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FISCAL FEDERALISM, INFORMATION AND GLOBAL PUBLIC GOODS WITH INCREASING COST

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Fiscal federalism, information and global public goods with increasing cost

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Abstract

The paper suggests that both the governments' struggle against Mafia and migrants monitoring may be regarded and analysed as public goods characterized by non constant production cost. The model considers a federation of States with two tiers of Government: the central and the local. The public good provision could be attributed to the Centre or to local governments. The cost to produce a given amount of public good is affected by the jurisdiction's type (high or low) and by the quantity of good. The model analyses at first the full information scenario, then the full information hypothesis is relaxed and it is assumed that the Central Government lacks information about the jurisdiction's type. The Centre, aiming at welfare maximization, has to find out the efficient way to redistribute resources among the members of the federation. The information setting deeply affects the outcome; it is shown when and under which conditions a second best outcome is reachable.

Keywords: global public good, asymmetric information, adverse selection, redistribution **JEL Codes:** H21, H41, H70

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1. Introduction¹

Modern fiscal federalism is based on funding schemes intended to induce efficiency in the local governments' provision of public goods through a system of incentives. The goal consists on granting a minimum level for public services locally provided but also in providing incentives for higher level of quality. We refer to a local public good when the economic jurisdiction coincides with the administrative jurisdiction. However when the local public good is characterized by externalities that affect other jurisdictions then we refer to it as "global public good". With reference to the latter we observe that the economic jurisdiction exceeds the administrative one. A global public good may be produced at central or local level but nonetheless it is consumed by all the members of the federation in the same amount.

Global public goods are not infrequent in real world. As an example we may refer to the pollution control or to the scientific research. Perhaps a more interesting example of global public good is represented by the struggle against Mafia. The Mafia phenomenon was at the beginning circumscribed to Sicily, but Mafia economic and criminal interests have spread out and at present it might be considered "global", even trespassing the Italian borders. The struggle against Mafia and other criminal organization is carried on by the Governments' justice. The administration of justice is, according to the Italian legal order, attributed to the central government level. However its provision is regionally produced by courts of law, judges, public prosecutor, and police. On the contrary in Germany the judicial function is attributed to Länder except for the supreme court (Fossati et al. 2000). As well in Switzerland the justice administration has a local character. Everyday public debate submits the discussion about the insufficient funding devoted to the "justice" implying inefficiencies and sub-optimal provision of the public good (quantity and quality below a minimum desirable level). Since the problem of lack of resources devoted to justice varies according to jurisdictions' utility, it emerges that the matter is locally differently perceived and worth. The local level of justice might be considered good enough in some areas whereas in some other it might be considered insufficient in terms of both quality and quantity. According to these empirical evidence it could be taken into account the hypothesis of a unique judicial system that is locally financed and provided. The struggle against Mafia represents a global public good in the sense that the overall quantity consumed by each government (or citizen) equalizes the quantity produced on the whole.

Also the prevention of immigration could be regarded as a pure public good at least within the Schengen area. The Schengen agreement, signed at present by 28 European Countries, requires the removal of systematic controls between the participating Countries. As a consequence, once immigrants enter the Schengen area, regardless to the entering point, they are able to freely move over. Therefore all the Countries members of the Schengen agreement face the same risk regardless to the border that has been violated. The border police service is a good locally provided, but it represents a pure public good inside the Schengen area.

Public goods models are used to explain central government's economic policies, and the literature has shown much interest in examining how decentralized Nash equilibrium might approach Pareto efficiency with appropriate incentive schemes under different information requirements. Even if (Williams 1966) claims that "the complex interactions that occur even in highly simplified situations make it impossible to predict a priori whether undersupply or oversupply will generally result", with perfect information the standard literature assess that when a public good is privately provided, then the level of its provision turns to be at a lower level with respect to the optimal socially desirable one. However, in the context of fiscal federalism income redistribution might be ineffective, since (Warr 1983) shows that the overall level of public good individually supplied might be independent from income redistribution. The neutrality theorem has been originally discussed by (Kemp 1984), which extends the theorem to the

¹ This paper develops the study already presented at the XX Siep Conference (2008); the present analysis focuses on global public goods, whereas the other work referred to local public goods.

case of more than one public good, and by (Bergstrom et al. 1986) which "analyze the extent to which government provision of a public good "crowds out" private contributions". At any rate, the discussion has highlighted that: i) individuals must behave as atomistic utility maximizers, ii) the redistribution of income has to take place among current contributors of the public good, and iii) individuals must face an identical constant prices.

Recent and growing literature on fiscal federalism relates with the implications of information asymmetry when local jurisdictions face different cost for the provision of public good (Cornes and Silva, 2002; Huber and Runkel, 2006). Our model shows a close relation with the work of (Huber and Runkel 2006), but with important differences: first of all we consider what we call here global public goods while they focus their analysis on the local public good, secondly they assume a separable utility function, whereas our analysis is more general and the separability condition is not required. Thirdly, they implicitly assume the same utility function for any jurisdictional type, while in the present paper the utility is allowed to vary from region to region according to the jurisdiction's type (high or low cost).

Hence the immigration control and the struggle against Mafia (this latter with reference to just a few of Countries) are "to all intents and purposes" locally provided public goods. In turn it implies local variable costs that depend on several country-specific parameters. In particular we assume that the cost function which characterizes the production of the global public good depends on a cost parameter (that varies according to the jurisdiction's type) and on the level of good supplied. Thus we adopt a fairly general cost function $e_i = E^i(x_i, \mathcal{G}_i)$ for the public good x, where θ is the cost parameter, i=low(l), high(h) indicates the jurisdiction's type; we account for two types of good: a private and a public one.

We assume that regional utility directly represents the preferences of citizens, since the local governments aim at individualistic utility maximization; central government uses the redistribution of resources among the members of the federation to maximize the social welfare which is given, as usual, by the sum of regional utility.

As far as the informational structure is concerned, the centre knows that there are different types of regions characterized by different cost, income and utility. However, it lacks information concerning the type to which each region belongs. Thus, the central government's key informational problem concerns the regional costs and quantities with regard both to the public and the private good. Indeed we assume that the centre can observe the expenditure levels but neither the costs nor the outputs associated with those expenditure levels.

In comparison with the current literature, the present paper contributes to the topic in two ways. Firstly we are able to highlight the conditions which call for a transfer from the high to the low cost region or vice versa in a fairly general setting, where no particular assumptions on utility and cost functions are required. Secondly, in an asymmetric information setting, we show how central governments might be limited by incentive compatibility constraints.

The paper is organized as follows: in Section 2 the model is presented and the receiving region is identified. In Section 3 asymmetry of information is examined. Finally, in Section 4 some concluding remarks are presented.

2. The model

Suppose an economic federation consisting of two tiers of government: a central government (the State) and a given number of regional governments. We assume that each region provides the following two goods: the private good y and the public good x. The production cost for the private good y is identical among jurisdictions and set, for simplicity, equal to 1. On the other hand the cost for the public good x differs according to the jurisdiction's type. We distinguish between the low cost region's type and the high one, denoting the former by the l index and the latter by the h index. The federation comprises L>1 (l=1...L) number of low cost type identical regions and H>1 (h=1...H) of high cost identical regions. The type $i \in \{l, h\}$ region faces an expenditure cost $E^i(\theta_i, x_i)$ on x which depends and increases both on the quantity of the public good x_i provided, and on the θ_i cost parameter, assuming $\theta_h > \theta_l$. The latter characteristic is rendered explicit by the following derivatives: $E_x^i; E_\theta^i > 0$; $E_{xx}^i; E_{x\theta}^i \ge 0$ (the subscript indicates the variable with respect to which the E cost function has been derived, either at first or second order). The maximization problem that faces the region type $i \in \{l, h\}$ is given by $Max \ u^i = U^i(y_i, X)$, where

 $X = \sum_{s}^{L+H} x^{s}$ subject to the budget constraint $R^{i} + \tau^{i} = y_{i} + E^{i}$, where R^{i} is the region's *i* income and τ^{i} is a lump-sum

transfer (either positive or negative) set by the central government. We adopt standard assumptions for the U(·) function: it is increasing in *y* and *X* and strictly quasiconcave, as well as that all goods are normal. In order to maximize the utility function subject to the budget constraint, the region chooses the amount of *y* and *x* to be provided, so that the correspondent FOCs are²

$$U_x^i = U_y^i E_x^i \text{ or equivalently } -SMS_{x,y}^i = E_y^i$$
$$R^i + \tau^i = y^i + E^i(\mathcal{G}^i, x^i)$$

Using the implicit function theorem, the optimal values³ for x and y can be defined:

$$J^{i}(x^{i}, X_{-i}, y^{i}) = E_{x}^{i} \Longrightarrow \begin{cases} y^{i^{*}} = D_{y}^{i}(\mathcal{G}^{i}, R^{i} + \tau^{i}, X_{-i}) \\ x^{i^{*}} = D_{x}^{i}(\mathcal{G}^{i}, R^{i} + \tau^{i}, X_{-i}) \end{cases}$$
(1)

To let the analysis as simple as possible, let's assume there are just two jurisdictions different in type; the central government maximization problem turns out to be the following:

$$Max_{\tau} W[X, y^{i}, y^{j}] = U^{i}[X, y^{i}] + U^{j}[X, y^{j}]$$

subject to the following constraints:

$$X = x^{i} + x^{j} \qquad (associated lm: \sigma)$$

- budget constraint

$$y' + E'(\vartheta', x') = R' + \tau$$
 (associated lm: λ')

$$y^{j} + E^{j}(\mathcal{G}^{j}, x^{j}) = R^{j} - \tau$$
 (associated lm: λ^{j})

- non negativity constraints

$$x^i, y^i, x^j, y^j \ge 0$$

where *i* and *j* represents the regional type.

² The subscript indicates the derivative with respect to that variable, i.e., for instance $U_x \equiv \partial U(.)/\partial x$

³ Which represent as well the demand function along the optimal path

The standard condition required for efficiency, when a global public good is involved, is as usual:

$$\sum_{s=i}^{j} SMS_{X,y^{s}}^{s} = E_{x^{i}}^{i} = E_{x^{j}}^{j}$$

Only when this condition is met, the social welfare is maximized.

At first full information is assumed. The Central Government can transfer money from one jurisdiction to the other (under the constraint to satisfy the condition for public budget balance) pursuing the social welfare maximization, but, as it will be shown later on, this result cannot be reached even in the complete information scenario⁴.

At first the transfer sign has to be identified. Sticking with the full information hypothesis it emerges that the "necessary and sufficient" information to identify the transfer sign is provided by the marginal utility on good $y(U_y)$ and the marginal expenditure on the public good (E_x) .

A first best is attained when $U_y^i = U_y^j$ and $E_x^i = E_x^j$: if $U_y^i > U_y^j$ or $E_x^i < E_x^j$ then *i* has to be subsidized while *j* taxed, the opposite applies in the case that $U_y^i < U_y^j$ or $E_x^i > E_x^j$. Thus the global public good scenario presents this new insight. The transfer has to equalize at the margin the utility (with respect to the private good) and the expenditure (with respect to the public good) of the two regions. The intuition underlying the condition $E_x^i = E_x^j$ is straightforward: since good *x* is a global public good, then its production has to be set in order to minimize its producing cost, given the optimal amount of public good. In other words the production has to split between the two jurisdictions so to contain as much as possible the overall cost.

But things are more complicated than they appear. In fact, in the case of the global public good the condition required for the welfare maximization does not correspond to that of individualistic (Nash) utility maximization but they sensibly differ: the individual utility maximization requires $SMS_{x,y}^i = E_x^i$, whereas the welfare maximization

requires
$$\sum_{s=i}^{j} SMS_{X,y^s}^s = E_{x^i}^i = E_{x^j}^j$$

The implication is straightforward: when the good x is a global public good (even though locally provided), it is not possible to reach a first best by means of a transfer of money among the jurisdictions, even in presence of perfect information. Obviously a first best might be obtained by imposing the optimal expenditure E_x^{i*} and y_i^{*} , i=l,h to each jurisdiction which is tantamount to say that a Leviathan sets (and forces) the optimal values suppressing the regional autonomy and considering the jurisdictions as a whole with the center.

Giving up with this "first best hypothesis", a second issue should be investigated: assuming a Nash behaviour among the regions, it is still possible to improve the social welfare (even in a second best scenario) by a money transfer among the regions and, in the case it is, what should be the sign of that transfer?

To answer this question the optimal values for *x* and *y* have to be taken into account:

$$x_{i} = \xi^{x_{i}} \left(R_{i}, \theta_{i}, R_{j}, \theta_{j} \right), y_{i} = \xi^{y_{i}} \left(R_{i}, \theta_{i}, R_{j}, \theta_{j} \right)$$

$$(18)$$

From them (see appendix for details) and by differentiating the first order equation system, the values can be

found for the following rate of variation: $\frac{dy_i}{dR_i}$; $\frac{dy_j}{dR_i}$; $\frac{dy_j}{d\theta_i}$; $\frac{dx_i}{d\theta_i}$; $\frac{dx_i}{dR_i}$; $\frac{dx_j}{dR_i}$; $\frac{dx_i}{d\theta_i}$; $\frac{dx_i}{d\theta_i}$; $\frac{dx_j}{d\theta_i}$; $\frac{dx_j}{d\theta_i$

⁴ At least without more assumptions.

It can be shown that a money transfer between the two type of local governments would yield the following ratio in terms of utility change:

$$\frac{dU^{i}}{dU^{j}} = \frac{-U_{y}^{i}\left\{1 + E_{x}^{i}\left(\frac{\partial x_{j}}{\partial R_{i}} - \frac{\partial x_{j}}{\partial R_{j}}\right)\right\}}{U_{y}^{l}\left\{1 + E_{x}^{j}\left(\frac{\partial x_{i}}{\partial R_{j}} - \frac{\partial x_{i}}{\partial R_{i}}\right)\right\}}$$
(19)

The sign of $\frac{dU^n}{dU^l}$ can turn to be greater, lower or equal to zero. The transfer sign is determined according to the

sign of the ratio: if $\frac{dU^h}{dU^l} > 0$ the transfer moves from the high cost region to the low one, if $\frac{dU^h}{dU^l} < 0$ the opposite

applies. The transfer that follows the afore-mentioned rule permits the improvement of the general welfare, assuming the Nash behaviour of local governments.

Another aspect concerning the ratio of eq.19 deserves attention; if the second derivative of the cost function for good *x*, for both types of jurisdictions, is assumed equal to zero ($E_{xx}^{l} = E_{xx}^{h} = 0$), then fixed prices for the public good are implicitly set and eq.19 becomes

details)

It clearly emerges that the ratio $\frac{dU^i}{dU^j}$ will be always positive. This result implies that it is possible to improve

the social welfare transferring money from the high cost region to the low cost one.

To be noted that the term $[E_x^j - E_x^i]$ is positive when the *j* region price for the public good is greater than *i*. In that case both dU^i and dU^j are positive, otherwise both dU^i and dU^j are negative.

This statement shows a very important consequence for the equilibrium. In fact, it turns out that without any central authority intervention, the local jurisdictions find it profitable to autonomously proceed with money transfers from the high cost type to the low cost one. In other words, in the presence of a global public good and linear prices for its production, then the Nash behaviour approaches the social welfare goal. In fact a money transfer increases the utility of the receiver but also that of the donor's. This result coincides with that provided by Buchholz and Konrad (1995).

However, even sticking with the assumption of non constant production cost (i.e., with $E_x^i > 0; E_{xx}^i > 0; E_x^i \neq E_x^j$) the outcome of a autonomous money transfer among governments is still a possible scenario. Let consider the local government *i* indirect utility:

 $V^{i}(e^{i},\tau,x^{j},\mathcal{G}^{i}) = \max U[(R^{i}-\tau-e^{i}),(B(\mathcal{G}^{i},e^{i})+x^{j})]$

where $B(\mathcal{G}^{i}, e^{i}) = x^{i}$ is obtained by inverting the cost function $e^{i} = E^{i}(\mathcal{G}^{i}, x^{i})$, with $B_{e}(\mathcal{G}^{i}, e^{i}) > 0; B_{ee}(\mathcal{G}^{i}, e^{i}) \le 0$.

By the indirect utility it is possible to derive the condition that benefit government i from a autonomous money transfer to j when governments act according to the Nash rule.

Assuming for simplicity a money transfer equal to 1 ($d\tau = 1$) the condition can be written as:

$$dx^{j} > \frac{U_{\tau}^{i} + [U_{e}^{i} - U_{B}^{i}B_{e}]de^{i}}{U_{x^{j}}^{i}}$$

It turns immediately clear that the afore mentioned condition is rarely met, but nonetheless it represents a possible outcome: the local jurisdictions *i* might find it profitable to autonomously proceed with money transfers to *j* so that to improve its own utility and in so doing improving also the other region utility and logically also the social welfare. Even though a social optimum is not reachable by this mechanism, at least a general welfare improvement is observable.

On the opposite assuming both $E_{xx}^{l} = E_{xx}^{h} = 0$ and $E_{x}^{l} = E_{x}^{h}$ (that makes the model to converge to the case in which regions face an identical constant prices) then an income redistribution would be ineffective. According to the Warr (1983) Neutrality Theorem a redistribution among jurisdictions would not affect the overall level of public good individually supplied. Furthermore even the individual consumption of the private good would remain constant regardless to the income redistribution.

3. Global public good and asymmetry of information

In this new scenario we assume that the center knows both the utility and the cost functions of jurisdictions, but information about the quantity provided for the public and the private good is not available. The center observes the expenditure on the private good (y) and the expenditure on the public good (E) that local jurisdictions face, but quantities are unverifiable. To this extent incentive compatible constraints have to be taken into account in the decisional process. The maximization problem for the central government can be set as follows:

$$MaxW[X, y', y'] = U'[X, y'] + U'[X, y']$$

subject to the following constraints:

$$X = x^{i} + x^{j}$$
 (associated lm: σ)

- budget constraint

$$y^{i} + E^{i}(\mathcal{G}^{i}, x^{i}) = R^{i} + \tau \text{ (associated Im: } \lambda^{i})$$
$$y^{j} + E^{j}(\mathcal{G}^{j}, x^{j}) = R^{j} - \tau \text{ (associated Im: } \lambda^{j})$$

- incentive compatible constraints

$$U^{i}[X, y^{i}] \ge U^{i} \left\{ \left(x^{j} + \psi^{i}_{x} [E^{j}(x^{j}, \mathcal{G}^{j}), \mathcal{G}^{i}] \right), y^{j} \right\}_{\mathbb{R}^{i} > \mathbb{R}^{j}} \qquad \text{(associated lm: } \mu^{i} \text{)}$$

$$U^{j}[X, y^{j}] \ge U^{j} \left\{ \left(x^{i} + \psi_{x}^{j}[E^{i}(x^{i}, \mathcal{G}^{i}), \mathcal{G}^{j}] \right), y^{i} \right\}_{R^{j} > R^{i}} \qquad (\text{associated Im: } \mu^{j})$$

$$(22)$$

- non negativity constraints

$$x^{i}, y^{j}, x^{j}, y^{j} \ge 0$$
(23)

Let's assume that the center knows that the sign of the transfer τ has to be set greater than zero: $\tau > 0$. This latter implies that the *j* region is taxed while the *i* region is subsidized. This assumption allows us to set $\mu^i = 0$ given that the first (eq.21) incentive compatibility constraint is not binding. In fact provided that region *i* receives the subsidy while region *j* is taxed, the Lagrange multiplier μ^i assumes the zero value because the *i* region has no advantages to misrepresenting its type declaring to be the other type.

The Central Government knows that the local behaviour meet the Nash rule, and each jurisdiction reacts to the other's behaviour. As a consequence the central government is forced to change the contract terms: both the receiver and the donor will be submitted to audit and forced to show a well defined expenditure on both the private and the public good. Each region will be free to opt for paying the tax τ and show the expenditure E_x^{j} or receive a subsidy equal to τ and show the expenditure E_x^{i} . The afore mentioned drawback directly derives from the equation system provided in eq.18 and the fact that at the individual level utility is maximized setting $SMS_{x,y}^{i} = E_x^{i}$.

The FOCs for this new maximization problem are reported in appendix.

From b.2 and b.3 we derive the crucial information that the center is avoided to reach the equivalence, at the margin, of regions' public good production cost. Since the transfer moves, by assumption, from region j to region i, it means that, before the transfer is set, both the following statement take place:

 $E_{x^i}^i > E_{x^j}^j$ and $U_{y^i}^i < U_{y^j}^j$.

In the complete information scenario the transfer's goal was to meet the conditions:

 $E_{x^{i}}^{i} = E_{x^{j}}^{j}$ and $U_{y^{i}}^{i} = U_{y^{j}}^{j}$.

B.2 and b.3 say that the condition $E_{x^i}^i = E_{x^j}^j$ is not reachable when the information is incomplete. The second best equilibrium requires that the difference between marginal expenditure *E* can only be reduced by the transfer. It unavoidably follows up that the receiving region *i* receives a lower transfer as consequence of the incentive compatibility constraints. Therefore the receiving region will have a lower disposable income and the contributing a higher disposable income with respect to the first best scenario. Denoting by * the first best (perfect information) and by ° the values in the asymmetric information case, then

$$\tau^* > \tau^\circ; \ x^{i^*} > x^{i^\circ}; \ y^{i^*} > y^{i^\circ}; \ x^{j^*} < x^{j^\circ}; \ y^{j^*} < y^{j^*}$$

Another important result is provided by the condition:

$$\frac{U_X^i}{U_{y^i}^i - \mu U_{y^i}^j} + SMS_{X,y^j}^j = E_{x^j}^j$$
(24)

Looking at the first component of the left hand side $(\frac{U_X^i}{U_{y^i}^i - \mu U_{y^i}^j})$, it clearly emerges that it is greater than

 SMS_{X,y^i}^i given that $U_{y^i}^i > U_{y^i}^i - \mu U_{y^i}^j$. This condition obviously diverges from that of first best. The fact that $\frac{U_X^i}{U_{y^i}^i - \mu U_{y^i}^j} > SMS_{X,y^i}^i$ implies a greater value for the overall amount of good X, but this condition is counterbalanced by the fact that the component at the right hand side of eq.24 is lower with respect to that of the

"optimal value", i.e., $E_{x^j}^{j^*} > E_{x^j}^{j^\circ}$ (as showed above).

Any assessment concerning the amount of the global public good X produced is prevented. Intuitively we may predict that when the transfer moves from the high cost region to the low cost one then $X^* > X^\circ$. On the other hand when the transfer has the opposite sign, we might expect $X^* < X^\circ$. In fact in the first case the low cost region receives a lower amount of transfer with respect to the complete information scenario, whereas in the second case the low cost region undergoes a lower taxation, and we know that the low cost region is able to produce the same amount of public good at a lower price with respect to the high cost one.

4. Concluding remarks

The issue addressed concerns the best way to fund and provide pure public goods assuming that their production cost is non constant and depending on a parameter cost that varies according to the jurisdiction type and to the level of good supplied. In particular we refer to governments struggle against Mafia and immigration containment. With reference to the former it is possible to find Countries characterized by a local provision (e.g., Germany and Switzerland) and Countries where the justice is administrate by the centre (e.g., Italy). The control of the illegal Mafia activities represents a pure public good in the Samuelsonian sense, since the overall consumption of it by each Country (or region) is equal to the sum of the overall provision.

The latter refers to the Schengen area. Because of the Schengen Agreement it is possible to consider all the countries members of the treaty as a whole, but its borders are controlled by single member Sates. The control activity of immigration carried on by each border's State affects the utility of all the members of the Schengen Agreement. The borders' control in order to avoid immigration (that is a public good) is locally provided but equally consumed inside the Schengen Area.

The afore-mentioned public goods show variable costs in their production depending on both the regional characteristic and the quantity produced.

Aiming at welfare maximization, what is the best option to be adopted between central and local for their provision? The answer is not that neat. The central provision seems to be (at first glance) Pareto superior. In fact, from a theoretical point of view it could be possible to reach a first best outcome. However this result is obtainable under the strong assumption of full information with reference to individual preferences, cost function and income.

The Oates' decentralization theorem (Oates 1972) suggests that central provision is subject to the condition of a uniform provision (among all the local jurisdictions) of the public good. Obviously in the present model the uniform provision has to be intended in terms of expenditure rather than quantity. Because of that unnecessary constraint a central provision can't be anything else than a second best. A trivial objection is that nationally provided services do not necessarily have to be standardized, but nonetheless is clear that central provision is subject (in practice) to a wider number of constraints with respect to local provision. Another support for local provision is provided by (Stigler 1957) which suggests as justification the asymmetric information that characterizes the market. Hence Stigler assumes that the central government knows local preferences less precisely (with a random error) with respect to the local governments. As a consequence the centre decision rule results biased implying either over or under provision of the public good. In our opinion the Stigler's information assumption well fit the local private information with respect both to the production cost and preferences.

Unfortunately, with non constant production cost, even the local provision of public good shows drawbacks. In particular, as it is proven in this paper, a first best outcome in the production of a global public good is unreachable even under perfect information when the local governments behave according to the Nash rule. However a money redistribution may allow for social welfare improvement, under the condition that the redistribution follows the rule provided. In the limiting case of constant and equal price for the public good, (Warr 1983) showed the ineffectiveness of any redistribution policy (among those jurisdictions that voluntarily contribute to the provision of public good). On the other hand, assuming constant prices that vary among jurisdictions, we find, as expected, the result suggested by (Buchholz and Konrad 1995): the Nash behaviour approaches the social welfare goal, given that the jurisdiction with a low productivity has an incentive to make large unconditional transfers to the other jurisdiction.

Moving back to our assumption of a non constant price for the public good it could emerge the quite unexpected result that a Nash voluntary transfer among jurisdictions (and the consequent social welfare improvement) is still a possible scenario. The necessary condition (presented at the end of section 2) might be difficult to be met, but nonetheless it would allow for a social welfare improvement without requiring any central authority intervention (recalling however that, by an autonomous governments' transfer, only a welfare improvement, and not its maximization, is hopefully the expected result). A Nash voluntary transfer takes place when the local government income reduction, which in turn implies a loss in terms of utility, is more than compensated by a utility gain originated by the overall public good provision. Individuals' cross elasticities of marginal utility with respect to income and expenditure make it possible.

If we move to asymmetry of information, it follows that the central government is forced to pose very strict conditions in order to render the contract enforceable and get a social welfare improvement (still in a second best scenario). Both the receiver and the donor have to be submitted to audit and forced to show a well defined expenditure on both the private and the public good.

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Appendix 1

Central Government maximization problem with full information $MaxW[X, y^i, y^j] = U^i[X, y^i] + U^j[X, y^j]$

subject to the following constraints:

$$\begin{split} X &= x^{i} + x^{j} & \text{(associated lm: } \sigma \text{)} \\ \text{- budget constraint} & \\ y^{i} &+ E^{i}(\mathcal{G}^{i}, x^{i}) = R^{i} + \tau & \text{(associated lm: } \lambda^{i} \text{)} \\ y^{j} &+ E^{j}(\mathcal{G}^{j}, x^{j}) = R^{j} - \tau & \text{(associated lm: } \lambda^{j} \text{)} \\ \text{- non negativity constraints} & \\ x^{i}, y^{i}, x^{j}, y^{j} \geq 0 \end{split}$$

where *i* and *j* represents the regional type.

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 $\overline{U_x^i} + U_x^j + \sigma = 0 \tag{a.1}$

$$-\sigma - \lambda^{i} E_{x^{i}}^{i} \leq 0, \ x^{i} \geq 0, \ x^{i} (\frac{\partial L}{\partial x^{i}}) = 0$$
(a.2)

$$-\sigma - \lambda^{j} E_{x^{j}}^{j} \le 0, \ x^{j} \ge 0, \ x^{j} (\frac{\partial L}{\partial x^{j}}) = 0$$
(a.3)

$$U_{y^{i}}^{i} - \lambda^{i} \leq 0, \ y^{i} \geq 0, \ y^{i} (\frac{\partial L}{\partial y^{i}}) = 0$$
(a.4)

$$U_{y^{j}}^{j} - \lambda^{j} \leq 0, \ y^{j} \geq 0, \ y^{j} (\frac{\partial L}{\partial y^{j}}) = 0$$
(a.5)

$$\lambda^i - \lambda^j = 0 \tag{a.6}$$

$$y^{i} + E^{i}(\mathcal{G}^{i}, x^{i}) - R^{i} - \tau = 0$$
 (a.7)

$$y^{j} + E^{j}(\mathcal{G}^{j}, x^{j}) - R^{j} + \tau = 0$$
(a.8)

Appendix 2

From the first order conditions we derive the best reply function for the two type of jurisdictions. Solving the simultaneous system of equations so determined, it is possible to obtain the Nash (general) equilibrium values:

$$\begin{cases} x_i = \xi^{x_i} (R_i, \theta_i, R_j, \theta_j) \\ y_i = \xi^{y_i} (R_i, \theta_i, R_j, \theta_j) \\ x_j = \xi^{x_j} (R_i, \theta_i, R_j, \theta_j) \\ y_j = \xi^{y_j} (R_i, \theta_i, R_j, \theta_j) \end{cases}$$

differentiating the system of first order conditions⁵ we get: $\left(\left(\begin{array}{c} \hline \\ \hline \\ \end{array} \right) \right) = \left[\begin{array}{c} \hline \\ \hline \\ \end{array} \right]$

$$\begin{cases} \left\{ \frac{\partial \left[\frac{U_x^i}{U_y^i} \right]}{\partial x_i} - E_{xx}^i \right\} dx_i + \frac{\partial \left[\frac{U_x^i}{U_y^j} \right]}{\partial x_j} dx_j + \frac{\partial \left[\frac{U_x^i}{U_y^j} \right]}{\partial y_i} dy_i = E_{x\theta}^i d\theta_i \\ \left\{ \frac{\partial \left[\frac{U_x^j}{U_y^j} \right]}{\partial x_i} dx_i + \left\{ \frac{\partial \left[\frac{U_x^j}{U_y^j} \right]}{\partial x_j} - E_{xx}^j \right\} dx_j + \frac{\partial \left[\frac{U_x^j}{U_y^j} \right]}{\partial y_i} dy_j = E_{x\theta}^j d\theta_j \\ E_x^i dx_i + dy_i = dR_i - E_{\theta}^i d\theta_i \\ E_x^j dx_j + dy_j = dR_j - E_{\theta}^j d\theta_j \end{cases} \end{cases}$$

Defining:

Defining:

$$A = \frac{\partial \left[\frac{U_x^i}{U_y^i}\right]}{\partial x_i} = \frac{\partial \left[\frac{U_x^i}{U_y^j}\right]}{\partial x_j} = \frac{U_{xx}^i U_y^i - U_x^i U_{yx}^j}{\left[U_y^i\right]^2} < 0 \quad \& \quad B = \frac{\partial \left[\frac{U_x^j}{U_y^j}\right]}{\partial x_i} = \frac{\partial \left[\frac{U_x^j}{U_y^j}\right]}{\partial x_j} = \frac{U_{xx}^j U_y^j - U_x^j U_{yx}^j}{\left[U_y^j\right]^2} < 0$$

$$C_i = \frac{\partial \left[\frac{U_x^i}{U_y^j}\right]}{\partial y_i} = \frac{U_{xy}^i U_y^i - U_x^i U_{yy}^i}{\left[U_y^i\right]^2} > 0 \quad \& \quad C_j = \frac{\partial \left[\frac{U_x^j}{U_y^j}\right]}{\partial y_j} = \frac{U_{xy}^j U_y^j - U_x^j U_{yy}^j}{\left[U_y^j\right]^2} > 0$$
we obtain the matrix:

$$H = \begin{vmatrix} A - E_{xx}^{i} & A & C^{i} & 0 \\ B & B - E_{xx}^{j} & 0 & C^{j} \\ E_{x}^{i} & 0 & 1 & 0 \\ 0 & E_{x}^{j} & 0 & 1 \end{vmatrix}$$

$$\begin{cases} \frac{U_{x}^{i}}{U_{y}^{j}} - E_{x}^{i} = 0\\ \frac{U_{x}^{j}}{U_{y}^{j}} - E_{x}^{j} = 0\\ y_{i} + E^{i} = R_{i}\\ y_{j} + E^{j} = R_{j} \end{cases}$$

the determinant of which is: $|H| = -E_x^i C_i (B - E_{xx}^i) + (A - E_{xx}^i)(B - E_{xx}^j) - AB - E_x^j C_j (A - E_{xx}^i - C_i E_x^i) > 0$

$$\begin{aligned} \frac{dx_i}{dR_i} &= -\frac{C_i}{|H|} (B - E_{xx}^j - C_j E_x^j) > 0; \\ \frac{dx_i}{dR_j} &= \frac{C_j}{|H|} A < 0; \\ \frac{dx_j}{dR_i} &= -\frac{C_j}{|H|} (A - E_{xx}^i - C_i E_x^i) > 0; \\ \frac{dy_i}{dR_i} &= -\frac{AE_{xx}^j + E_{xx}^i (B - E_{xx}^j)}{|H|} - \frac{C_j E_x^j (A - E_{xx}^i)}{|H|} > 0; \\ \frac{dy_i}{dR_j} &= -\frac{C_j}{|H|} A E_x^i > 0 \end{aligned}$$

Let's first assume that a income variation in region *i* occurs. a) $dR_i \neq 0 \& dR_i = 0$

$$dU^{i} = \left[U_{x}^{i} (\frac{dx_{i}}{dR_{i}} + \frac{dx_{j}}{dR_{i}}) + U_{y}^{i} \frac{dy_{i}}{dR_{i}} \right] dR_{i} = \left[U_{y}^{i} E_{x}^{i} (\frac{dx_{i}}{dR_{i}} + \frac{dx_{j}}{dR_{i}}) + U_{y}^{i} \frac{dy_{i}}{dR_{i}} \right] dR_{i} = U_{y}^{i} \left[E_{x}^{i} (\frac{dx_{i}}{dR_{i}} + \frac{dx_{j}}{dR_{i}}) + \frac{dy_{i}}{dR_{i}} \right] dR_{i}$$

$$dU^{i} = U_{y}^{i} \left[\left(E_{x}^{i} \frac{dx_{i}}{dR_{i}} + \frac{dy_{i}}{dR_{i}} \right) + E_{x}^{i} \frac{dx_{j}}{dR_{i}} \right] dR_{i} \Rightarrow$$

$$E_{x}^{i} \frac{dx_{i}}{dR_{i}} + \frac{dy_{i}}{dR_{i}} = \left[-E_{x}^{i} \frac{C_{i}}{|H|} (B - E_{xx}^{j} - C_{j} E_{x}^{j}) \right] + \left[-\frac{AE_{xx}^{j} + E_{xx}^{i} (B - E_{xx}^{j})}{|H|} - \frac{C_{j} E_{x}^{j} (A - E_{xx}^{i})}{|H|} - \frac{C_{j} E_{x}^{j} (A - E_{xx}^{i})}{|H|} \right] =$$

$$= \frac{-E_{x}^{i} C_{i} (B - E_{xx}^{j} - C_{j} E_{x}^{j}) - AE_{xx}^{j} - E_{xx}^{i} (B - E_{xx}^{j}) - C_{j} E_{xx}^{j} (A - E_{xx}^{i})}{|H|} = 1$$

$$= \frac{-E_{x}^{i} C_{i} (B - E_{xx}^{j} - C_{j} E_{x}^{j}) - AE_{xx}^{j} - E_{xx}^{i} (B - E_{xx}^{j}) - C_{j} E_{xx}^{j} (A - E_{xx}^{i})}{|H|} = 1$$

$$= \frac{-E_{x}^{i} C_{i} (B - E_{xx}^{j} - C_{j} E_{x}^{j}) - AE_{xx}^{j} - E_{xx}^{i} (B - E_{xx}^{j}) - C_{j} E_{xx}^{j} (A - E_{xx}^{i})}{|H|} = 1$$

$$= \frac{-E_{x}^{i} C_{i} (B - E_{xx}^{j} - C_{j} E_{x}^{j}) - AE_{xx}^{j} - E_{xx}^{i} (B - E_{xx}^{j}) - C_{j} E_{xx}^{j} (A - E_{xx}^{i})}{|H|} = 1$$

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 $\frac{dX}{dR_i} = \frac{dx_i}{dR_i} + \frac{dx_j}{dR_i} = -\frac{C_i}{|H|} (B - E_{xx}^j - C_j E_x^j) + \frac{C_i}{|H|} B = \frac{C_i}{|H|} (E_{xx}^j + C_j E_x^j) > 0, \text{ and also knowing that}$ $\frac{dy_i}{dR_i} > 0, \text{ then } \frac{dU^i}{dR_i} > 0, \text{ and therefore } 1 > -E_x^i \frac{dx_j}{dR_i}. \text{ Which in turn implies that:}$ $\frac{dU^i}{dR_i} = U_y^i (1 + E_x^i \frac{dx_j}{dR_i}) > 0$

Assuming now that only region *j* faces a income variation:

b)
$$d\mathbf{R}_{i} = 0 \& d\mathbf{R}_{j} \neq 0$$

 $dU^{i} = U_{y}^{i} \left[E_{x}^{i} \left(\frac{dx_{i}}{dR_{j}} + \frac{dx_{j}}{dR_{j}} \right) + \frac{dy_{i}}{dR_{j}} \right] dR_{j} = U_{y}^{i} \left[E_{x}^{i} \frac{dx_{i}}{dR_{j}} + \frac{dy_{i}}{dR_{j}} \right] dR_{j} + U_{y}^{i} E_{x}^{i} \frac{dx_{j}}{dR_{j}} dR_{j}$
 $E_{x}^{i} \frac{dx_{i}}{dR_{j}} + \frac{dy_{i}}{dR_{j}} = \left[E_{x}^{i} \frac{C_{j}}{|H|} B \right] \left[-\frac{C_{j}}{|H|} BE_{x}^{i} \right] = 0$
 $\frac{dU^{i}}{dR_{j}} = U_{y}^{i} E_{x}^{i} \frac{dx_{j}}{dR_{j}} > 0$

Similarly to the previous case, let's define the region's *j* utility variation:

$$\frac{dU^{j}}{dR_{j}} = U_{y}^{j} \left(1 + E_{x}^{j} \frac{dx_{i}}{dR_{j}}\right) > 0 \quad \& \quad 1 > -E_{x}^{j} \frac{dx_{i}}{dR_{j}}$$

whereas assuming $dR_{i} \neq 0 \& dR_{j} = 0$
$$\frac{dU^{j}}{dR_{i}} = U_{y}^{j} E_{x}^{j} \frac{dx_{i}}{dR_{i}} > 0$$

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Considering the utility variation for both regions when a money transfer from *i* to *j* occurs:

We assume that region *i* is characterized by higher cost (with respect to region *j*) in providing the public good; i.e., $E_x^i > E_x^j$; $E_{xx}^i > E_{xx}^j$. When a transfer between the two regions occurs, it is tantamount to say that both regions face a income variation, equal in absolute value, but different in sign: dR=dR_i=-dR_j>0. Let's first assume a income variation in region *i*:

The utility of *j* varies accordingly:
$$dU^{j} = \left[U_{y}^{j} \left(1 + E_{x}^{j} \frac{dx_{i}}{dR_{j}} \right) \right] dR_{j} + \left[U_{y}^{j} E_{x}^{j} \frac{dx_{i}}{dR_{i}} \right] dR_{i}.$$

Thus:
$$dU^{j} = U_{y}^{j} \left[1 + E_{x}^{j} \frac{dx_{i}}{dR_{j}} - E_{x}^{j} \frac{dx_{i}}{dR_{i}} \right] dR \Rightarrow dU^{j} = U_{y}^{j} \left[1 + E_{x}^{j} \left(\frac{dx_{i}}{dR_{j}} - \frac{dx_{i}}{dR_{i}} \right) \right] dR$$

We know that $\frac{dx_i}{dR_j} - \frac{dx_i}{dR_i} < 0$. Thus a money transfer from *i* to *j* will cause a utility increase to the *j* region if:

$$U_{y}^{j} + U_{y}^{j} E_{x}^{j} \left(\frac{dx_{i}}{dR_{j}} - \frac{dx_{i}}{dR_{i}} \right) > 0 \implies 1 > E_{x}^{j} \left(\frac{dx_{i}}{dR_{i}} - \frac{dx_{i}}{dR_{j}} \right) > 0, \text{ or equivalently if: } E_{x}^{j} < \frac{1}{\frac{dx_{i}}{dR_{i}} - \frac{dx_{i}}{dR_{i}}}$$

A money transfer from *i* to *j* is suitable to produce a utility increase in *j* when the utility increase in the latter (region *j*) originated by the larger consumption of the private good *y* is greater than the utility loss consequent to the diminished contribution in the public good X by region *i*

Similarly
$$dU^{i} = -\left[U_{y}^{i}\left(1 + E_{x}^{i}\frac{dx_{j}}{dR_{i}}\right)\right]dR + \left[U_{y}^{i}E_{x}^{i}\frac{dx_{j}}{dR_{j}}\right]dR$$

 $dU^{i} = -U_{y}^{i}\left[1 + E_{x}^{i}\left(\frac{dx_{j}}{dR_{i}} - \frac{dx_{j}}{dR_{j}}\right)\right]dR$,
but considering that $\frac{dx_{j}}{dR_{i}} - \frac{dx_{j}}{dR_{j}} < 0$,

we may conclude that a money transfer from i to j produces a utility increase for region i when:

$$1 + E_x^i \left(\frac{dx_j}{dR_i} - \frac{dx_j}{dR_j} \right) < 0, \text{ that is equivalent to say:} \quad 1 < E_x^i \left(\frac{dx_j}{dR_j} - \frac{dx_j}{dR_i} \right)$$
$$E_x^i > \frac{1}{\frac{dx_j}{dR_i} - \frac{dx_j}{dR_i}}.$$

Thus, to sum up, a money transfer from a region to the other determines a ratio of utility variation equal to:

$$\frac{dU^{j}}{dU^{i}} = -\frac{U_{y}^{i}}{U_{y}^{b}} \frac{1 + E_{x}^{j} \left(\frac{dx_{i}}{dR_{j}} - \frac{dx_{i}}{dR_{i}}\right)}{1 + E_{x}^{a} \left(\frac{dx_{j}}{dR_{i}} - \frac{dx_{j}}{dR_{j}}\right)}$$

Appendix 3

Central Government maximization problem with asymmetric information: focs $U^{i} + U^{j} + \sigma + \mu U^{j} = 0$

$$U_{x}^{i} + U_{x}^{j} + \sigma + \mu U_{x^{j}}^{j} = 0$$
(b.1)

$$-\sigma - \lambda^{i} E_{x^{i}}^{i} - \mu [U_{x^{i}}^{j} + U_{\psi^{j}}^{j} \psi_{e^{i}}^{j} E_{x^{i}}^{i}] \le 0, \ x^{i} \ge 0, \ x^{i} (\frac{\partial L}{\partial x^{i}}) = 0$$
(b.2)

$$-\sigma - \lambda^{j} E_{x^{j}}^{j} \le 0, \ x^{j} \ge 0, \ x^{j} (\frac{\partial L}{\partial x^{j}}) = 0$$
(b.3)

$$U_{y^{i}}^{i} - \lambda^{i} - \mu U_{y^{i}}^{j} \le 0, \ y^{i} \ge 0, \ y^{i} (\frac{\partial L}{\partial y^{i}}) = 0$$
(b.4)

$$U_{y^{j}}^{j} - \lambda^{j} + \mu^{j} U_{y^{j}}^{j} \le 0, \ y^{j} \ge 0, \ y^{j} (\frac{\partial L}{\partial y^{j}}) = 0$$
 (b.5)

$$\lambda^i - \lambda^j = 0 \tag{b.6}$$

$$y^{i} + E^{i}(\mathcal{G}^{i}, x^{i}) - R^{i} - \tau = 0$$
 (b.7)

$$y^{j} + E^{j}(\mathcal{G}^{j}, x^{j}) - R^{j} + \tau = 0$$
(b.8)

$$U^{j}[X, y^{j}] - U^{j}\left\{\!\left(\!x^{i} + \psi_{x}^{j}[E^{i}(x^{i}, \mathcal{G}^{i}), \mathcal{G}^{j}]\right)\!, y^{i}\right\} \ge 0, \ \mu^{j} \ge 0, \ \mu^{j}(\frac{\partial L}{\partial \mu^{j}}) = 0$$
(b.9)