A MEASURE OF ITALIAN LOCAL GOVERNMENT SPENDING EFFICIENCY. THE CASE OF TRANSPORT RELATED EXPENDITURE

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Abstract

This article analyses the relative efficiency of Italian provincial capital using panel data (2000 - 2007), looking also into its main determinants and to its impact on the local housing market quoatations. To that end a unique longitudinal dataset will be constructed including financial and qualitative information about the other 7.500 Italian municipalities provided by the Ministry of the Interior. Information about the main sources of (in)efficiency are important, since they can provide useful insights for local policy-makers and for defining accountability measures by both higher levels of governments and residents (voters). Although the analysis is carried out for all expenditure chapters, our discussion will focus on one particular area of public goods provision, namely transport related expenses. The analysis is performed in two stages: first, we use Data Envelopment Analysis (DEA) to calculate an index of local municipalities efficiency, in particular the methodology proposed by Simar and Wilson [1998, 2000] will be used to estimate a "bias corrected" measure of efficiency; second, a parametric approach will be used to evaluate the determinants of efficiency and its impact of the housing market quotations. As an output measure we compute a composite local government indicator of municipal performance. This allows assessing the extent of possible municipal improvement relative to the "best practice" frontier. We focus, in particular, on transport related expenses and identify performance measures in line with the objective of the paper. We expect to be able to identify where it be possible to improve performance without necessarily increasing municipal spending. In a second stage efficiency scores are investigated in order to identify elements that play a major role in determining single input slacks in relation to transport related expenditure.

1 Introduction

2 Efficiency of local government and housing market

3 The Construction of the Efficiency Index

We will measure efficiency by data envelopment analysis (DEA hereafter). To that end, each provincial capital will be treated as a decision-making unit that provides local services under the behavioural assumption that each of them operates in order to minimise the level of inputs given the level of output (input approach), or alternatively, that operates in order to maximise the output given the inputs (output approach). According to these simplified assumptions, therefore, we assume that the aggregated output of the local authorities is the result of the following production function:

$$y_{it} = f(\mathbf{x}_{it}; \beta)h(\mathbf{z}_{it}; \gamma)exp(v_{it} + u_i) \quad i = 1, 2, ..., N \text{ and } t = 1, 2, ..., T. \tag{1}$$

where N is the number of local authorities, T the number of years, y_{it} is the aggregated output, $\mathbf{x}_{it}$ is a $(L \times 1)$ vector of inputs, $\mathbf{z}_{it}$ is a $(M \times 1)$ vector of environmental variables, β a vector of technology parameters, γ is the vector of coefficients on the environmental variables. For simplicity, and with little loss of generality, we assume separability between $f(.)$, which describes the technology, and $h(.)$ which represents the way in which the environment affect the output. Since we are estimating a "frontier" production function, the error term has two components: the idiosyncratic error $v_{it} \sim i.i.d.(0, \sigma_v^2)$, which accounts for the statistical noise in the production function, and the inefficiency error component u_i, which is assumed to satisfy the restriction $u_i \leqslant 0$ and can be associated to the managerial inefficiency specific to each local authority, that can not be observed directly but only inferred as a residual. In this case, since we are conducting a short term analysis, it is possible to assume that u_i is time invariant.

Given the previous assumptions about the behaviour of the local authorities, the first step is the estimation of the "gross" level of efficiency $e_{it} = \frac{y_{it}}{f(x_{it},\beta)}$ that corresponds to the distance between the actual level of output attained by the local authority i in the year t and the maximum output attainable given the inputs employed in the production. DEA is a non-parametric estimator of e_{it}, no assumptions about the shape of $f(.)$ are necessary, and the convexity of the production set is the only restriction that needs to be imposed. Moreover DEA is a powerful estimator in case of multidimensional production frontier since allows us to avoid contrived forms of output aggregation. On the other hand, a large number of observations is an important prerequisite for a meaningful analysis in case of a multidimensional production frontier. This problem will be discussed again later on.

In the case of the input approach, let e_{it}^{DEA} be the solution of the following linear program:[1]

$$\underset{\phi,\lambda}{min}\ \phi \quad s.t.\ \ \phi\mathbf{x_{it}} \geqslant \mathbf{X_t}\lambda; \quad \mathbf{Y_t}\lambda \geqslant \mathbf{y_{it}}; \quad \lambda \geqslant 0; \quad \iota'\lambda = 1 \tag{2}$$

Then e_{it}^{DEA} is the efficiency score for the council i in period t. It satisfies: $e_{it}^{DEA} \in (0, 1]$, with a value of 1 indicating a point on the frontier and hence a technically efficient council, according to Debreu [1951], Farrell [1957] definition. The linear program in (2) is usually solved by using a pooled approach where only one production frontier is estimated and each municipality is compared also with itself in another year. In this way it is possible to use all the $N \times T$ observations.

Consistency is the most important property of an estimator, that is an estimator of an unknown parameter is consistent when it converges to the true value of that parameter as the sample size increases. There is no reason to use an inconsistent estimator since in that case increasing the amount of data would not allow of getting close to the true value that one wants to estimate. In nonparametric statistics, it is quite difficult to prove convergence of an estimator as well as to obtain its rate of

[1] In (2) $\mathbf{x_{it}}$ is the matrix of input of council i at time t, $\mathbf{X_t}$ is the matrix of inputs of all councils, $\mathbf{Y_t}$ is the matrix of outputs of all councils, λ is a vector of optimal weights attached to the peers of local government i; ι is a vector of ones, the last constraint is important for imposing variable returns to scale.

convergence. Recently, however, it has been found that e_{it}^{DEA}, as a non-parametric estimator of Debreu-Farrell measure of technical efficiency, is biased upwards since it does not converge toward e_{it}. In particular, Kneip et al. [1998] showed that:

$$e_{it}^{DEA} = e_{it} + O_p(n^{-\frac{2}{l+q+1}}) \tag{3}$$

where n is the number of observed production plans, l is the number of inputs, and q is the number of outputs in DEA. Which means, with some abuse of language, that as we increase the sample size DEA converges toward the true value of efficiency plus something that corresponds to the bias, and that the rate of convergence is $n^{\frac{2}{l+q+1}}$. Hence it appears that the higher the number of the inputs and/or the outputs, the slower the convergence rate, this means that when $l + q$ is grater than three, like in this case, our estimates can be very imprecise unless a very large quantity of data is also available since the rate of convergence is slower than the standard $\sqrt{n}$.

In this study, although more than 500 observations are available, the bootstrap procedure developed by Simar and Wilson [1998, 2000] will be used to estimate a "bias corrected" measure of efficiency ($\widetilde{e}_{it}^{DEA}$) along with its interval of confidence at the 95% level of significance in order to study the statistical properties of the efficiency estimates. In the second stage the impact of CPA and other exogenous environmental variables on local government's efficiency is evaluated through the estimation of the following empirical model derived directly from the base model in (1):

$$\frac{y_{it}}{f(\mathbf{x}_{it}, \beta)} = h(\mathbf{z}_{it}; \gamma)exp(v_{it} + u_i) \tag{4}$$

After replacing $\frac{y_{it}}{f(\mathbf{x}_{it},\beta)}$ with the bias corrected DEA measure of efficiency $\widetilde{e}_{it}^{DEA}$, and assuming for simplicity a Cobb-Douglas functional form for $h(.)$ the final empirical model to estimate the impact of CPA and other environmental variables becomes:

$$\widetilde{e}_{it}^{DEA} = \prod_{m=1}^{M} z_{itm}^{\gamma_m} \times \exp{(v_{it} + u_{it})} \tag{5}$$

where M is the number of environmental variables.

4 The Data

The choice of the variables apt to measure the output of the local government's activity is, in general, a very difficult exercise. It is very important to stress the we had to do some simplifications in order to deal with the usual tread-off between accuracy and the curse of dimensionality that undermine the validity of DEA as a non-parametric estimator of efficiency.

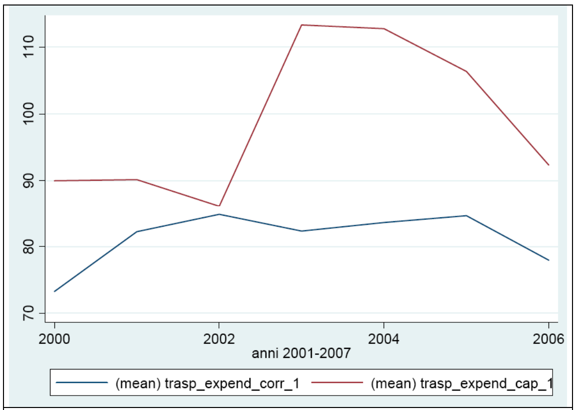
Considering the particular nature of the decision making units, the basic idea is to measure not only the "quantity" but also the "quality" of the output achieved by municipalities in the transport sector. To that end a subset of transport indicators (see Table 1) published by ISTAT [2010] have been chosen considering that they are broadly accepted by the local governments as measures of output quality and are fully comparable both across time and between local authorities.

As far as the input side is concerned the choice of the right variables is much less problematic: data from municipal budget accounts, published online by the Ministry of the Interior, have been used to define the inputs in terms of current and capital expenditure (real euro per capita) related to the transport service.

Table 1: Output and input variables, descriptive statistics without outliers.

Variables	**Mean**	**Standard deviation**		
		overall	*between*	*within*
OUTPUT				
Posti km per 1000 abitanti	1.99	1.01	1.02	0.14
n. di incidenti per mille abitanti	6.56	2.16	2.13	0.90
Km rete per 100 Km2 di sup. comunale	151	86	88	10
Vetture per 10000 abitanti	6.55	2.83	2.92	0.68
n. di fermate per km2 di sup. comunale	4.26	3.65	3.75	0.18
INPUT				
Spesa settore trasporti corrente pro capite	78.88	34.67	31.98	15.38
Spesa settore trasporti c/cap. pro capite	98.93	90.51	73.77	69.92
Popolazione residente	91159	80736	84625	2137

Table 1 reports some descriptive statistics of our input and output variables, it is important to stress that they do not include the two following groups of municipalities: Roma, Milano, Torino, Genova, Napoli, and Catania excluded because differently from most of the other provincial capitals provide the underground services; 2) Aosta, Bari, Bergamo, Cagliari, Cosenza, Firenze, La Spezia, Messina, Pescara Rimini, Siena, Trapani, Trento, Trieste and Venezia excluded because they are outliers in one or more output and input variables.



Red = cpital expenditure; Blue = Current expenditure.
Euro per capita

Figure 1: Current and Capital expenditure related to the transport sector, average across municipalities

Figure 1 reports the path followed by the current and the capital expenditure averaged across municipalities between 2000 and 2006, the maximum time span of our dataset. It is important to note that capital expenditure exhibits a huge increase after 2002 followed by a sudden decrease in the subsequent years. This particular path cast some doubts about the inclusions of capital expenditure among the inputs, therefore efficiency indices will be also computed excluding the capital expenditure from the production function.

5 Efficiency Indices in the Transport Sector

Using a sample made up of 534 production plans including 90 provincial capitals over on average time span of six years, DEA bias-corrected efficiency indices along with their 95% interval of confidence have been computed following the bootstrap methodology discussed above. Subsequently a sample of bias corrected measure of efficiency will be constructed using only those estimates considered statistically significant according to the following criteria (that are required to hold simultaneously): 1) the mean-square error of the unbiased DEA estimator should be smaller than the mean-square error of the biased estimator of efficiency; 2) each bias corrected measure of efficiency should be found in the same quartile considering the distribution of the lower and the upper bound of the 95% interval of confidence constructed around our bias corrected estimates of efficiency.

Table 2: Sample of statistically significant bias corrected indices of efficiency

Years	Total observations $\tilde{e}_{it}^{DEA}$	Input Approach		Output Approach	
		Statist. significant	%	Statist. significant	%
2000	55	53	96	45	81
2001	78	71	91	58	74
2002	79	77	97	62	78
2003	75	71	94	57	76
2004	82	75	91	67	81
2005	79	74	93	59	74
2006	86	79	91	70	81
Total	534	500	93	418	78

As reported in Table 3 we started with 534 bias corrected indices of efficiency, then 93% of total indices score resulted statistically significant in case of input approach, and 78% of total indices score resulted statistically significant in case of output approach.

Figure 2 report the density of the bias corrected indices of efficiency obtained using the input and the output approach. Although the two distributions looks

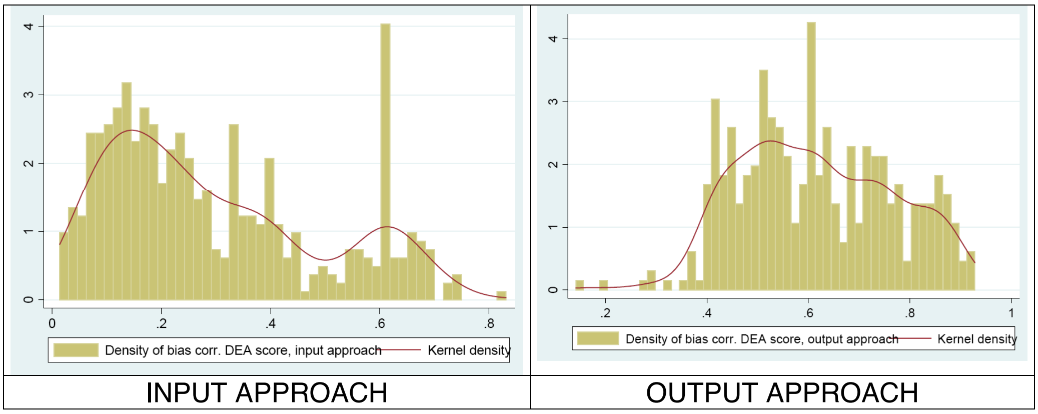


Figure 2: Density of bias corrected DEA indices of efficiency (input and output approach, only statistically significant observations.

very different the Spearman correlation between the two rankings of municipalities is 0.84.

Figure 3 and 4 report the DEA score (both in case of input and output approach) considering only the statistically significant DEA indices. In Figure 3 it is possible to register a decreasing trend in efficiency both in case of input and output approach. In Figure 4, where regions are ordered form north to south, only in case of the output approach it is possible to note an outstanding difference between the north and the south of the country, that respectively exhibit the highest and lowest levels of efficiency.

A final issue is the degree to which other aspects related to the quality of the urban transport service, which have not been included in the production function because can not be interpreted as outputs, affects the efficiency scores. This is important because the ability to provided the transport service efficiently may be subject to external constraints like the numerousness of the resident population, the passengers demand, the density of vehicles, the presence of pedestrian precinct etc. Therefore this group of variables can be used to explain, at least partially, why some municipality are more efficient than others.

This issue can be addressed estimating the followig log-linearised version of the model reported in (5):

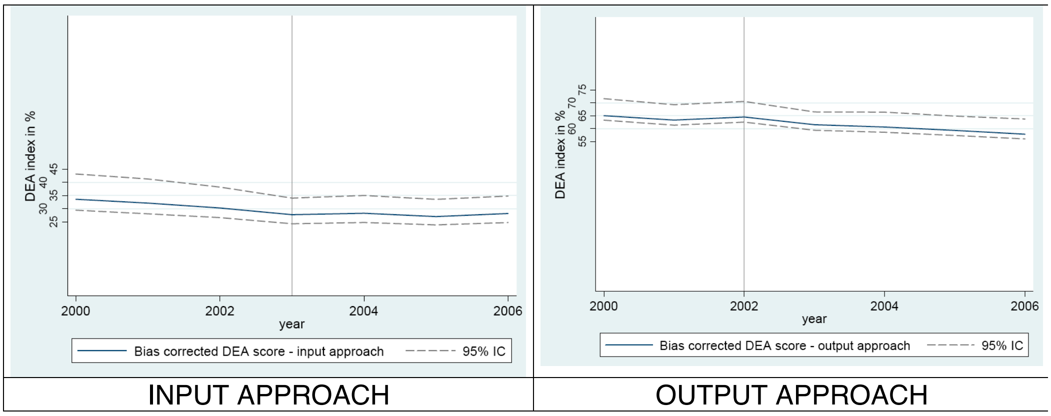


Figure 3: Average DEA scores (input and output approach) across years, only statistically significant indices, year 2001-2006. Note: Efficiency scores on the Y-axis are in percentage.

$$\log(\widetilde{e}_{it}^{DEA} \times 100) = \sum_{m=1}^{M} \gamma_m \log z_{itm} + \underbrace{\eta_t}_{year\ dummies} + u_i + v_{it} \tag{6}$$

that corresponds to a linear FE-panel data model where η_t corresponds to a set of year dummies, then dummies specific to each municipality can be used to control for the impact of the unobserved managerial efficiency "u_i". The simplest consistent estimator of the model in (6) is the "*within the group*" that eliminates u_i form the model thereby avoiding any cumbersome assumption about it.

Point estimates for the parameters of the model in (6) are reported in Table 3: the first two columns display the results in relation to efficiency indices obtained including the capital expenditure in the production function, whereas the last two columns report the estimates related to efficiency indices computed using a production function where the only input is the current expenditure. Efficiency in the delivery of the local transport service seems to be negatively affected by the following variables: the population (a symptom of negative return to scale), the introduction of limited traffic zones, the number of paying car parks, and councils run by centre-right parties. Instead the number of pedestrian precinct and the council run by local parties seems to be positively correlated with efficiency.

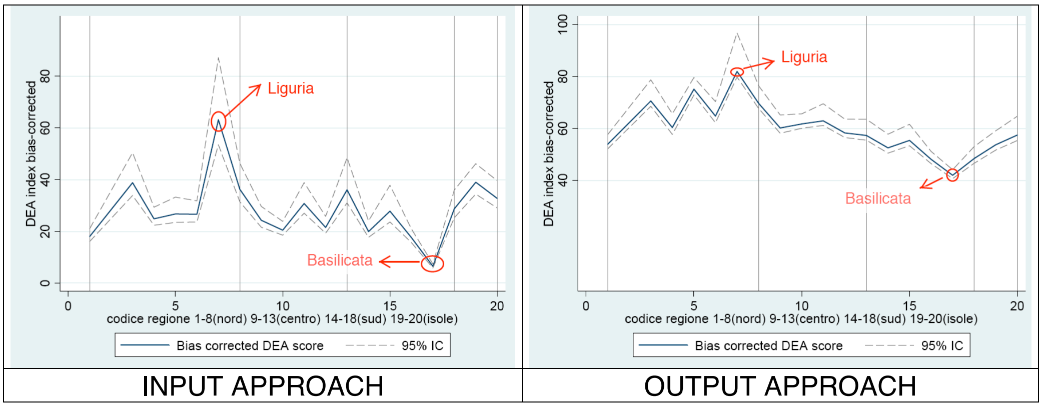


Figure 4: Average DEA scores (input and output approach) across geographical regions, only statistically significant indices. Note: Efficiency scores on the Y-axis are in percentage; first block of codes = northern regions, second block of codes = central regions, third block of codes = southern regions, fourth block of codes = islands

6 The Impact of Efficiency on the Housing Market Quotation

The final issue addressed in the paper is the evaluation of the impact that the level of efficiency in the provision of local transport services might exert on the quotations of the housing market. The first step in this analysis will be the estimation of the following FE panel data model:

$$y_{it} = \beta x_{it} + \eta_t + u_i + v_{it} \tag{7}$$

where the dependent variable will correspond to the housing market quotations, expressed in euros per square meter, in three zones: city centre, semi-central zone and suburban zone. Then in each zone four types of properties are considered: residential homes, shops, parks, offices, and warehouses. Finally x_{it} will correspond to DEA efficiency indices related to the municipal transport service, η_t is a set of year dummies, u_i is a set of municipal dummies, and finally v_{it} is the $i.i.d.$ random shock. Similarly to the model in (6) also the model in (7) can be estimated using the "within the group" estimator in order to avoid particular assumptions bout u_i

Table 3: Point estimate of the impact of the environmental variables on efficiency in the provision of local transport services.

Variabili indipendenti	**Fuzione di produzione completa**		**Fuzione di produzione solo spesa corr.**	
	Input Approach	*Output Approach*	*Input Approach*	*Output Approach*
pop. residente	-2.782 *[5.069]*	-1.070 *[0.871]*	-11.565 *[3.532]****	-1.366 *[0.630]***
passeggeri annui trasportati dai mezzi di trasporto pubblico per abitante	-0.376 *[0.457]*	0.133 *[0.069]**	0.158 *[0.438]*	0.132 *[0.053]***
Disponibilità di aree pedonali m2 per 100 abitanti	0.348 *[0.098]****	0.054 *[0.017]****	0.149 *[0.112]*	0.029 *[0.015]**
Densità zone traffico limitato (ZTL) km2 per 100 km2 di superfici e comunale	-0.459 *[0.196]***	-0.036 *[0.026]*	-0.135 *[0.177]*	-0.007 *[0.028]*
Stalli parcheggi scambio con trasporto pubblico per 1000 autovetture circolanti	-0.139 *[0.119]*	0.036 *[0.018]**	-0.191 *[0.129]*	0.018 *[0.018]*
Stalli sosta a pagamento su strada per 1000 autovetture circolant	-0.169 *[0.077]***	-0.115 *[0.029]****	-0.045 *[0.076]*	-0.019 *[0.021]*
autovetture per 1.000 abitanti	-0.582 *[3.916]*	-0.476 *[0.768]*	-6.589 *[3.605]**	-0.345 *[0.495]*
Giunta di centro – destra (dummy) *Variabile omessa giunta di c-sinisra*	-0.050 *[0.121]*	-0.039 *[0.017]***	-0.265 *[0.111]***	-0.028 *[0.015]**
Giunta retta da liste civiche (dummy) *Variabile omessa giunta di c-sinisra*	1.570 *[0.305]****	0.061 *[0.041]*	1.103 *[0.722]*	-0.015 *[0.036]*
Approvazione del Piano Urbano del Traffico (PUT), dummy	-0.127 *[0.350]*	0.056 *[0.068]*	0.015 *[0.471]*	0.038 *[0.044]*
Veicoli per km2 di superficie comunale	-0.915 *[5.566]*	0.734 *[0.932]*	4.995 *[3.436]*	0.538 *[0.722]*
Motocicli per 1.000 abitanti	-0.291 *[1.178]*	0.020 *[0.260]*	-0.531 *[1.120]*	-0.048 *[0.201]*
Densità di piste ciclabili km per km2 di superficie comunale	-0.118 *[0.090]*	-0.003 *[0.014]*	-0.169 *[0.159]*	-0.005 *[0.011]*
Observations	224	190	229	218
R-squared	0.23	0.37	0.37	0.36

*Robust standard errors in brackets, * significant at 10%; ** significant at 5%; *** significant at 1%*
Coefficient can be interpreted as elsticity values

that now captures the unobserved heterogeneity in the housing market quotations.

Tables 4, 5, and 6 report the point estimates for the parameter β respectively for the four types of properties in the central, semi-sentral and suburban zone.

All tables reports empirical evidence in favour of the hypothesis that housing market quotations are positively affected by a higher efficiency in the provision of the local transport services. In particular this result is quite robust in relation to residential homes and offices when efficiency is measured using the input approach.

Table 4: Point estimates of the impact of efficiency on the housing market quotations: city centre

Variabili indipendenti	**Fuzione di produzione completa**				
	Residenziale	*Commerciale*	*Parcheggi*	*Terziario*	*Produttivo*
DEA eff. index bias corr. - input approach	2.063 *[1.142]**	-0.545 *[1.785]*	0.834 *[0.925]*	2.596 *[1.185]***	0.130 *[0.625]*
DEA eff. index bias corr. - output approach	2.801 *[5.080]*	-6.010 *[8.524]*	-1.017 *[3.940]*	4.358 *[4.140]*	-1.527 *[2.198]*
Observations - input approach	353	341	200	338	221
Number of codice istat – input approach	85	84	81	83	56
R-squared - input approach	0.31	0.03	0.1	0.12	0.02
Observations – output approach	293	281	165	279	177
Number of codice istat - output approach	82	81	75	80	53
R-squared - output approach	0.30	0.02	0.07	0.07	0.04
	Fuzione di produzione con solo le spese correnti				
	Residenziale	*Commerciale*	*Parcheggi*	*Terziario*	*Produttivo*
DEA eff. index bias corr. - input approach	1.290 *[2.840]*	-5.929 *[5.357]*	1.495 *[3.877]*	2.847 *[3.012]*	-0.265 *[2.030]*
DEA eff. index bias corr. - output approach	1.906 *[4.331]*	-1.615 *[9.048]*	0.435 *[8.117]*	4.715 *[5.045]*	-1.704 *[3.319]*
Observations - input approach	365	350	208	349	228
Number of codice istat – input approach	85	84	81	83	56
R-squared - input approach	0.33	0.04	0.11	0.11	0.05
Observations – output approach	334	320	182	320	210
Number of codice istat - output approach	84	83	78	82	54
R-squared - output approach	0.32	0.03	0.06	0.11	0.02

*Robust standard errors in brackets, * significant at 10%; ** significant at 5%; *** significant at 1%*

Table 5: Point estimates of the impact of efficiency on the housing market quotations: semi-central zone

Variabili indipendenti	**Fuzione di produzione completa**				
	Residenziale	*Commerciale*	*Parcheggi*	*Terziario*	*Produttivo*
DEA eff. index bias corr. - input approach	2.009	-0.471	1.483	1.206	0.004
	*[1.040]**	*[0.905]*	*[0.607]***	*[0.996]*	*[0.726]*
DEA eff. index bias corr. - output approach	4.491	1.446	3.264	3.886	-1.025
	[3.099]	*[2.432]*	*[1.555]***	*[2.378]*	*[2.119]*
Observations - input approach	332	330	183	319	256
Number of codice istat - input approach	78	78	74	76	62
R-squared - input approach	0.39	0.05	0.21	0.32	0.04
Observations - output approach	275	273	152	265	211
Number of codice istat - output approach	75	75	70	73	59
R-squared - output approach	0.39	0.05	0.22	0.32	0.03
	Fuzione di produzione con solo le spese correnti				
	Residenziale	*Commerciale*	*Parcheggi*	*Terziario*	*Produttivo*
DEA eff. index bias corr. - input approach	1.448	0.352	2.535	3.652	1.291
	[1.885]	*[2.195]*	*[2.275]*	*[2.070]**	*[1.773]*
DEA eff. index bias corr. - output approach	0.308	0.784	6.213	5.110	-0.367
	[3.095]	*[4.024]*	*[4.078]*	*[3.207]*	*[2.918]*
Observations - input approach	343	340	191	330	261
Number of codice istat - input approach	79	79	75	77	63
R-squared - input approach	0.38	0.05	0.21	0.34	0.05
Observations - output approach	312	309	166	300	240
Number of codice istat - output approach	77	77	72	75	61
R-squared - output approach	0.36	0.05	0.18	0.31	0.05

*Robust standard errors in brackets, * significant at 10%; ** significant at 5%; *** significant at 1%*

7 Conclusions

Table 6: Point estimates of the impact of efficiency on the housing market quotations: suburbanzone

Variabili indipendenti	**Fuzione di produzione completa**				
	Residenziale	*Commerciale*	*Parcheggi*	*Terziario*	*Produttivo*
DEA eff. index bias corr. - input approach	2.232 *[1.093]*****	0.322 *[0.515]*	0.689 *[0.665]*	1.137 *[0.510]*****	0.287 *[0.429]*
DEA eff. index bias corr. - output approach	6.736 *[4.543]*	1.740 *[1.914]*	1.975 *[1.711]*	2.482 *[1.852]*	-0.272 *[0.960]*
Observations - input approach	337	331	186	306	308
Number of codice istat - input approach	82	83	75	77	77
R-squared - input approach	0.41	0.18	0.24	0.32	0.09
Observations - output approach	281	274	155	256	258
Number of codice istat - output approach	79	80	70	75	74
R-squared - output approach	0.42	0.17	0.23	0.28	0.08
	Fuzione di produzione con solo le spese correnti				
	Residenziale	*Commerciale*	*Parcheggi*	*Terziario*	*Produttivo*
DEA eff. index bias corr. - input approach	6.429 *[2.303]******	3.487 *[1.495]*****	4.127 *[2.662]*	4.095 *[1.970]*****	1.229 *[1.000]*
DEA eff. index bias corr. - output approach	4.108 *[4.328]*	3.586 *[2.852]*	3.860 *[2.539]*	5.977 *[2.995]*****	-0.029 *[1.386]*
Observations - input approach	348	339	193	316	316
Number of codice istat - input approach	81	81	75	77	75
R-squared - input approach	0.42	0.19	0.25	0.34	0.10
Observations - output approach	317	311	169	287	286
Number of codice istat - output approach	81	82	72	77	76
R-squared - output approach	0.39	0.17	0.20	0.3	0.11

*Robust standard errors in brackets, * significant at 10%; ** significant at 5%; *** significant at 1%*

References

Debreu, G. (1951). The coefficient of resource utilization. *Econometrica*, 19(3):273–292.

Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society*, 120(3):253–281.

ISTAT (2010). Indicatori sui trasporti urbani, anno 2008. *www.istat.it*.

Kneip, A., Park, B. U., and Simar, L. (1998). A note on the convergence of nonparametric DEA estimators for production efficiency scores. *Econometric Theory*, 14:783–793.

Simar, L. and Wilson, P. W. (1998). Sensitivity analysis of efficiency scores: How to bootstrap in nonparametric frontier models. *Management Science*, 44(1).

Simar, L. and Wilson, P. W. (2000). A general methodology for bootsrapping in non-parametric frontier. *Journal of Applied Econometrics*, 27(6).