REGULATING UNVERIFIABLE QUALITY BY FIXED-PRICE CONTRACTS

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Abstract

We apply the idea of relational contracting to a very simple problem of regulating a single-product monopolistic firm when the regulatory instrument is a fixed-price contract, and quality is endogenous and observable, but not verifiable. We model the interaction between the regulator and the firm as a dynamic game, and we show that, provided both players are sufficiently patient, there exist self-enforcing regulatory contracts in which the firm prefers to produce the quality mandated by the regulator, while the regulator chooses to leave the firm a positive rent as a reward to its quality choice. We also show that the socially optimal self-enforcing contract implies a distortion from the second best, which is greater the more impatient is the firm and the larger is the (marginal) effect of the contractual price on the profits the firm would make by deviating from the offered contract. Whenever the punishment profits are strictly positive, even if the firm were infinitely patient, the optimal contract would ensure a Ramsey condition but with positive profits to the firm. Our result also illustrates that, whenever the firm's output has some unverifiable component, optimal regulatory lag in fixed-price contract should be reduced to limit the reward of the firm's opportunistic behaviour.

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1 Introduction

The quality of goods and services provided by regulated firms is an extremely sensitive issue: indeed, how to regulate quality has been a subject widely explored since the early days of the economics of regulation (Spence, 1975, Sheshinsky, 1976). Quality has many distinctive features: for instance, it may be difficult to observe, it has a non-deterministic component, consumers' preferences towards it may be difficult to observe. Our focus here is on unverifiability: unverifiability occurs whenever a variable cannot be proven in front of a court and, as a consequence, cannot be contracted upon. In regulated industries it is often the case that a quality dimension of the regulated firm's output is not verifiable: possible examples are courtesy to the customers, voltage of electricity provided in a particular moment, noise of a call, and so forth.

When quality is not verifiable, the regulated firm cannot simply be *directly* rewarded or penalized for the levels of service quality provided. This implies that the regulatory tools developed by the existing literature and commonly applied in practice may turn out not to be effective. Theoretically, optimal contracts under asymmetric information may provide the firm incentives to supply quality which are intrinsically in conflict with those to reduce cost (Laffont and Tirole, 1993). On more applied grounds, the regulatory instruments commonly in use, such as quality standards and links between the quality provided and his allowed revenues or prices, are typically able to influence only those quality dimension which are readily verifiable (Waddams Price et al., 2008; De Fraja and Iozzi, 2008).

In this paper we suggest an alternative way of regulating quality, based on the idea of *relational contracts*. These are informal agreements and unwritten codes of conduct that are sustained by the value of future relationships, and are applicable in the cases where the outcome of a repeated relationship is based on some unverifiable variables. This type of contract fits in naturally with the nature of the interaction between the regulator and the regulated firm. This is a relationship in which the regulator enjoys a quite large discretion.[1] Also, the two parties typically interact over time

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[1]Beesley and Littlechild (1989) clearly illustrates this point. On p 458, they state that "when setting X [which, under price cap regulation, is the annual rate of allowed price change, i.e. the parameter limiting the price level and dynamics (our note)] initially there are many degrees of freedom. [...] There is nothing unique, optimal, or mechanical about the initial choice of X. When X is reset, there are significantly fewer degrees of freedom.

in a repeated way, and are both well informed on many variables affecting the outcome of the relationship, even if part of this knowledge cannot be proven in court or be written in a contract.

We apply the idea of relational contracting to a very simple problem of regulating a single-product monopolistic firm when the regulatory instrument is a fixed-price contract, and quality is endogenous and observable, but not verifiable. We model the interaction between the regulator and the firm as a dynamic game, and we show that, provided both players are sufficiently patient, there exist self-enforcing regulatory contracts in which the firm prefers to produce the quality mandated by the regulator, while the regulator chooses to leave the firm a positive rent as a reward to its quality choice. We also characterize the optimal contract, that is the self-enforcing agreement which produces the highest social welfare. We show that, under normal circumstances, this contract implies distortion from the second best. We find that this distortion is greater the more impatient is the firm and the larger is the (marginal) effect of the contractual price on the profits the firm would make by deviating from the offered contract. Under some cost and demand conditions, even if the firm were infinitely patient, the optimal contract grants positive profits to the firm even if entailed a Ramsey condition of tangency between the isowelfare and the isoprofit.

When one interprets the discount factor as resulting from the frequency of interactions between the players and the probability of continuation of the game, this paper makes a contribution also to the issue of optimal contract length. In the context of regulation, our result suggests that, together with accounting for the effects on cost-reducing effort and profits, the frequency of the price revisions should be higher the more relevant is the issue of unverifiability. This is because a shorter regulatory lag reduces the reward for opportunistic behaviour by the firm and allows the regulator to elicit "better" price and quality pairs. A similar indication comes also in the case of procurement contracts, which our paper contributes to study once one focuses to the case of repeated purchases and reads the discount factor as depending also on the contract length and on the probability of continuation of the game (possibly because of some unspecified tendering procedure): our result suggests that a higher frequency of the competitive tendering and

Nevertheless, there invariably *are* degrees of freedom open to the regulator". Notice that Littlechild was the Director General of OFFER, the UK energy regulator, from 1989 to 1998. See also Olson and Richards (2003).

some form of favouritism towards the incumbent firm when re-tendering (future) contracts increases the value of adhering to the current contract and results in a price and quality pair which is closer to the one most socially preferred.

This paper is clearly related to the literature on quality regulation, recently presented in the excellent survey by Sappington (2005). Price cap regulation plans give the firm insufficient incentive to deliver the socially optimal level of service quality. A few recent papers have successufully solved this problem by incorporating some quality index into the price cap formula (Currier, 2007; De Fraja and Iozzi, 2008; Weisman, 2005). All these schemes however need quality to be observable and verifiable. Unverifiability of quality is explicitly taken into account by the optimal regulation literature which shows that providing incentives for cost reduction conflicts with the objective of stimulating the quality provision. The optimal cost-reimbursing schemes is typically low-powered (Laffont and Tirole, 1993) and higher quality is obtained by reducing (increasing, respectively) prices when quantity and quality are complements (substitutes) in the consumers preferences (Lewis and Sappington, 1992). Auray at al. (2008) extend the analysis of incentives in quality regulation to a dynamic framework, albeit restricting the analysis to the case of observable quality. More closely related to our paper is Dalen (1997), who analyses a two-period model in which the regulator uses past information revealed to re-write the contract in the second period: the use of incentive schemes with lower power reduces the firm's informational rent and therefore mitigates the incentive for the firm to increase the current inefficiency to secure future rents (the so-called ratchet effect).

This paper is also related to the recently growing and cross-field literature on relational contracts (Hviid, 2000; MacLeod, 2007). The cooperative profits the firm enjoys in our paper when delivering the required quality are a simplified form of the stationary contract characterised by Levin (2003) in dealing with a simultaneous repeated game with problems of moral hazard and hidden information. Our paper is also linked to the literature on repeated procurement with non-contractible elements, which shows that an optimal strategy for the buyer to enforce unverifiable quality is leaving future rents to the contractor (Kim, 1998; Doni, 2006). This literature has also shown that shorter contracts may increase the possibility of collusion

between bidders (Kremer et al., 2006; Sasaki and Strausz, 2008). Both these issues are analysed in connection in Calzolari and Spagnolo (2009); they show that, in order to increase the value of the contract to the firm and induce it to deliver a "good" level of the non-contractible variable, it is optimal to reduce the contract length and limit the competition for the contract, possibly facilitating collusion.

The rest of the paper is organised as follows. Section 2 presents the model. The equilibria of the static and the dynamic game are characterized in section 3. Section 4 contains two examples that shed further light on the nature of the optimal contract arising in the dynamic game equilibrium. Section 5 concludes.

2 The Model

We analyse an infinite horizon game in which two parties, a regulator and a monopolistic firm, interact at dates $t = 0, 1, \ldots, \infty$. Let δ be the discount factor common to the firm and the regulator.

The monopolist produces one good, whose demand is given by $x(p, q)$, with p denoting the price of the good and q its quality; we assume that $p \in \mathbb{R}_+$ and $q \in Q \equiv [\underline{q}, \overline{q}] \subseteq \mathbb{R}_{++}$. The demand function is assumed to have standard properties: for all quality levels q, it is continuous and twice differentiable, with $\frac{\partial x}{\partial p} < 0$ and $\frac{\partial x}{\partial q} > 0$ whenever $x > 0$.

The firm's technology is described by the cost function $c(x, q)$, which satisfies, plausibly, $\frac{\partial c}{\partial x} > 0$ and $\frac{\partial c}{\partial q} > 0$. To avoid corner solutions, we assume that $\lim_{q \to \underline{q}} \frac{\partial c}{\partial q} = 0$ and that $\lim_{q \to \overline{q}} c(x, q) = +\infty$: a marginal increase of quality is costless when quality is at its minimum and maximal quality is infinitely costly. The firm's profits are therefore given by $\pi(p, q) = x(p, q)p - c(x(p, q), q)$.

The regulator's objective function is given by the social welfare function $V(p, q)$ which is simply assumed to be continuously differentiable and satisfy the following plausible conditions: $\frac{\partial V(.)}{\partial p} \leq 0$, and $\frac{\partial V(\cdot)}{\partial q} > 0$.[2] The social

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[2] We purposely do not impose any further restrictions on the consumers and regulator's preferences. A less general but still natural setting would be with many consumers with quasi-linear preferences and a benevolent regulator (see Bös, 1981; and De Fraja and Iozzi, 2008). In this setting, the regulator's objective function is the unweighted sum of individuals' utility, measured by the area below the demand curve and above the price, summed over the various goods. Of course, all our results would be valid also in this setting.

value of not having the good produced by the firm is equal to V_0; we assume that having the good produced is always beneficial for the society, so that $V(p, q) > V_0$ for any value of p and q.

The dynamic game we consider is an infinite repetition of the following sequential stage game:

Stage 1: the regulator makes an offer $\mathcal{F} = \{p', q'\}$ in which it asks the firm to produce a good of quality q' and sets the market price p' at which the good has to be sold in the market;

Stage 2: the firm chooses whether or not to accept the contract; if the firm does not accept the contract, the game ends, otherwise the game proceeds to the following stage;

Stage 3: the firm chooses the effective quality level q''; at the end of this stage the regulator observes q'', and the payoffs $V(p', q'')$ and $\pi(p', q'')$ are realized.

Observe that, because of the assumption on the regulator's reservation value, the regulator will always make offers such that $\pi(p', q') \geq \pi_0$, where π_0 denotes the firm's reservation profits, which we normalise to zero. This implies that the second stage of the game can be ignored in the rest of the analysis, since the firm will never find it profitable to reject the offer and quit the game.

We analyse a repeated game of perfect monitoring. However, despite the realisation of price and quality being fully observable by both players, quality is not enforceable in a court of law, in that the regulator cannot impose any directly enforceable penalty on the firm when it observes $q' \neq q''$.

Before proceeding into the analysis of the game, we state the following:

Definition 1. *Let the following definitions hold:*

  a) for any price p, let $\widehat{q}(p) = \arg\max\limits_{q} \ \pi(p, q)$;

  b) for any price p, let $\widehat{\pi}(p) = \pi(p, \widehat{q}(p))$;

  c) let p^0 be the smallest price such that $\widehat{\pi}(p^0) = 0$.

In words, $\widehat{q}(p)$ is the quality level that delivers the highest profit to the firm for any possible price. We assume it to exist and be unique. Similarly,

$\widehat{\pi}(p)$ is the profit the firm can make, for any given price, when it optimally chooses its quality. Also, p^0 is the price which ensures that the firm obtains zero profits when it freely chooses its quality level, given this price. If more than one such price exist, p^0 denotes the smallest.

We also state:

Definition 2. *Let p^R and q^R be the pair of price and quality which solves the following problem:*

$$\max_{\{p,q\}} \quad V(p,q) \tag{1}$$
$$\text{s.t.} \quad \pi(p,q) \geq 0$$

In words, p^R and q^R is the Ramsey price and quality pair which maximises (static) social welfare subject to a nonnegativity constraint on the firm's profits. It is easy to show that, at p^R and q^R, the following holds:

$$\frac{\partial V/\partial p^R}{\partial V/\partial q^R} = \frac{\partial \pi/\partial p^R}{\partial \pi/\partial q^R}, \tag{2}$$

and that, at p^R and q^R, the nonnegativity constraint holds as an equality. We assume p^R and q^R to exist, to be unique and different from p^M and q^M, where p^M and q^M are the profit maximising price and quality values.

Figure 1 gives an illustration of these Definitions. It depicts the price-quality cartesian plane; the solid curves are the isoprofit lines and the dashed curves are the isowelfare lines, upward sloping because welfare is increasing in quality and decreasing in price.[3] The pair $\{p^M, q^M\}$ is the profit maximising price and quality pair and $\{p^R, q^R\}$ is the second best optimal pair: at this point, the zero-profit isoprofit line is tangent to the isowelfare map. At prices p^0 and p^1, the firm, freely choosing its quality level, selects $\widehat{q}(p^0)$ or, respectively, $\widehat{q}(p^1)$: at the price and quality pair $\{p^0, \widehat{q}(p^0)\}$ the firm makes zero profits.

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[3]If we had assumed that consumers' preferences are as in the previous footnote, the quasi-concavity of $V(p,q)$ shown in the Figure would reflect the quite natural (but not always satisfied) property of consumers' preferences that their willingness to pay for increases in quality is higher when quality is low than when quality is already high (for further discussion, De Fraja and Iozzi, 2008).
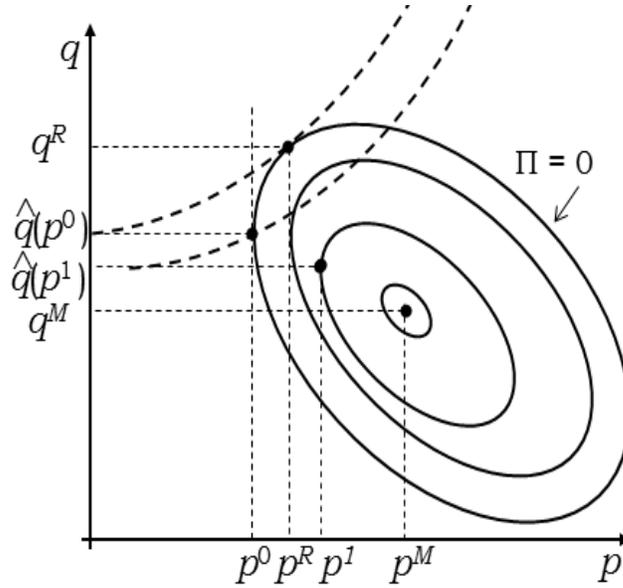
Fig. 1 - The static game

3 Equilibrium

3.1 The static game

We start by noting that, in a static context, unverifiability of the quality provided by the firm implies that the regulator cannot enforce the second best quality level. In other words, since the regulator can only observe but not punish any choice of quality other than the mandated level, we are back in the context of price regulation with endogenous quality, firstly analysed by Spence (1975). It is then straightforward to characterize the equilibrium of the stage game described above. In the last stage of the game, for any price mandated by the regulator, the firm chooses the profit maximizing quality level $\widehat{q}(p)$. Anticipating this, in the first stage the regulator makes an offer $\mathcal{F}^S \equiv \{p^S, \widehat{q}(p^S)\}$, where p^S comes as the solution of the following

problem:[4]

$$\max_{p} \quad V(p,q) \tag{3}$$
$$\text{s.t.} \quad \pi(p,q) \geq 0$$
$$q = \widehat{q}(p).$$

The properties of this equilibrium price are described in the following Proposition:

Proposition 1. *The price p^S offered by the regulator in the static equilibrium of the stage game has the following features:*

- $p^S = p^0$ *whenever* $-\frac{\partial V}{\partial p^S} > \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S}$, *which implies* $\widehat{\pi}(p^S) = 0$, *and*

- $p^S > p^0$ *whenever* $-\frac{\partial V}{\partial p^S} = \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S}$, *which implies* $\widehat{\pi}(p^S) > 0$.

Proof. To solve problem (3), set up the Lagrangean incorporating the second constraint

$$L = V(p, \widehat{q}(p)) + \mu\widehat{\pi}(p).$$

FOCs are:

$$\frac{\partial L}{\partial \mu} = \widehat{\pi}(p^S) \geq 0; \quad \mu \geq 0 \quad \mu\widehat{\pi}(p^S) = 0$$

and

$$\frac{\partial L}{\partial p^S} = \frac{\partial V}{\partial p^S} + \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S} + \mu\frac{\partial \widehat{\pi}}{\partial p^S} = 0.$$

If $\mu = 0$, then $\widehat{\pi}(p^S) \geq 0$ and $\frac{\partial V}{\partial p^S} + \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S} = 0$. Instead, if $\mu > 0$, then $\pi(p^S, \widehat{q}(p^S)) = 0$ and $\frac{\partial V}{\partial p^S} + \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S} + \mu\frac{\partial \widehat{\pi}}{\partial p^S} = 0$. Since $\widehat{\pi}(p)$ is monotonically increasing in p whenever $p < p^M$, this establishes the result. $\square$

Proposition 1 illustrates that the optimal static offer is such that the firm may obtain strictly positive profits.[5] When the optimal offer implies strictly positive profits, it equalizes the marginal negative direct effect on welfare of a price increase with the marginal positive indirect effect, due to an increase

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[4]We take $\widehat{q}(p^S)$ as the quality level included in the offer $\mathcal{F}^S$ only for the sake of definiteness; indeed, any quality level could be part of such an offer because, in this static setting, the regulator anticipates that the firm will always choose its profit maximizing quality level and that it cannot prevent or punish this behaviour.

[5]Note that the solution to problem (3) need not be unique. In case of multiple solutions, for reasons that will be clearer thereafter, we select p^S as the solution giving the firm the lowest profit.

in the quality provision (i.e. $-\frac{\partial V}{\partial p^S} = \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S}$). Notice that, in this case, a marginal increase in the price necessarily induces an increase in the quality provided by the firm. On the other hand, when the optimal offer implies zero profits, the direct positive effect on welfare of a price reduction would outplay the effect going through a change in quality; however, the non-negativity constraint on the firm's profits limits a further price decrease. Notice that in this case, at the equilibrium, the sign of the marginal change in quality due to a marginal price change ($\frac{\partial \widehat{q}(p^S)}{\partial p^S}$) is indeterminate. It should be also noted that, as already pointed out in the existing literature (Spence, 1975; and De Fraja and Iozzi, 2008), equilibrium quality is always underprovided, in the sense that there always exists a Pareto improving increase in quality. On the other hand, no clear-cut conclusion can be reached on the magnitude of p^S relatively to the second-best price: it is indeed even possible that the equilibrium price is above the second best price.
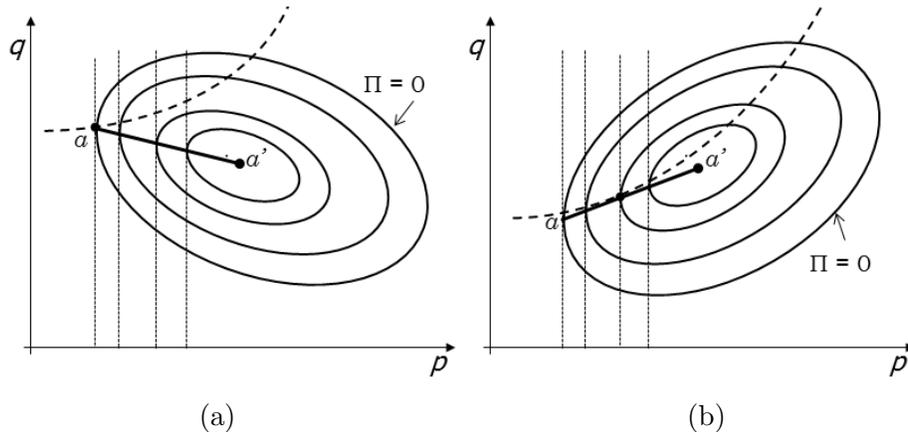

Fig. 2 - The optimal static contract

The equilibrium of the stage game is illustrated in Figure 2. In both panels, the locus aa' is made of the optimal quality choices for the different price levels, i.e. $\widehat{q}(p)$. Taking this as a constraint, the regulator chooses its optimal one-shot price p^S to maximise social welfare. Depending on the local relative slope of iso-welfare ($-\frac{\partial V}{\partial p^S}/\frac{\partial V}{\partial \widehat{q}(p^S)}$) and the sign of $\frac{\partial \widehat{q}(p^S)}{\partial p^S}$ which determines the two possible situations in Proposition 1, the optimal price may be given by a tangency condition between the isowelfare and the locus aa', as in panel (a), or may be a corner solution, as in panel (b). Clearly, given the many possible shapes the locus aa' can take on, restrictions are necessary to ensure that the solution to the regulator's problem is unique or, more restrictively, exists altogether.

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3.2 The dynamic game

In this section, we study the dynamic game given by an infinite repetition of the sequential stage game discussed in the previous section. We illustrate that a relational contract ensures a social welfare higher than the one which would prevail in a static context, and show the nature of the price and quality pair which are chosen under this contract to maximise the social welfare.

A regulatory relational contract under unverifiable quality is a strategy profile such that, given the offer $\mathcal{F}^C \equiv \{p^C, q^C\}$, the parties take the following actions in each period

- the regulator makes the offer $\mathcal{F}^C$;

- the firm chooses q^C.

This regulatory relational contract is self-enforcing if the strategy profile is a perfect equilibrium of the repeated game.

The definition leaves undefined two elements of the players' strategies, the offer $\mathcal{F}^C$ and the parties' behaviour off the equilibrium path. We make them precise concentrating on the following trigger strategies with Nash reversion for the players:

- **regulator**: the regulator begins the game by making the firm an offer $\mathcal{F}^C$ and keeps making this offer if the firm has always chosen quality q^C in previous periods; otherwise, it reverts indefinitely to its equilibrium strategy in the stage game;

- **firm**: the firm chooses the quality q^C whenever the regulator has offered $\mathcal{F}^C$ in the first stage of the same period; otherwise, it reverts indefinitely to its equilibrium strategy in the stage game.

Notice the somehow different nature of the two strategies, due to the sequential nature of the stage game; while a choice of the quality level different from q^C is detected by the regulator only in the following period, an offer different from $\mathcal{F}^C$ by the regulator is immediately observed by the firm and triggers a reaction in the same period it is made.

The Folk theorem ensures the existence of an equilibrium in these trigger strategies, provided that the players are sufficiently patient.[6] Formally, this requirement of "sufficient patience" is equivalent to the following condition:

$$\frac{1}{1-\delta}\pi(p^C, q^C) \geq \widehat{\pi}(p^C) + \frac{\delta}{1-\delta}\widehat{\pi}(p^S) \tag{ICF}$$

Observe also that, in principle, an incentive compatibility constraint needs to hold also for the regulator. However, the regulator's IC is also always satisfied provided it gains from offering $\mathcal{F}^C$, that is $V(p^C, q^C) \geq V(p^S, \widehat{q}(p^S))$. Indeed, there is no short-term gain for the regulator in deviating from its trigger strategy, because this is observed and punished by the firm in the same period before payoffs are realized.

Condition ICF simply imposes limits on the nature of the offer $\mathcal{F}^C$. However, since it is the regulator to choose the offer, it will select the socially optimal among the ones which ensure that the ICF holds. We now turn to studying the characteristics of the regulator's *optimal* offer which ensures the self-enforcing nature of the relational contract. We let $\mathcal{F}^* \equiv \{p^*, q^*\}$ be such an offer; it comes as the solution to the following problem:[7]

$$\max_{\{p,q\}} \sum_{t=0}^{\infty} \delta^t V(p,q) = \frac{1}{1-\delta} V(p,q) \tag{4}$$

$$\text{s.t. } \frac{1}{1-\delta}\pi(p,q) \geq \widehat{\pi}(p) + \frac{\delta}{1-\delta}\widehat{\pi}(p^S)$$

We can now state the main result of the paper:

Proposition 2. *The price and quality pair $\{p^*, q^*\}$ solving problem (4) satisfies the following conditions:*

$$\frac{\partial V/\partial p^*}{\partial V/\partial q^*} = \frac{\partial \pi/\partial p^*}{\partial \pi/\partial q^*} - (1-\delta)\frac{\partial \widehat{\pi}/\partial p^*}{\partial \pi/\partial q^*} \tag{5}$$

and

$$\frac{1}{1-\delta}\pi(p^*, q^*) = \widehat{\pi}(p^*) + \frac{\delta}{1-\delta}\widehat{\pi}(p^S). \tag{6}$$

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[6]Sorin (1995) proves that the Folk theorem proved by Fudenberg and Maskin (1986) for simultaneous repeated game also applies to sequential repeated games provided that full dimensionality condition (FDC) holds. This requires that the convex hull of the set of the feasible payoff vectors of the stage game must have dimension equal to the number of players, or equivalently a nonempty interior. FDC is clearly satisfied in our model. Abreu et al. (1994) and Wen (1994, 2002) further weaken this requirements.

[7]Notice that, in problem (4) the single-period participation constraint for the firm is implied by the ICF and is therefore omitted.

Proof. The Lagrangian and the FOC's of the problem defined by (4) are the following:

$$L = \frac{1}{1-\delta} V\left(p^*, q^*\right) + \lambda \left[\frac{1}{1-\delta} \pi\left(p^*, q^*\right) - \widehat{\pi}(p^*) - \frac{\delta}{1-\delta} \widehat{\pi}(p^S) \right] \quad (7)$$

$$\frac{\partial L}{\partial p^*} = \frac{1}{1-\delta} \frac{\partial V\left(.\right)}{\partial p^*} + \lambda \left[\frac{1}{1-\delta} \frac{\partial \pi\left(.\right)}{\partial p^*} - \frac{\partial \widehat{\pi}\left(.\right)}{\partial p^*} \right] = 0 \quad (8)$$

$$\frac{\partial L}{\partial q^*} = \frac{1}{1-\delta} \frac{\partial V\left(.\right)}{\partial q^*} + \lambda \left[\frac{1}{1-\delta} \frac{\partial \pi\left(.\right)}{\partial q^*} \right] = 0 \quad (9)$$

$$\frac{\partial L}{\partial \lambda} = \frac{1}{1-\delta} \pi(p^*, q^*) - \widehat{\pi}(p^*) - \frac{\delta}{1-\delta} \widehat{\pi}(p^S) \geq 0;$$
$$\lambda \geq 0; \lambda \frac{\partial L}{\partial \lambda} = 0. \quad (10)$$

From (8) and (9), it follows that $\lambda > 0$; if this were not the case, we would have that $\frac{\partial V\left(.\right)}{\partial p^*} = \frac{\partial V\left(.\right)}{\partial q^*} = 0$, which clearly contradicts the hypothesis that the first best is out of reach. Therefore, $\frac{\partial L}{\partial \lambda} = 0$ in (10), which gives (6). Also, dividing (8) by (9), we get (5). $\square$

Conditions (5) and (6) define the optimal equilibrium price and quality pair and illustrate the way it departs from the Ramsey condition. Condition (5) differs from the standard Ramsey condition (2) because of the second term in the RHS, which in turn depends, first, on the discount factor and, second, on the marginal effect on the deviation profits of a change in the contractual price. To interpret (5), note that, in the Ramsey condition of tangency, the marginal rate of substitution between price and quality is equated between the regulator and the firm. On the contrary, here, the regulator finds it optimal to offer a price lower than the one which would ensure the tangency between isoprofit and isowelfare (at the minimum profit level for the firm). This is because it takes into account the fact that the higher is the price offered, the higher are the firm's profits in case of deviation and, therefore, the risk of a firm's deviation. The greater is this effect the smaller will be the regulator's willingness to substitute away price with quality. Clearly, these considerations play a role in the regulator's choice of the optimal contract which is more important the more the firm is "tempted" to deviate, that is when the firm is the less patient. This implies that the

distortion from a Ramsey tangency condition typical of the optimal offer is greater the smaller is the firm's discount factor. Only if the firm were infinitely patient and/or the effect of the optimal price on the deviation profits were null, the optimal contract would correspond to a tangency condition between the isowelfare and the isoprofit, as with the standard Ramsey condition.

On the other hand, condition (6) illustrates the level of the profits the regulator has to ensure to the firm. These increase not only with the profits the firm obtains by deviating from the regulator's offer, but also with the profits the firm would obtain in the punishment phase. Combining conditions (5) and (6), it is also possible to see that, whenever the punishment phase entails strictly positive profits for the firm, even if the firms were infinitely patient, the best possible contract would satisfy the Ramsey tangency condition, though on an isoprofit corresponding to strictly positive profit.[8] Proposition 2 illustrates how the optimal offer delivers an outcome which differs the more from the second best the smaller is the discount rate. The discussion of the nature of the discount rate is useful in drawing a connection between this paper and other related fields of analysis.

The frequency of interactions positively affects the value of the discount rate, since a high value of δ is associated to a low value of the per-period interest rate. In our setting an increase in the frequency of interactions is equivalent to a reduction in the time between two successive price (and quality) determination, something which is normally referred to as regulatory lag. The existing literature illustrates that, when the firm's cost is endogenous, there is a trade-off in setting the regulatory lag: the longer is the regulatory lag, the higher is the incentive the firm has to undertake cost-reducing efforts, but also the greater the allocative inefficiency arising from the firm's excessive profits (Armstrong et al., 1994; and Armstrong et al., 1995). Our results suggest the existence of a further determinant in

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[8]This result depends on our choice to concentrate on trigger strategies with Nash reversion. Alternatively, we could have focused on players carrying out optimal penal codes that involve *stick and carrot* strategies (Abreu, 1986 and 1988), which entails that the punishment is not forever. These strategies are optimal in the sense that the punishment is calculated so that the discounted present value of the payoffs after a deviation is zero; this clearly implies that the player obtains negative profits for the period immediately after the deviation. Since these strategies allow such a harsh punishment, our optimal offer would entail a second best outcome (with zero profit) with an infinitely patient firm. However, the severity of the punishment would clearly conflict with the firm's participation constraint.

setting the regulatory lag. Since our equilibrium outcome becomes closer to a Ramsey solution as δ increases, our results suggest that, in regulatory settings in which unverifiability plays a role, the shorter is the regulatory lag the greater is the regulator's ability to induce a second best outcome.

On the other hand, our game is formally equivalent to another game of indeterminate length in which δ would reflects both the players' intertemporal preferences *and* the probability of continuation of the game in the following period. This interpretation of the discount factor enables to interpret our model as a model of procurement in case of a single producer and repeated purchases. The existing literature has analysed the issue of the optimal length of contracts. A classical result, non confined to procurement, is that contract length depends on the severity of the problem of investment specificity (Hart and Moore, 1988; and, more recently, Guriev and Kvasov, 2005). The length of procurement contracts has effects on efficiency similar to the one described above for the case of regulation. Moreover, with many bidders and a dynamic auction, contract length (i.e., in this case, auction frequency) affects the possibility of collusion between bidders (Kremer et al., 2006; Sasaki and Strausz, 2008). As in the case of the regulatory lag discussed above, our paper illustrates that, in case of repeated purchases, there is scope for some favouritism towards the incumbent firm when re-tendering the contract, in that this increases the value of adhering to the offered contract and results in a price and quality pair which is close to the one most socially preferred. This is in line with Calzolari and Spagnolo (2009), who additionaly find that a reduction in the contract length, in increasing the probability of collusion, increases the value of the contract to the firm and, therefore, induces it to deliver a "good" level of the non-contractible variable.

4   Two examples with specific functional forms

To illustrate further the nature of the optimal offer to be made to the firm and to shed light on the effect on it of the time preferences and of the interplay between the "cooperative" profits and, in contrast, the profits arising during the deviation and the punishment phase, we use two examples with specific functional forms. In particular, the first example includes an optimal punishment with zero-profit while in the second the punishment profit is positive.

4.1 Example 1

Demand function is given by $x(p,q) = (4+q) - p$. Social welfare, defined as aggregate consumers' surplus, is given by

$$V(p,q) = \frac{1}{2}(4+q-p)^2. \tag{11}$$

The firm's cost function is $c(q,x) = (1+q^2)x$, and profits are given by

$$\pi(p,q) = (4+q-p)(p-(1+q^2)). \tag{12}$$

The monopolist unconstrained profit maximising choices are given by $p^M = \frac{23}{8}$ and $q^M = \frac{1}{2}$, which give profits equal to $\pi^M = \frac{169}{64}$. For any given price, the optimal quality choice for the firm is

$$\widehat{q}(p) = -\frac{4}{3} + \frac{1}{3}p + \frac{1}{3}\sqrt{13 - 5p + p^2} \tag{13}$$

The (static) second best price and quality pair, found by solving problem (3), is such that

$$p^R = \frac{5}{4} \text{ and } q^R = \frac{1}{2} \tag{14}$$

which give social welfare equal to $V(p^R, q^R) = \frac{169}{32}$ and profits equal to 0. Notice that the socially optimal quality level is also chosen by the unconstrained monopolist, something which the previous literature has already recognised to be possible (see, for instance, Tirole, 1989). The punishment price and quality pair is $p^P = 1$ and $q^P = 0$ with $\pi(p^P, q^P) = 0$.

Unfortunately, it is not possible to solve for the optimal price and quality pair analitically and we then have to resort to numerical methods. Table 1 provides the values of p^*, q^* and p^P for different values of the discount factor δ; the same Table also provides the equilibrium level of static profits and social welfare.

These same values are illustrated in Figure 3. The points lying southwest to the (static) socially optimal pair $\{p^R, q^R\}$ are the optimal offer, $\{p^*, q^*\}$, drawn for different values of δ. The higher is δ, the closer to the second best are both price and quality included in the optimal contract; on the other hand, the lower is δ, the closer are the contractual pairs to the pair $\{1, 0\}$, the equilibrium of the static game and also the offer by which the regulator may punish any deviation from the optimal contract. Through the optimal offer when $\delta = 0.5$, we draw both the isowelfare and the isoprofit to illustrate the distortion (i.e. the difference between the firm's and regulator's marginal rate of substitution) typical of the optimal contract.

Table 1: Optimal contract in Example 1

δ	p^*	q^*	p^P	$\pi(p^*, q^*)$	$\pi(p^*, \widehat{q}(p^*))$	$V(p^*, q^*)$
0.1	1.0344	0.0637	1	0.0918	0.1020	4.5884
0.2	1.0658	0.1233	1	0.1548	0.1935	4.6739
0.3	1.0948	0.1792	1	0.1933	0.2762	4.7568
0.4	1.1216	0.2321	1	0.2108	0.3514	4.8374
0.5	1.1466	0.2821	1	0.2101	0.4201	4.9158
0.6	1.1699	0.3297	1	0.1933	0.4833	4.9922
0.7	1.1917	0.3751	1	0.1625	0.5417	5.0669
0.8	1.2123	0.4185	1	0.1192	0.5958	5.1398
0.9	1.2317	0.4601	1	0.0646	0.6461	5.2113
0.999	1.2498	0.4996	1	0.0007	0.6926	5.2806

This example neatly illustrates the nature of the optimal offer. This has two important features: first of all, it entails a distortion from the second-best in that the marginal rate of substitution between price and quality is different between the firm and the regulator. As shown in Proposition 2, this distortion is greater the lower is the value of δ and the larger is the marginal effect of a change in the contractual price on the deviation profits. In this example though, both distortions tend to disappear as the value of δ goes to 1: the more patient is the firm, the smaller are the extra profit necessary to convince it to adhere to the optimal contract and, also, the higher is the gain from the deviation. This result however depends on the possibility to punish the firm with a zero-profit contract in case of deviation: the following example illustrates the relevance of the punishment phase for the level of social welfare the optimal contract is able to deliver.

4.2 Example 2

Demand function is given by $x(p, q) = 2q - qp$. This implies that aggregate consumers' surplus is given by

$$V(p, q) = \frac{1}{2}(2 - p)^2 q \tag{15}$$

The firm's cost function is $c(q, x) = q^2 + \frac{1}{2}x$, so that its profits are given by

$$\pi(p, q) = p\left(2q - qp\right) - q + \frac{1}{2}qp - q^2; \tag{16}$$
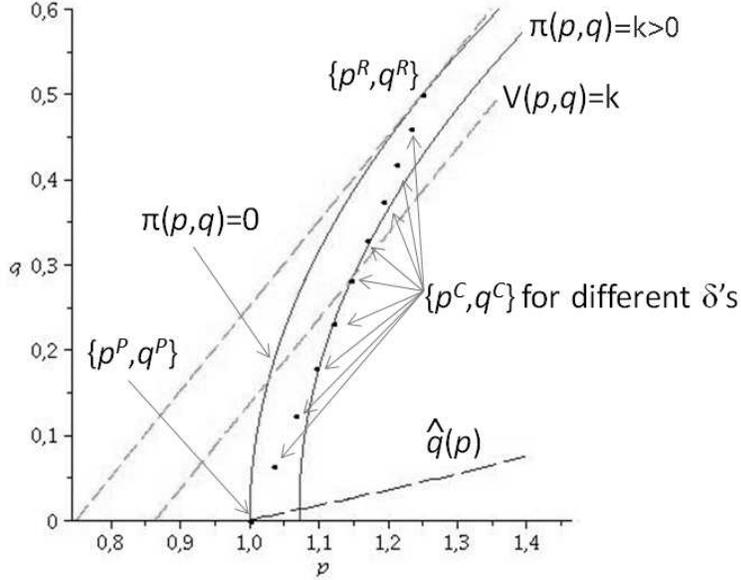
Fig. 3 - Example 1

The monopoly unconstrained price and quality are given by $p^M = \frac{5}{4}$ and $q^M = \frac{9}{32}$, which give profits equal to $\pi^M = \frac{81}{1024}$. For any given price, the optimal quality choice for the firm is

$$\widehat{q}(p) = \frac{5}{4}p - \frac{1}{2}p^2 - \frac{1}{2} \tag{17}$$

By solving the problem (3), we find that the (static) second best price and quality pair is given by

$$p^R = \frac{7}{8} \quad \text{and} \quad q^R = \frac{27}{64}, \tag{18}$$

which gives rise to social welfare equal to $V(p^R, q^R) = \frac{2187}{8129}$ and, clearly, profits equal to zero. As in the previous example, we have to resort to numerical methods, whose results are given in Table 2, identical in its nature to the previous one.

These values are illustrated in Figure 4, where the same notation as in the previous Figure is used.

The main result from Table 2 is that, when the "punishment" brings positive profits to the firm, the second best is never enforceable; as a matter of fact, for any δ, the optimal price and quality are always distorted respectively upward and downward. In particular, it is possible to see that, even thought the firm was infinitely patient ($\delta = 1$) (or the frequency of the

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price revision is sufficiently high), the quality-adjusted Ramsey condition of tangency is satisfied though with a strictly profit and an isowelfare always lower than the second best value. This means that the punishment is not harsh enough to ensure the second best and time-preference adjustment itself cannot eliminate distortions as in the Example 1. Social welfare indeed is always lower than his second best value.

Table 2: Optimal contract in Example 2

δ	p^*	q^*	p^P	$\pi(p^*, q^*)$	$\pi(p^*, \widehat{q}(p^*))$	$V(p^*, q^*)$
0.1	0.9492	0.2695	0.8750	0.05458	0.0557	0.1488
0.2	0.9617	0.2906	0.8750	0.05486	0.0575	0.1566
0.3	0.9693	0.3066	0.8750	0.05429	0.0585	0.1629
0.4	0.9747	0.3201	0.8750	0.05333	0.0592	0.1683
0.5	0.9789	0.3319	0.8750	0.05214	0.0598	0.1730
0.6	0.9823	0.3426	0.8750	0.05079	0.0602	0.1774
0.7	0.9852	0.3524	0.8750	0.04933	0.0606	0.1815
0.8	0.9877	0.3615	0.8750	0.04778	0.0609	0.1852
0.9	0.9898	0.3700	0.8750	0.04617	0.0612	0.1888
0.999	0.9917	0.3780	0.8750	0.04451	0.0614	0.1922

5   Conclusions

This paper tackles the issue of how regulation authorities might induce a regulated firm to provide unverifiable (then ex ante not contractible) quality. Although the recent literature has successfully showed that enforcing a socially optimal level of quality is possible by incorporating some sophisticated instruments, we show that the main problem of unverifiability could be solved even by a standard (easy to apply) fixed-price contract. Our paper defines the optimal fixed-price contract the regulator needs to offer a regulated firm when the quality is endogenous, observable but not verifiable. We suggest that, using the discretionary powers of the regulator and exploiting the repeated nature of the interaction between the regulator and the firm, there exist self-enforcing agreements which may help overcoming the difficulties due to the unverifiable nature of quality. We show that, in an infinitely repeated contractual relation, if the regulator rewards the firm by means of a high regulated price when it delivers a mandated quality level

and punishes it when it deviates from such a level by reducing the regulated price in future periods, the optimal contract improves upon the level of static social optimum. This contract however typically entails distortions from the quality and price of second best, unless the punishment is so harsh to induce zero profit. What this paper predicts in terms of regulation policy is that sufficiently short contracts including harsh punishments could induce the regulated firm to deliver a price and quality combination sufficiently close to the second best.
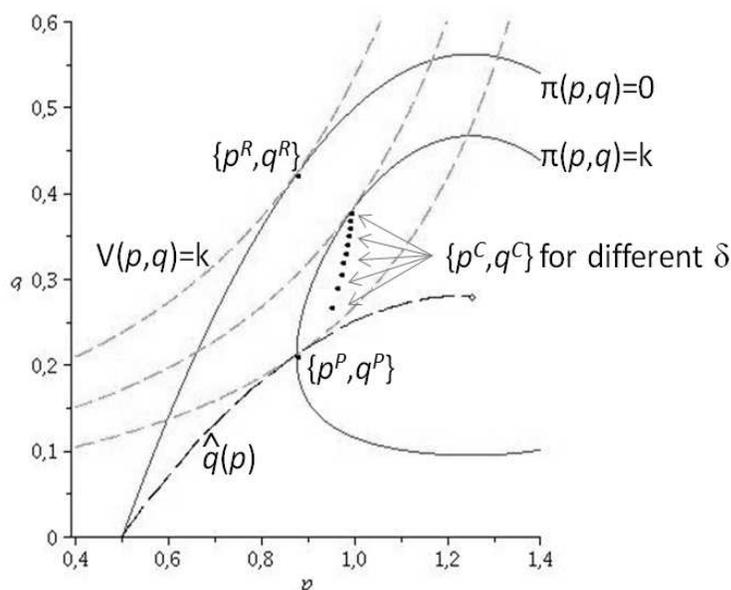

Fig. 4 - Example 2

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