XXII CONFERENZA

NUOVE FRONTIERE DELL'INTERVENTO PUBBLICO
IN UN MONDO DI INTERDIPENDENZA

Pavia, Università, 20-21 settembre 2010

COOPERATIVE BEHAVIOUR IN MUTUAL INSURANCE

FRANCESCA BARIGOZZI, GIUSEPPE PIGNATARO

Cooperative Behaviour in Mutual Insurance

Francesca Barigozzi[*]and Giuseppe Pignataro[†]
University of Bologna

This draft: September 2010

Abstract

Mutual insurance companies have significant market shares in the insurance industries and may constitute innovating responses to social needs and to the crisis of the welfare state. In this paper we examine consumers' behaviour in a risk pooling arrangement investigating their strategic interaction when a non-cooperative and a cooperative option in the choice of a preventive effort exist. The effort is a self-insurance measure and is consumers'private information. When cooperation prevails it guarantees an efficient reduction of the ex-post random premium in the mutual. We show that a limited size of the mutual arrangement is necessary for cooperation to be sustainable.

JEL classification: D82, I11, I18.

Keywords: Mutual arrangement, self insurance, cooperation, internalizing premium.

1 Introduction

Since the late Middle Ages insurance contracts were mainly defined as mutual compensation schemes. If for example one of a number of participating merchants lost a ship or cargo, the remainder would contribute to paying for the loss. From then on, this sharing rule emerges as the most interesting feature of the mutual form, called *a participating policy* (see Picard, 2009). The owners of the business are those that hold participating policies. The benefits for owners in the company are very similar to the benefits offered by stockholders although they get an insurance policy to go along with their benefits. The payout under mutual policies comprises two parts; an indemnity and a dividend (positively or negatively related). Each year the members of the mutual risk arrangement contribute whatever amount is needed to meet the losses insured by the pool. This is usually in the form of an initial (partial) contribution followed by later

[*]Department of Economics, University of Bologna and CHILD, P.zza Scaravilli 2, 40126 Bologna (Italy). E-mail: francesca.barigozzi@unibo.it

[†]Department of Economics, University of Bologna, P.zza Scaravilli 2, 40126 Bologna (Italy). E-mail: giuseppe.pignataro@unibo.it

'calls' if needed to maintain the common fund. It follows that in a mutual arrangement insurance premium is *random* since it always guarantees *ex-post* budget balance between contributions from the policyholders and indemnities paid to those who experienced the loss.

From Borch (1962) on, theorists have taken great care over the comparison in terms of optimal risk sharing performance between stock and mutual insurance companies (see Eeckhoudt and Kimball, 1992; Doherty and Dionne,1993). Stock insurance companies associated with risk neutrality can spread systematic risk over all investors in the capital market. In the case of mutuals instead, a risk pool can spread the systematic risk of the pool only across its membership, and thus the pool involves a suboptional sharing of systematic risk. That is, the policyholders share their risk exposures through the organization but retain exposure to the risk of the pool through their equity positions.

However, there exist some advantages offered by the risk pooling arrangements to offset their inferior risk-sharing capacity. Mayers and Smith (1986, 1988) have shown that mutual forms of organization are sometimes efficient at controlling expropriatory behavior of owners and managers. In the asymmetric information context, Smith and Stutzer (1995) modeled an ex-ante moral hazard situation in which loss probabilities are determined by a policyholder action selected after a state of nature has been revealed. They contemplate states of nature that commonly expose groups of policyholders, so their optimal policy depends on the group loss as well as the individual idiosyncratic loss. Taking into account the role of the background risk, Ligon and Thistle (2005) report that the significant market share of the mutual companies in the insurance industries can be explained through some advantages in solving asymmetric information problems. In particular,their organizational forms allow for a better target in models where adverse selection is present.

On this view, Lee and Ligon (2001) search whether these advantages can be re-proposed looking at self-protection (ex-ante moral hazard) issue[1]. They adopt a non-cooperative solution among members of the risk pooling arrangement showing that a full coverage is induced even in the presence of moral hazard. In fact uncertainty due to the mutual arrangement exists even with full coverage so that policyholders still exert a positive effort. The impact of moral hazard on preventive effort may contribute to define an ideal optimal size of the pool. Ligon and Thistle (2008) discover instead that under certain conditions stock insurance company may be preferred to mutual insurance one. This happens because the former may offer deterministic policies which yield at least as much expected utility to policy-holders as mutual insurance and earn positive expected returns which better performs in addressing the balance between risk bearing and moral hazard.

In our paper we focus on the case of self-insurance. The preventive effort is consumers' private information. Our investigation is particularly related to Lee and Ligon (2001) who, contrary to us, analyzed the case of self-protection. How-

[1] See Ehrlich and Becker (1972) for the definition of consumers' self insurance and self-protection.

ever we innovate with respect to their model since we distinguish between a non-cooperative and a cooperative options on the effort choice. We investigate how consumers' effort choice affects the random premium, an issue almost neglected in the literature of the mutual insurance. We develop the non-cooperative and the cooperative solutions in a mutual agreement and show that they generate quite different performances in the pool.

The internalization of the impact of the effort on the random premium was traditionally proposed in the non-cooperative option and it consists of an evaluation of the effect of individual's *own effort* on the premium. However, in a mutual agreement it may not be quite realistic to assume that individuals are always looking out for themselves and that they do so by weighing costs and benefits of their own behavior. Mutual arrangements may get an incredible opportunity to market their own business model and benefit from a desire of some customers to conduct their business in a more ethical and cooperative way. In particular, due to their role of owner-policyholder in the firm, individuals can cooperate in a manner that contributes to the others' welfare. Cooperation therefore implies a complete internalization of the impact of the effort choice of the whole pool of policyholders on the random premium. With respect to the non-cooperative solution it guarantees a secure reduction on ex-post premium and a positive impact on marginal utility of aggregate consumption in both state of the world.

We show that cooperation imposes a positive externality among members in the pool and that it becomes sustainable for a limited size of the arrangement. The intuition being that the positive impact of cooperation on the premium is decreasing in the pool size. Indeed small size mutual insurance firms are an empirically relevant phenomenon (see Smith and Stutzer,1995; Mayers and Smith, 2002; Nekby, 2004).

In the last part of the paper we want to prove that, when cooperation in the mutual company is sustained as an equilibrium, then the mutual arrangement allows consumers to get a higher welfare than with a stock insurance in a competitive market (this would be in contrast with Ligon and Thistle, 2008).

The structure of the article is as follows. Section 2 develops the model discussing the first-best and the stock-insurance case with self insurance. Section 3 proposes the basic analysis of the mutual agreement investigating the potential difference determined by the partial and complete internalization of the effort on the random premium in the non-cooperative and cooperative strategy respectively. It also shows that cooperation is sustainable for a limited size of the pool. Section 4 gets the flavour on the potential welfare-improving property of cooperation in the mutual arrangement with respect to the stock insurance case. However this extension is left for future (although imminent) research. Concluding remarks follows in the last section.

2 The model

There are n individuals in the society who are assumed to be identical. They have initial wealth w and face a probability of loss p of size $L(e)$ with independently and identically distributed risks. The loss $L(e)$ is a function of individuals' nonnegative effort level e such that $L'(e) < 0$. From Ehrlich and Becker (1972), a consumer's effort decreasing the amount of the loss is a self-insurance measure. The level of effort e is consumers' private information and it is exerted before the risk realizes. Each member's utility is represented by a strictly increasing von Neumann-Morgenstern utility function $U(w)$ which is differentiable at least two times, with $U'(w) > 0$, $U''(w) < 0$. It is assumed to be additively separable in the utility from money and in the cost of effort such that $C(e)$ denote the disutility of effort with $C'(e) > 0$ and $C''(e) > 0$.

2.1 First-best

Here we show the first-best of the previously depicted situation. In the case where the consumers' effort is observable to the insurance firms, the latter solve the following program:

$$\max_{e,q} EU = pU\left(w - L(e) - pqL(e) + qL(e)\right) + (1-p)\,U\left(w - pqL(e)\right) - C(e) \quad (1)$$

where q is the insurance coverage, with $0 \leq q \leq 1$. Consumers receive $qL(e)$ in the case of the loss and $pqL(e)$ is the premium when the market is competitive (zero profit condition). Let us define aggregate consumption in the two state of the world as $W_L = w - L(e) - P + qL(e)$ and $W_0 = w - pqL(e)$. Then, the optimal choice of coverage q follows from the FOC:

$$p(1-p)U'\left[W_L\right]L(e) = p\left(1-p\right)U'\left[W_0\right]L(e) \quad (2)$$

Therefore, $U'\left[W_L\right] = U'\left[W_0\right]$ implying $q^{FB} = 1$. Consequently, the optimal choice of effort e^{FB} is derived by:

$$e^{FB} \text{ such that: } E\left[U'(W)\right]\left(-pL'(e)\right) = C'(e) \quad (3)$$

which is the first-best level of effort, useful in the next sections for further comparisons. Note that aggregate consumption in first best is:

$$W = w - pL(e^{FB})$$

The right hand side of (3) shows the marginal benefit and the left hand side the marginal cost of the effort. Note that, in the first-best, consumers perfectly internalize the beneficial effect of the effort on the premium. In particular they take into account that a higher effort, by decreasing the premium, has a positive impact on marginal utility of both the possible states of health. Marginal benefit is increasing in p and in effectiveness of the self-insurance technology $-L'(e)$. Of course in the first-best consumers' surplus is maximized.

2.2 Stock insurance with self-insurance

In this subsections we consider a competitive market with standard stock-insurers offering contracts to consumers when moral hazard is an issue. The timing of actions is such that first, the insurance firms propose the contract, second the consumers accept the contract and choose the effort level, finally the risk realizes.

Suppose that stock insurance companies offer contract (P, q) where P is the premium and q, as before, is the cost sharing parameter. Consumers receive $qL(e)$ in the case of the loss. Again, since the market for stock insurers is competitive, the premium is $P = pqL(e)$.

Given the insurance contract (P, q), the representative consumer's expected utility is:

$$EU = pU\left[w - L(e) - P + qL(e)\right] + (1-p)\,U\,(w - P) - C(e) \tag{4}$$

Note that the optimal effort level is:

$$\arg\max_{e} EU(e; P, q)$$

this means that, under moral hazard, the effort level is calculated *given* the contract (P, q):

$$e^*(q) : -(1-q)L'(e)pU'(W_L) = C'(e) \tag{5}$$

Obviously, if $q = 1$ then the effort is zero so that full insurance is not the optimal coverage. By comparing (3) and (5) we observe that in the latter FOC consumers do not internalize the positive impact the effort has on the premium. In fact, in the l.h.s. of (5) only the positive effect the effort has on the loss, when the latter occurs (and aggregate consumption is W_L), is taken into account.

In the first step, stock insurance solves the following program:

$$\begin{aligned} &\max\nolimits_q EU = pU\left(W_L\right) + (1-p)\,U\left(W_0\right) - C(e) \\ &s.t. : P = pqL(e) \\ &-(1-q))L'(e)pU'(W_L) - C'(e) = 0 \qquad (IC) \end{aligned} \tag{6}$$

where (IC) is the consumer's incentive constraint. Solving program 6 we observe that the optimal level of coverage q is lower than 1 (partial coverage) which implies that the usual trade-off between optimal incentives and risk-sharing arises.

3 Mutual insurance

Suppose that the n identical individuals create a mutual arrangement where the indemnity paid by the pool in the case of the individual's loss is $qL(e)$. Again q is the percentage of the loss reimbursed to the policyholder. Let us call k the number of consumers that experience the loss among the n identical consumers in the pool: $k \in \{0, ..., n\}$. In mutual insurance the premium is not fixed but it depends on k, which means that the premium is random.

Definition 1 *The pooling arrangement is such that the total amount of indemnities to be paid to policyholders in the pool ($kqL(e)$) is equally shared among the n members of the mutual. Thus, the premium is:* $\frac{kqL(e)}{n}$.

The timing of actions for policyholders in the mutual insurance is the following:

- The percentage of the loss reimbursed to policyholders, q, is chosen cooperatively.
- policyholders choose the effort level cooperatively or non-cooperatively
- the risk (and thus the number of individuals experiencing the loss) realizes.

3.1 Non-cooperative and cooperative strategy in mutual insurance

As was mentioned before, policyholders' choice of effort can be cooperative or not. The non-cooperative view represents the standard approach in mutual arrangements (see Lee and Ligon 2001) and implies that consumers only internalize the effect of *their own effort* on the random premium of the mutual policy. In other words individuals neglect the "social benefit" of the effort on the aggregate loss (see below).

In the non-cooperative case the representative consumer i's expected utility given k is:

$$\begin{aligned} EU_i^{NC}(k) = pU\left(w - \frac{q}{n}\left(L(e_i) + (k-1)L(e_{-i})\right) - L(e_i) + qL(e_i)\right) \\ + (1-p)\, U\left(w - \frac{kq}{n}L(e_{-i})\right) - C(e_i) \end{aligned} \tag{7}$$

where e_i is the effort exerted by consumer i and e_{-i} is the effort exerted by the others $n-1$ consumers in the pool. If the policyholder is one of the k individuals suffering the loss, the premium she considers is $\frac{q}{n}\left(L(e_i) + (k-1)L(e_{-i})\right)$, that is, the premium also depends on her effort. Whereas, if she is one of the $n-k$ individuals not suffering the loss, the premium she considers is $\frac{kq}{n}L(e_{-i})$, that is the premium does not depend on her effort. Note that the total amount of premiums collected in the pool ($p\frac{q}{n}\left(L(e_i) + (k-1)L(e_{-i})\right) + (1-p)\frac{kq}{n}L(e_{-i})$) exactly covers the total amount of the indemnities paid to the k individuals experiencing the loss. This represents a standard property of mutual insurance: ex-post profits are always zero. Note that, in the non-cooperative case, each policyholder only internalizes part of the effect of the effort on the premium, i.e., $\frac{q}{n}L(e_i)$ and only in the case where the loss occurs. While, taking into account $\frac{kq}{n}L(e)$ in both states of nature would correspond to internalizing the whole effect of the effort on the premium.

We innovate with respect to the previous literature on mutual insurance by arguing that, given the double role of *shareholders-policyholders* exerted by pool members, individuals are able to understand the advantage of cooperation by choosing an optimal level of effort: they take into account the "social benefit" of the effort on the aggregate loss. Put differently, policyholders can correctly evaluate the beneficial effect that the effort has on the random premium in the mutual policy. In the cooperative case the representative consumer's expected utility given k is:

$$EU(k) = pU\left(w - L(e) - \frac{kqL(e)}{n} + qL(e)\right) + (1-p)\,U\left(w - \frac{kqL(e)}{n}\right) - C(e) \tag{8}$$

where e is the effort exerted by all the cooperating consumers in the pool.

By investigating the non-cooperative and the cooperative behavior by policyholders in the pool we will obtain conditions such that the cooperative strategy is sustained as an equilibrium.

In the following subsections, we will analyze the non-cooperative game first, then we will show the policyholders payoff under cooperation, and, finally, we will prove that cooperation can be an equilibrium.

3.2 Non-cooperative strategy in mutual insurance

With non cooperative policyholders the representative consumer i's expected utility given k is expressed in (7) above.

Let us call $b(k; n-1; p)$ the binomial probability of k losses with $n-1$ individuals with probability of loss p each. Moreover, aggregate consumptions in the two states of nature are $W_{Li}^{NC} = w - \frac{q}{n}\left(L(e_i) + (k-1)L(e_{-i})\right) - L(e_i) + qL(e_i)$ and $W_0^{NC} = w - \frac{kq}{n}L(e_{-i})$, the representative consumer's expected utility then is:

$$EU_i = \sum_{k=0}^{n-1} b(k; n-1; p)\{pU\left(W_{Li}^{NC}\right) + (1-p)\,U\left(W_0^{NC}\right)\} - C(e_i) \tag{9}$$

Moving backward, in the second step the representative consumer chooses her own effort. Note that the optimal effort level is:

$$\arg\max_{e_i} EU(e_i, q, e_{-i})$$

In particular:

$$e_i^{**}(n, q, e_{-i}) : \tag{10}$$

$$\sum_{k=0}^{n-1} b(k; n-1; p)\{pU'\left[W_{Li}^{NC}\right]\left(-(1-q)L'(e_i) - \frac{q}{n}L'(e_i)\right)\} = C'(e_i)$$

Interestingly, from (10) and contrary to (5), we see that under a mutual agreement the effort is positive even with full-insurance ($q = 1$). The reason is that here policyholders internalize part of the effect of the effort on the premium, that is $\frac{q}{n}L(e_i)$.

By expressing the binomial distribution:

$$e_i^{**}(n, q, e_{-i}) :$$

$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\{pU'\left[W_L^{NC}\right](-(1-q)L'(e_i) - \frac{q}{n}L'(e_i))\}$$

$$= C'(e_i)$$

With identical agents the equilibrium is symmetric and $e_i = e_{-i} = e$, or:

$$e^{**}(n, q) : \tag{11}$$

$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\{pU'\left[W_L^{NC}\right](-L'(e)(1-q+\frac{q}{n})\}$$

$$= C'(e)$$

We can now consider the first step of the game: since they always act cooperatively in the first stage, policyholders in the mutual insurance solves:

$$\max_q \textstyle\sum_{k=0}^{n-1} \frac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\{pU\left[W_L^{NC}\right] + (1-p)U\left[W_0^{NC}\right]\} - C(e)$$

$$s.t. : \textstyle\sum_{k=0}^{n-1} \frac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\{pU'(\left[W_L^{NC}\right](-L'(e)(1-q+\frac{q}{n})\} \\ = C'(e) \tag{12}$$

Where now $W_L^{NC} = w - \frac{kq}{n}L(e^{**}) - (1-q)L(e^{**})$ and $W_0^{NC} = w - \frac{kq}{n}L(e^{**})$.

Lemma 1 *With self-insurance and policyholders acting non-cooperatively in the second stage, the mutual agreement offers a full coverage contract.*

Proof. 1) By Envelope theorem, such that $max_e U(e^{**}(q), q) = U^*(q)$, we can write the FOC of (9) with respect to q as follows:

$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\{pU'\left[W_L^{NC}\right]\left(\frac{(n-k)}{n}L(e^{**})\right)$$

$$+ (1-p)U'\left[W_0^{NC}\right]\left(-\frac{k}{n}L(e^{**})\right)\} = 0$$

2) We may exactly proceed following Proposition 1 in Lee and Ligon (2001), such that:

$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\left\{pU'\left[W_L^{NC}\right]\left(\frac{(n-k)}{n}L(e^{**})\right)\right\}$$

$$= \sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\left\{(1-p)U'\left[W_0^{NC}\right]\left(\frac{k}{n}L(e^{**})\right)\right\}$$

then substituting:

$$\begin{aligned} &\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^k (1-p)^{n-k} L(e^{**}) \left\{ U' \left[W_L^{NC} \right] \right\} \\ = &\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^k (1-p)^{n-k} L(e^{**}) \left\{ U' \left[W_0^{NC} \right] \right\} \end{aligned}$$

which implies that it must be $U' \left[W_L^{NC} \right] = U' \left[W_0^{NC} \right]$. This is possible if $q = 1$.
■

From (10) recall that the optimal effort choice $e^{**}(q)$ is positive even when $q = 1$. Thus, Lemma 1 shows that a non-cooperative strategy among individuals brings to full coverage $q = 1$ and to the positive effort $e_{NC}^{**}(n, 1)$. Note that this is in contrast with standard results in the stock insurance market (see 5). In particular:

$$e_{NC}^{**}(n,1): \tag{13}$$

$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1} (1-p)^{n-k-1} \left[pU' \left(w - \frac{k}{n} L(e_{NC}^{**}) \right) \left(-\frac{1}{n} L'(e_{NC}^{**}) \right) \right]$$
$$= C'(e_{NC}^{**})$$

The policyholder's payoff as a result of the non-cooperative strategic choice can be expressed as:

$$EU_{NC} = \sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1} (1-p)^{n-k-1} U \left(w - \frac{kL(e_{NC}^{**})}{n} \right) - C(e_{NC}^{**})$$

3.3 Cooperative strategy in mutual insurance

Under cooperation the representative consumer i's expected utility given k is expressed in (8). Interestingly:

Remark 1 *According to the law of large number, as $n \to \infty$, we obtain that:*

$$\frac{k}{n} \to p$$

Proof. See the appendix 6.1. ■

This result implies that, in the case of a very large pool, the random premium in the mutual company $\frac{kqL(e)}{n}$ tends to the fixed premium $P = pqL(e)$ given in the standard insurance company context (see subsection 2.2 before), thus nullifying the premium uncertainty in the risk-pooling:

Remark 2 *When the number of individuals insured in the mutual company is infinite, the premium converges to the fixed (independent from k) premium $pqL(e)$.*

Thus, in the limit and if the market for stock insurers is competitive, stock and mutual insurance are characterized by the same premium structure[2]. Moreover, if the effort is observable, the stock and the mutual insurer both leads to the first best level of effort for $n \to \infty$.

Considering the binomial probability, expected utility is:

$$EU = \sum_{k=0}^{n-1} b(k; n-1; p)\{pU\left(W_L^C\right) + (1-p)\,U\left(W_0^C\right)\} - C(e) \tag{14}$$

where $W_L^C = w - \frac{kq}{n}L(e) - L(e) + qL(e)$ and $W_0^C = w - \frac{kq}{n}L(e)$
The optimal effort choice here is:

$$\arg\max_e EU(e; n, q)$$

in particular it is:

$$e^{***}(n, q) :$$
$$\sum_{k=0}^{n-1} b(k; n-1; p)\{p(U'\left[W_L^C\right](-L'(e) + \frac{(n-k)}{n}qL'(e))$$
$$+(1-p)U'\left[W_0^C\right]\left(-\frac{kqL'(e)}{n}\right)\} = C'(e)$$

while rearranging and expliciting the binomial distribution:

$$e^{***}(n, q) : \tag{15}$$
$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!}p^k(1-p)^{n-k}\{-(1-q)L'(e)pU'(W_L^C)$$
$$-E\left[U'(W^C)\right]L'(e)\frac{k}{n}q\} = C'(e)$$

By comparing $e^{***}(n, q)$ with $e^{**}(n, q)$ in **(13)** and $e^*(q)$ in **(5)**, we note that the new term $-E\left[U'(W^C)\right]L'(e)\frac{k}{n}q$ appears in the marginal benefit of the effort in expression (15) such that the amount of effort exerted by consumers increases in this case. Such a term represents the positive impact that a higher effort has on the random premium and, thus, on marginal utility of aggregate consumption in both states of the world. It is decreasing in n, implying that the higher is the pool size and the lower is the benefit of cooperation in the mutual insurance.

[2] Note that, for the definition of mutual agreement (ex-post profits are zero for every realization of k), competition among mutual insurers has no impact on mutuals profits.

We can now consider the first step of the game: since they always act cooperatively in the first stage, policyholders in the mutual insurance solves:

$$\max_q \sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\{pU\left(W_L^C\right)+(1-p)\,U\left(W_0^C\right)\}-C(e)$$

$$\begin{aligned} s.t.: &\textstyle\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\{-(1-q)L'(e)pU'\left[W_L^C\right] \\ &-E\left[U'(W^C)\right]\tfrac{k}{n}qL'(e)\}=C'(e) \end{aligned} \tag{16}$$

Lemma 2 *With self-insurance and policyholders acting cooperatively in the second stage, the mutual agreement offers a full coverage contract.*

Proof. 1) From **16,** by Envelope theorem, we can write the FOC with respect to q as follows:

$$\sum_{k=0}^{n-1} \frac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\{pU'\left[W_L^C\right]\left[\frac{(n-k)}{n}L(e^{***})\right]$$
$$+(1-p)\,U'\left[W_0^C\right]\left[-\frac{k}{n}L(e^{***})\right]\}=0$$

2) As above,

$$\begin{aligned} &\sum_{k=0}^{n-1} \frac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\left\{pU'\left[W_L^C\right]\left[\frac{(n-k)}{n}L(e^{***})\right]\right\} \\ =\ &\sum_{k=0}^{n-1} \frac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\left\{(1-p)\,U'\left[W_0^C\right]\left[\frac{k}{n}L(e^{***})\right]\right\} \end{aligned}$$

then substituting,

$$\begin{aligned} &\sum_{k=0}^{n-1} \frac{(n-1)!}{(k-1)!(n-k-1)!} p^{k}(1-p)^{n-k}L(e^{***})\left\{U'\left[W_L^C\right]\right\} \\ =\ &\sum_{k=0}^{n-1} \frac{(n-1)!}{(k-1)!(n-k-1)!} p^{k}(1-p)^{n-k}L(e^{***})\left\{U'\left[W_0^C\right]\right\} \end{aligned}$$

which implies that it must be $U'\left[W_L^C\right]=U'\left[W_0^C\right]$. This is possible for $q=1$. ■

From (15) recall that the optimal effort choice $e^{***}(q)$ is positive even when $q=1$. Thus, as Lemma 1, Lemma 2 shows that $e_C^{***}(n,1)$ is positive in case of full coverage in a risk pooling arrangement.

From Lemma 2 the optimal effort becomes:

$$e_C^{***}(n,1): \tag{17}$$
$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\left[U'\left(w-\frac{k}{n}L(e_C^{***})\right)\left(-\frac{k}{n}L'(e_C^{***})\right)\right]$$
$$=C'(e_C^{***})$$

Equations (13) and (17) show that, as we expected, $e^{**}_{NC}(n,1) < e^{***}_{C}(n,1)$. In fact, in (13), the term p and $-\frac{1}{n}L'(\cdot)$ appear in the marginal benefit of the effort, meaning that policyholders takes into account only a part of the impact of the effort on the premium and only in the event of the loss. On the contrary, in (17), policyholders internalize the impact of the effort on total losses of the pool $\left(-\frac{k}{n}L'(\cdot)\right)$, and they do it in both states of natures.

From the two lemmas and from (13) and (17):

Corollary 1 *With both cooperative and non-cooperative effort strategies in the second stage of the game, the optimal coverage in mutual insurance is full coverage. However policyholders exert a higher level of effort in the case of cooperation than in the case of non-cooperation.*

The policyholder's payoff as a result of the cooperative strategic choice can be expressed as:

$$EU_C = \sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1} U\left(w - \frac{k}{n}L(e^{***}_C)\right) - C(e^{***}_C) \quad (18)$$

3.4 Cooperation as an equilibrium

We want to prove that cooperation in the second stage of the game can be an equilibrium. Suppose that the optimal cooperative choice of q has been already taken by the mutual in the previous stage. Under full coverage ($q = 1$) the policyholder's optimal choice of effort when she deviates from the cooperative strategy can be obtained from (9) as follows:

$$\begin{aligned}\max_{e_i^D} EU_i &= \sum_{k=0}^{n-1} b(k;n-1;p)\{pU\left[w - \frac{1}{n}\left(L(e_i^D) + (k-1)L(e^{***}_C)\right)\right] \quad (19)\\ &\quad + (1-p)\,U\left[w - \frac{k}{n}L(e^{***}_C)\right]\} - C(e_i^D)\end{aligned}$$

where e_i^D is the effort choice in case of deviation when the other policyholders choose e^{***}_C. Thus, the deviation effort is given by:

$$e_i^D(n,1): \quad (20)$$

$$\sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1}\left[pU'\left[w - \tfrac{1}{n}\left(L(e_i^D) + (k-1)L(e^{***}_C)\right)\right]\left(-\tfrac{1}{n}L'(e_i^D)\right)\right]$$
$$= C'(e_i^D)$$

which is lower than the cooperative effort choice, i.e., $e_i^D < e^{***}_{NC}$. We can now derive the policyholder's payoff in case of deviation:

$$EU_i^D = \sum_{k=0}^{n-1} \tfrac{(n-1)!}{(k-1)!(n-k-1)!} p^{k-1}(1-p)^{n-k-1} U\left(w - P^D\right) - C(e_i^D) \quad (21)$$

where the expected random premium P^D obtained in case of the deviation strategy is:

$$P^D = p\frac{1}{n}\left(L(e_i^D) + (k-1)L(e_C^{***})\right) + (1-p)\frac{k}{n}L(e_C^{***}) \tag{22}$$

Let us consider the difference between P^D and the premium when consumers choose cooperatively the effort $\frac{k}{n}L(e_C^{***})$. We easily see that:

$$P^D - \frac{k}{n}L(e_C^{***}) = \frac{p}{n}\left[L(e_i^D) - L(e_C^{***})\right] > 0 \tag{23}$$

Thus, on the one side deviation allows the policyholder to choose a lower effort and then to pay a lower disutility. On the other side deviation implies that a higher premium has to be paid.

Interestingly, as (23) shows, the lower is n, the higher is the advange of cooperation (a lower premium is paid by policyholders under cooperation).

Summarizing, cooperation is sustainable if:

$$\sum_{k=0}^{n-1} b(k; n-1; p)U\left[w - P^D\right] - C(e_i^D)$$
$$\leq$$
$$\sum_{k=0}^{n-1} b(k; n-1; p)U\left[w - \frac{kq}{n}L(e_C^{***})\right] - C(e_C^{***})$$

where:

$$\begin{aligned} P^D &> \frac{kq}{n}L(e_C^{***}) \\ e_i^D &< e_C^{***} \end{aligned}$$

From the previous discussion we can state:

Proposition 1 *The cooperative choice of the effort e_C^{***} is a Nash equilibrium and no profitable deviations is possible if the pool in the mutual arrangement is small enough and if $C'(\bullet)$ is low.*

Remark 3 *The maximum number of individuals in the pool compatible with cooperation is:*

$$\hat{n} \quad \text{such that } \sum_{k=0}^{n-1} b(k; n-1; p)U\left[w - P^D\right] - C(e_i^D) =$$
$$\sum_{k=0}^{n-1} b(k; n-1; p)U\left[w - \frac{kq}{n}L(e_C^{***})\right] - C(e_C^{***})$$

$\hat{n}$ is unique since the two payoff functions (18) and (21) are monotonous in n, as well as their difference.

Remark 4 *We showed that cooperation in the effort choice can be sustained as an equilibrium only if the numbers of individuals in the pool is sufficiently low, this is an argument in favor of mutual insurers characterized by small size.*

4 Stock insurance vs mutual agreements

It has been shown that consumers' welfare is higher in the case of a competitive stock insurance market than in the case of mutual agreements (See Ligon and Thistle 2008). Their model considers ex-ante moral hazard. We want to prove that, when cooperation in the mutual company is sustained as an equilibrium (thus when the mutual has a small size), then the mutual agreement allows consumers to get a higher welfare than a stock insurance in a competitive market. To prove that we should compare consumers' payoff in Subsection 2.2 with consumers' payoff as expressed in 18. This will take the second part of the paper.

5 Concluding remarks

In this paper, we investigate the policyholders' non-cooperative and cooperative behavior in mutual arrangement focusing on the case where a self-insurance measure is available to consumers. Differences in the non-cooperative and cooperative strategies strictly depend on the (partial or whole) internalization of the effect that individuals' effort choice may determine on the random premium. We show that cooperation realizes higher welfare with respect to the traditional non-cooperative option, and for a limited size of the pool, it becomes sustainable as an equilibrium. Future extensions on this line remains to be done.

6 Appendix

6.1 Proof of Remark 1

Consider the *Tchebycheff* 's inequality and apply it to the binomial distribution through *Bernoulli* Theorem (we are in the case of independently and identically distributed risks). Being k the number of times in which the event of a loss L of probability p is realized in n trials, and considering that for the binomial distribution the mean is $\mu = np$ and the variance is $\sigma^2 = np(1-p)$, then we easily obtain that:

$$\Pr\left\{\left|\frac{k}{n} - p\right| \geq \epsilon\right\} < \delta$$

where ϵ and δ are arbitrary small numbers. This expression reveals that the probability of a deviation from $\frac{k}{n}$ to p is at least equal, in absolute value, to ϵ and tends to zero when n grows infinitely. The convergence in probability can be realized even observing other different conditions to those observed in the Bernoulli Theorem.

References

[1] Borch, K. (1962), "Equilibrium in a reinsurance market", *Econometrica,* 30, 424–444

[2] Bourlès R. and D. Henriet, 2008. "Mutual Insurance With Asymmetric Information: The Case Of Adverse Selection," Working Papers halshs-00278178_v1, HAL

[3] Doherty, N. and Dionne, D. (1993), "Insurance with undiversifiable risk: Contract structure and organizational form of insurance firms", *Journal of Risk and Uncertainty*, 6, 187-203

[4] Eeckhoudt, L. and Kimball, M. (1992), "Background risk, prudence, and the demand for insurance. In G. Dionne (ed.), *Contributions to Insurance Economics*, Boston: Kluwer Academic.

[5] Ehrlich, I. and Becker, G. (1972), "Market insurance, self-insurance and self-protection", Journal of Political Economy, 80, 623-648

[6] Hansmann, H. (1985), "The organization of insurance companies: mutual versus stock", *Journal of Law, Economics and Organization*, 1, 125-153

[7] **Lee, W. and J, Ligon (2001), "Moral Hazard in risk pooling arrangements", The Journal of Risk and Insurance, 68(1), 175-190.**

[8] Ligon, J. and Thistle, P. (2005), "The formation of mutual insurers in markets with adverse selection", *Journal of Business*, 78, 529-555

[9] **Ligon, J. and Thistle, P. (2008), "Moral hazard and background risk in competitive insurance markets", Economica, 75, 700-709**

[10] Manning W.G., Newhouse, J.P., Duan, N., Keeler, E.B., Liebowitz, A. and Marquis, M.S. (1987), "Health insurance and the demand for health care; evidence from a randomized experiment", *American Economic Review*, 77, 251-77

[11] Mayers, D. and Smith, C. (1986), "Ownership Structure and Control: The Mutualization of Stock Life Insurance Companies", J*ournal of Financial Economics*, 16, 73-98

[12] Mayers, D. and Smith, C. (1988), "Ownership structure across lines of property-casualty insurance", *Journal of Law and Economics*, 31, 351-378

[13] Mayers, D. and Smith, C. (2002), "Ownership structure and control: property-casualty insurer conversion to stock charter", *Journal of Financial Services Research* (21), 117-144

[14] Nekby, L. (2004), "Pure versus mutual health insurance: Evidence from Swedish historical data", *The Journal of Risk and Insurance*, 71, 115-134

[15] Pauly, M. (1968), "The economics of moral hazard", *American Economic Review*, 58, 531-537

[16] Picard, P. (2009), "Participating insurance contracts and the Rothschild-Stiglitz equilibrium puzzle," Working Papers hal-00413825_v1, HAL.

[17] Rothschild, M. and Stiglitz J. (1976), "Equilibrium in Competitive Insurance Markets", *Quarterly Journal of Economics*, 90 (4), 629-649

[18] Smith, B. and Stutzer, M. (1990), "Adverse selection, aggregate uncertainty, and the role for mutual insurance contracts", *The Journal of Business,* 63 (4), 493-510

[19] Smith, B. and Stutzer, M. (1995), "A Theory of Mutual Formation and Moral Hazard with Evidence from the History of the Insurance Industry", *Review of Economic Studies*, 8 (2), 545-577

[20] Winter, R. (1992), "Moral hazard and insurance contracts", In G. Dionne (ed.), *Contributions to Insurance Economics*, Boston: Kluwer Academic