption and tax, respectively. Let us assume that the tax revenue is a constant fraction of output, i.e.

$$T = \tau Y, \tag{2.2}$$

with $\tau \in (0, 1)$ the tax rate, and the consumption is a constant fraction of disposable income, i.e.

$$C = a(Y - T), \tag{2.3}$$

with $a \in (0, 1)$ the marginal propensity to consume (MPC). It is immediate to check that $C = a(1 - \tau)Y$. Next, we assume that capital depreciates at the constant rate $\delta > 0$. This means that at each point in time a constant fraction of the capital stock wears out, and hence it can no longer be used in production. Consequently, the net increase in the stock of physical capital at a point in time equals gross investment less depreciation, i.e.

$$\dot{K} = I - \delta K, \tag{2.4}$$

where a dot over a variable denotes differentiation with respect to time. Since $I = S$, equations (2.1), (2.2) and (2.3) yield that the capital accumulation equation (2.4) takes the form

$$\dot{K} = (1 - a)(1 - \tau)Y - \delta K. \tag{2.5}$$

Regarding the environmental stock E, let us assume that its evolution over time is described by the following differential equation

$$\dot{E} = \alpha E + \phi T - \beta Y - \gamma C, \tag{2.6}$$

where α, ϕ, β and γ are some constants. Essentially, we are assuming that the instantaneous rate of change of the environmental stock is determined linearly by three forces.

In case there is no human economic activity, then E changes over time at the exponential rate α, with the parameter α positive, zero or negative according to whether the environment grows, remains unchanged or decays autonomously over time. In case there is human economic activity, then we have depletion of β units of E for every unit of the final good produced (the production of the final good causes external damages to the environment), and also that each unit of the final good consumed depletes γ units of the environmental stock. Furthermore, environmental programs, funded by the entire tax revenue, generate ϕ units of the environmental stock per unit of the tax spent. The government runs a balanced budget at any instant of time, the taxation revenue is costlessly collected and there are no government failures. It also seems reasonable to assume that $\phi > \beta$, because production externalities are typically unintentional whereas environmental actions are well planned and executed. Finally, note that equations (2.2) and (2.3) yield equation (2.6) to be rewritten as

$$\dot{E} = \alpha E + [(\phi + \gamma a)\tau - (\beta + \gamma a)]Y. \tag{2.7}$$

Since we want to frame the model in per capita terms, we define physical capital stock per capita, natural capital stock per capita, consumption per capita, and income per capita as k, e, c, $y = f(k, e)$, respectively. By differentiation we have that the instantaneous change in the physical capital stock per capita and in the natural capital stock at any moment are given by

$$\dot{k} = \frac{d(K/L)}{dt} = \frac{\dot{K}L - K\dot{L}}{L^2} \quad \Rightarrow \quad \dot{k} = \frac{\dot{K}}{L} - k\frac{\dot{L}}{L}, \tag{2.8}$$

$$\dot{e} = \frac{d(E/L)}{dt} = \frac{\dot{E}L - E\dot{L}}{L^2} \quad \Rightarrow \quad \dot{e} = \frac{\dot{E}}{L} - e\frac{\dot{L}}{L}. \tag{2.9}$$

Next, dividing by L both sides of equations (2.5) and (2.7) yield

$$\frac{\dot{K}}{L} = (1-a)(1-\tau)f(k,e) - \delta k, \tag{2.10}$$

$$\frac{\dot{E}}{L} = \alpha e + [(\phi + \gamma a)\tau - (\beta + \gamma a)]f(k,e), \tag{2.11}$$

Therefore, substituting (2.10) and (2.11) in (2.8) and (2.9), respectively, we obtain

$$\begin{cases} \dot{k} = (1-a)(1-\tau)k^{\eta}e^{\theta} - (\delta + \dot{L}/L)k, \\ \dot{e} = [(\phi + \gamma a)\tau - (\beta + \gamma a)]k^{\eta}e^{\theta} - (\dot{L}/L - \alpha)e. \end{cases} \tag{2.12}$$

Following Guerrini (2006), and contrary to what done by Tram-Nam (2001), let us assume the labor growth rate to be non constant over time but variable, i.e. $\dot{L}/L = n(t)$. More precisely, let $n(t)$ be controllable subject only to be between prescribed upper and lower limits, i.e. $\alpha \leq n(t) \leq M$, for all t, and such that there exists $\lim_{t\to\infty} n(t) = n_\infty < \infty$. Moreover, let us assume that $L(0) = 1$, and there exists

$$\lim_{t\to\infty} L(t) = L_\infty \leq \infty, \ L_\infty \neq 0.$$

Since $e^{\alpha t} < L(t) < e^{Mt}$, for all t, we can conclude that

$$L_\infty = \infty \ \text{ if } \alpha > 0, \quad 1 \leq L_\infty \leq \infty \ \text{ if } \alpha = 0, \quad 0 < L_\infty \leq \infty \ \text{ if } \alpha < 0.$$

Remark 1. In case of a constant rate of population, i.e. $\dot{L}/L = n$, then $L(t) = e^{nt}$, for all t, and so $L_\infty = \infty$ no matter who is α.

Remark 2. If $L_\infty < \infty$, then $n_\infty = 0$. This statement follows from the following result: "Let $\varphi : [x_0, +\infty) \to \mathbb{R}$ be a differentiable function such that there exist (finite or infinite) the limits $\lim_{x\to+\infty} \varphi(x) = l$, $\lim_{x\to+\infty} \varphi'(x) = n$. If l is finite, then $n = 0$."
Proof: By Lagrange's theorem, $\varphi(x+1) - \varphi(x) = \varphi'(\xi_x)$, for some $\xi_x \in (x, x+1)$. Since $\lim_{x\to+\infty} \xi_x = +\infty$, we have that $\lim_{x\to+\infty} \varphi'(\xi_x) = \lim_{x\to+\infty} \varphi'(x) = n$. Consequently, $\lim_{x\to+\infty} [\varphi(x+1) - \varphi(x)] = n$. The statement follows noting that l finite also implies $\lim_{x\to+\infty} [\varphi(x+1) - \varphi(x)] = 0$.

Setting $A = (1-a)(1-\tau) > 0$, $B = (\phi+\gamma a)\tau - (\beta+\gamma a)$, and replacing $\dot{L}/L = n(t)$ in (2.12), we have that the model's economy is described by the following system of differential equations

$$\begin{cases} \dot{k} &= Ak^\eta e^\theta - [\delta + n(t)]k, \\ \dot{e} &= Bk^\eta e^\theta - [n(t) - \alpha]e. \end{cases} \tag{2.13}$$

Given $k_0 = k(0) > 0$, and $e_0 = e(0) > 0$, this Cauchy problem has a unique solution, say $(k(t), e(t))$, defined on $[0, \infty)$ (see Birkhoff and Rota, 1978).

3. Long run sustainability conditions

In the conventional Solow model, there is no mention of environmental resources. The implicit assumption is that environmental stock is fixed and it does not depend on human activities. Thus, an economy is long run (infinite time horizon) sustainable so long as per capita consumption c equals or exceeds a given subsistence consumption level $\bar{c} > 0$. This condition can be written in terms of output per worker as $f(k(t), e(t)) = k^\eta e^\theta \geq \bar{c}/[a(1-\tau)]$, for all t. In particular, this requires k and e to be both at least positive. In the present model, the survival of the economy depends among other things on its ability to manage the environment, and life can only be sustained if human beings enjoy a sufficient amount of environment. Therefore, considering the environment as a private good, i.e. no joint consumption, then long run sustainability also requires that the environmental stock never falls below a minimum life-sustaining level $\bar{e} > 0$, i.e. $e(t) \geq \bar{e}$, for all t. We are now going to show that a necessary condition for the economy to be sustainable in the long run will depend crucially on the sign of B.

Proposition 1. *Let $B = 0$. For all t, the model's economy is described by*

$$e(t) = e_0 \xi^{\alpha t} L(t)^{-1}, \tag{3.1}$$

$$k(t) = \xi^{-\delta t} L(t)^{-1} \left\{ k_0^{1-\eta} + A e_0^{\theta} \left[\int_0^t \xi^{[\alpha\theta + (1-\eta)\delta] t} L(t)^{1-(\eta+\theta)} dt \right] \right\}^{\frac{1}{1-\eta}}, \quad (3.2)$$

where ξ denotes the exponential function.

Proof. Plugging $B = 0$ in (2.13) yields a dynamical system formed by the following two equations: $\dot{e} = -[n(t) - \alpha]e$, and $\dot{k} = Ak^{\eta}e^{\theta} - [\delta + n(t)]k$. A simple integration of the first linear differential equation gives $e(t) = e_0 \xi^{\alpha t} L(t)^{-1}$, which substituted in the second differential equation yields $\dot{k} = A \left[e_0 \xi^{\alpha t} L(t)^{-1} \right]^{\theta} k^{\eta} - [\delta + n(t)]k$. This is a Bernoulli equation, whose solution is known to be given by

$$k(t) = \xi^{-\int_0^t [\delta + n(t)] dt} \left\{ k_0^{1-\eta} + \left[\int_0^t A \left[e_0 \xi^{\alpha t} L(t)^{-1} \right]^{\theta} \xi^{(1-\eta) \int_0^t [\delta + n(t)] dt} dt \right] \right\}^{\frac{1}{1-\eta}},$$

$$= \xi^{-\delta t} L(t)^{-1} \left\{ k_0^{1-\eta} + A e_0^{\theta} \left[\int_0^t \xi^{[\alpha\theta + (1-\eta)\delta] t} L(t)^{1-(\eta+\theta)} dt \right] \right\}^{\frac{1}{1-\eta}}.$$

□

Proposition 2. *Let $B = 0$. Then the function $e(t)$ is monotone decreasing, and we have that $\lim_{t\to\infty} e(t) = 0$ if $\alpha > 0$, $\alpha < 0$, or $\alpha = 0$ and $L_\infty = \infty$, while $\lim_{t\to\infty} e(t) = e_0 L_\infty^{-1}$ if $\alpha = 0$ and $L_\infty < \infty$.*

Proof. Since $\dot{e} = -[n(t) - \alpha]e$, the first part of the statement follows immediately from the hypothesis that $n(t) > \alpha$, for all t. Next, set $\lim_{t\to\infty} e(t) = e_\infty$. If $\alpha = 0$, then (3.1) becomes $e(t) = e_0 L(t)^{-1}$, and so we have that $e_\infty = 0$ if $L_\infty = \infty$, and $e_\infty = e_0 L_\infty^{-1}$ otherwise. If $\alpha < 0$ or $\alpha > 0$, then from (3.1) and the inequality $L(t) > \xi^{\alpha t}$, for all t, we get $e_\infty = 0$. □

Proposition 3. *Let $B = 0$. Then the function $k(t)$ is convergent in the long run. We have that $\lim_{t\to\infty} k(t) = 0$ if $\alpha > 0$, $\alpha < 0$, or $\alpha = 0$ and $L_\infty = \infty$, while*

$$\lim_{t\to\infty} k(t) = \left[\frac{A e_0^{\theta}}{(1-\eta)\delta L_\infty^{\theta}} \right]^{\frac{1}{1-\eta}},$$

if $\alpha = 0$ and $L_\infty < \infty$.

Proof. Let us rewrite (3.2) as

$$k(t)^{1-\eta} = \frac{k_0^{1-\eta} + A e_0^{\theta} \left[\int_0^t \xi^{[\alpha\theta + (1-\eta)\delta] t} L(t)^{1-(\eta+\theta)} dt \right]}{\xi^{(1-\eta)\delta t} L(t)^{1-\eta}}. \quad (3.3)$$

Let $\alpha \geq 0$. As t grows to infinity, the right hand side of (3.3) leads to an indeterminate form since both its numerator and denominator will go to infinity. This fact is immediate for the denominator as $L_\infty \leq \infty$, $L_\infty \neq 0$, and $1 - \eta > 0$, while for the numerator it follows from the inequality

$$\int_0^t \xi^{[\alpha\theta+(1-\eta)\delta]t} L(t)^{1-(\eta+\theta)} dt \geq \int_0^t \xi^{[\alpha\theta+(1-\eta)\delta]t} dt = \frac{\xi^{[\alpha\theta+(1-\eta)\delta]t} - 1}{\alpha\theta + (1-\eta)\delta},$$

as $\alpha\theta + (1-\eta)\delta > 0$. Note that $L(t) \geq 1$ since $L(t) \geq \xi^{\alpha t}$, for all t. An application of Hopital's rule yields

$$\lim_{t\to\infty} k(t)^{1-\eta} = \frac{Ae_0^\theta}{(1-\eta)} \lim_{t\to\infty} \frac{1}{[\delta + n(t)]} \frac{\xi^{\alpha\theta t}}{L(t)^\theta} = \frac{Ae_0^\theta}{(1-\eta)(\delta + n_\infty)} \left[\lim_{t\to\infty} \frac{\xi^{\alpha t}}{L(t)}\right]^\theta.$$

In conclusion, it can be concluded from (3.3) and what done above that

$$\lim_{t\to\infty} k(t) = \left\{ \frac{Ae_0^\theta}{(1-\eta)(\delta + n_\infty)} \left[\lim_{t\to\infty} \frac{\xi^{\alpha t}}{L(t)}\right]^\theta \right\}^{\frac{1}{1-\eta}}. \tag{3.4}$$

The statement of our Proposition is now immediate if $\alpha = 0$ (with L_∞ finite or infinite), or if $\alpha > 0$ (it is $L_\infty = \infty$ since $L(t) > \xi^{\alpha t}$, for all t). Let us now consider the last case, i.e. $\alpha < 0$. As $t \to \infty$, the denominator of the right hand side of (3.3) is still divergent since $0 < L_\infty \leq \infty$, while the behavior of its numerator is undetermined. However, note that if it converges, or it does not exist, then we still have that $\lim_{t\to\infty} k(t)^{1-\eta} = 0$, whether if it is infinite, we may use again Hopital's rule and get the expression in (3.4). The statement will follow considering the two cases $L_\infty \neq 0$ and $L_\infty = \infty$, and recalling that $\xi^{\alpha t} \to 0$. □

Proposition 4. *Let $B < 0$. The function $e(t)$ is monotone decreasing. As t grows to infinity, $e(t)$ is convergent to zero if $\alpha > 0$, $\alpha < 0$, or $\alpha = 0$ and $L_\infty = \infty$, and it does not diverge to infinity if $\alpha = 0$ and $L_\infty < \infty$.*

Proof. The first part of the statement is immediate from $B < 0$ and $n(t) > \alpha$, for all t, since we have from (2.13) that $\dot{e} = Bk^\eta e^\theta - [n(t) - \alpha]e$. Note that this differential equation cannot be solved in terms of elementary functions, as done before. A common technique in this case is to compare the unknown solutions of the given equation with the known solutions of another, i.e. to use the so-called Comparison theorems. Using the following one (see Birkhoff and Rota, 1978) "if $u_i(t)$, $i = 1, 2$, is the solution of the Cauchy problem $\dot{u} = \varphi_i(t, u)$, $u(0) = u_0$, and $\varphi_1(t, u) \leq \varphi_2(t, u)$, for all (t, u), then $u_1(t) \leq u_2(t)$, for all t", together with the previous Propositions gives the statement of the second part of our Proposition. For example, from the inequality $\dot{e} = Bk^\eta e^\theta - [n(t) - \alpha]e < -[n(t) - \alpha]e = \dot{e}$, we obtain that $e(t) < e_0 \xi^{\alpha t} L(t)^{-1}$, for all t. Thus, we have the statement by Proposition 2. □

We can now state the following result.

Theorem 1. *If human activities have a net zero or negative effect on the environment in every time period, i.e. if $B \leq 0$, then the economy is unsustainable in the long run unless $\alpha = 0$ and $L_\infty < \infty$. If $B = 0$, $\alpha = 0$ and $L_\infty < \infty$, the economy is sustainable.*

Remark 3. In case of a constant population growth rate (recall $L_\infty = \infty$, for any α) Binh Tran-Nam (2001) showed that the economy is always unsustainable in the long run if $B \leq 0$. Moreover, a necessary condition for the economy to be sustainable in the long run is that $B > 0$, i.e. if human activities produce a net beneficial effect on the environment for every time period.

4. Tax rate and sustainability

Theorem 1 implies that a sufficient condition for the economy to be sustainable in the long run is that $B = 0$, $\alpha = 0$ and $L_\infty < \infty$, while a necessary condition is that $B > 0$, or $B < 0$, $\alpha = 0$ and $L_\infty < \infty$. Mathematically, this can be formulated as follows.

Lemma 1.

• *For any given tax rate* τ, $B \gtreqless 0$ *if and only if* $a \lesseqgtr (\phi\tau - \beta)/[\gamma(1-\tau)]$.

• *For any given MPC* a, $B \gtreqless 0$ *if and only if* $\tau \gtreqless (a\gamma + \beta)/(a\gamma + \phi)$.

Proof. The statement is immediate recalling that $B = (\phi + \gamma a)\tau - (\beta + \gamma a)$. □

Remark 4. Note that the number $(\phi\tau - \beta)/[\gamma(1-\tau)]$ is ≤ 0 if and only if $\tau \leq \beta/\phi$, it belongs to the interval $(0, 1)$ if and only if $\beta/\phi < \tau < (\beta + \gamma)/(\phi + \gamma)$, and it is ≥ 1 if and only if $\tau \geq (\beta + \gamma)/(\phi + \gamma)$.

Let us assume that the tax rate is given, and let $B > 0$. Bearing in mind that both tax rate and MPC must lie within the open interval $(0, 1)$, and that $\phi > \beta$, it can be seen from Lemma 1 and Remark 3 that if the tax rate is too small, i.e. $\tau \leq \beta/\phi$, then no positive MPC is sustainable, if the tax rate is sufficiently large, i.e. $\tau \geq (\beta+\gamma)/(\phi+\gamma)$, then a sustainable MPC can take any value in the interval $(0, 1)$, and if $\beta/\phi < \tau < (\beta + \gamma)/(\phi + \gamma)$, then $(0, (\phi\tau - \beta)/[\gamma(1-\tau)])$ is the set of sustainable MPCs.

We can now state the following Proposition.

Proposition 5.

- *For any given tax rate* $\tau \in (0, 1)$:
  - *i) if* $\tau \leq \beta/\phi$*, then the set of sustainable MPCs is empty if* $B > 0$*, while it is the interval* $(0, 1)$ *if* $B < 0$, $\alpha = 0$ *and* $L_\infty < \infty$;
  - *ii) if* $\beta/\phi < \tau < (\beta + \gamma)/(\phi + \gamma)$*, then the set of sustainable MPCs is the interval* $(0, (\phi\tau - \beta)/[\gamma(1-\tau)])$ *if* $B > 0$*, while it is* $((\phi\tau - \beta)/[\gamma(1-\tau)], 1)$ *if* $B < 0$, $\alpha = 0$ *and* $L_\infty < \infty$;
  - *iii) if* $\tau \geq (\beta+\gamma)/(\phi+\gamma)$*, then the set of sustainable MPCs is* $(0, 1)$ *if* $B > 0$*, while it is empty if* $B < 0$, $\alpha = 0$ *and* $L_\infty < \infty$.

- *For any given MPC* $a \in (0, 1)$*, the set of sustainable tax rates is the interval* $((a\gamma + \beta)/(a\gamma + \phi), 1)$ *if* $B > 0$*, and the interval* $(0, (a\gamma + \beta)/(a\gamma + \phi))$ *if* $B < 0$, $\alpha = 0$ *and* $L_\infty < \infty$.

- *If $B = 0$, $\alpha = 0$ and $L_\infty < \infty$, then, for any given tax rate, the set of sustainable MPCs reduces to $a = (\phi\tau - \beta)/[\gamma(1-\tau)]$. Similarly, for any given MPC, the set of sustainable tax rates consists of only one element, $\tau = (a\gamma + \beta)/(a\gamma + \phi)$.*

Remark 5. Some interesting things can be inferred from the previous Proposition. For example, if $B > 0$, we deduce that an increase (resp. a decrease) in the tax rate in the relevant range widens (reps. narrows) the choice of sustainable MPCs, or alternatively an increase (resp. a decrease) in the MPC narrows (resp. widens) the choice of sustainable tax rates. We also note that these results are intuitively clear. In fact, more (resp. less) resources are spent to repair the environment, then, keeping the economy sustainable, a larger (resp. smaller) fraction of the remaining output is available for consumption, or alternatively if a larger (resp. smaller) fraction of output is consumed, then the range sustainable tax rates becomes narrower (resp. wider). Similarly for the cases $B \leq 0$.

5. Conclusion

In this paper, we build a two-sector growth model with physical and natural capital accumulation in order to analyze the relationship between economic development and long run sustainability. Contrary to the existing literature, we assume that population growth rate is non constant but variable over time. In this framework, we find that the economy is sustainable in the long run if human activities have a net zero or positive effect on the environment. Moreover, for any given tax rate (or MPC, i.e. marginal propensity to consume), we derive the set of sustainable MPCs (or tax rates).

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Massimiliano Ferrara
Università Mediterranea di Reggio Calabria
Facoltà di Giurisprudenza, CdL in Scienze Economiche
Via dei Bianchi (Palazzo Zani) 2, 89100 - Reggio Calabria, Italy
E-mail address: massimiliano.ferrara@unirc.it

Luca Guerrini
Università di Bologna
Dipartimento di Matematica per le Scienze Economiche e Sociali
Viale Quirico Filopanti 5, 40126 - Bologna, Italy
E-mail address: guerrini@rimini.unibo.it