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NON-ANONYMOUS PRO-POOR GROWTH: A CHARACTERIZATION

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Non-anonymous pro-poor growth: a characterization

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PRELIMINARY DRAFT

1 Introduction

In this paper we focus on the measurement of non-anonymous pro-poor growth.

Eventful days rapidly following each other, such as the different phases of the recent economic crisis, have motivated a renewed and increasing interest, among economists and policy makers, on the issue of the measurement of growth and its distributional implications. This issue has been vastly studied in the economic literature and a particular branch has been developed in order to focus on the pro-poorness of growth (see Ferreira, 2010).

The Growth Incidence Curve (GIC), measuring the quantile-specific rate of economic growth in a given period of time (Ravallion and Chen, 2003; Son 2004), and its cumulative specification represent standard tools in evaluating growth from a distributional viewpoint. Building on the GIC, a variety of indices for the measurement of pro-poor growth have been proposed (see Grosse et al., 2008; Kakwani and Son, 2008; Kraay, 2006; Kakwani and Pernia, 2000; Essama-Nssah, 2005; Essama-Nssah and Lambert, 2009).

This literature has been recently criticized as being unable to capture the welfare effect of growth: it in fact compares the income levels of individuals in the same positions at time t and $t+1$; but these could be (and typically will be) different individuals. Therefore it is unable to track the (welfare relevant) individual movements from period t to period $t+1$. To capture this relevant effect, an increasing number of works has recently endorsed a non-anonymous approach to the evaluation of growth (Grimm, 2007; Bourguignon, 2011; Jenkins and Van Kerm, 2011; Palmisano and Peragine, 2011). These works provide formal derivations of partial dominance conditions that can be used to rank growth processes, but they do not characterize any synthetic index of non-anonymous pro-poor growth.

Hence, the aim of this paper is to propose an axiomatic characterization of an aggregate measure of non-anonymous pro-poor growth, that is, an approach taking into account the inequalities in the initial distribution of income. From a formal point of view, the contribution of our work to the existing literature is twofold. The first is to provide a unifying framework for the derivation of an absolute and a relative measure of individual growth. The second is represented by the aggregation procedure which represents a generalization of existing measures of growth.

With regard to the first aspect, we axiomatize two directional measures of income growth. Both measures satisfy Normalization, Monotonicity and Independence. Normalization and Monotonicity

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are common properties in the literature: the former implies that the index is equal to 0 if the initial and final level of income are the same; the latter implies that growth is increasing in second period incomes. The independence condition, instead, is a new property in this literature. It requires that adding a given amount of income to two individuals with the same initial level of income (but possibly different levels of second period incomes) affects their individual growth rates by the same amount. This independence condition is natural in the present context, as it implies that different effects of additions to second period income on individual growth can only be due to differences in initial income levels. Moreover, it enables us to obtain a unifying characterization for a relative and an absolute measure of individual growth. Standard Scale Invariance and Translation Invariance are then introduced to obtain the specific functional form of each measure of individual growth.

With regard to the second aspect, our aggregation procedure is closest to that proposed by Demuynck and Van de gaer (2011). They characterize a family of inequality adjusted growth imposing, in addition to Weak Decomposability, Decomposability with respect to Highest Growth (D-HG). D-HG implies that the income growth of the overall society, composed by n individuals, depends on the income growth of the group of the $n-1$ most growing individuals and the growth of the least growing individual. The measure of aggregate growth obtained is more sensitive to the increment of growth for individuals with lower individual growth than for individuals with higher individual growth, expressing, therefore, aversion toward inequality in the distribution of individual income growth.

We instead characterize an aggregation procedure imposing Rank Dependent Monotonicity (RDM), which allows to express aggregate growth as a function of the magnitude of growth experienced by each individual and the identity of that individual, which in our framework is given by his position in the initial distribution of income. We then impose Recursive Decomposability (RD), which implies that an overall measure of growth will not be sensitive to the order of aggregation. This property represents a new contribution in the literature on growth measurement. It is indeed a weak condition which turns out to be satisfied by most aggregate measures of growth. It requires that letting aggregate growth only depend on the aggregate growth of the $n-1$ group of initially poorest and on the growth of the initially richest individual is equivalent to letting aggregate growth depend only on the growth of the initially poorest individual and on the aggregate growth of the $n-1$ group of the initially richest. In addition, this property ensures that every group of the initially richest (or poorest) income recipients is strictly separable from anyone who is initially poorer (or richer). Hence aggregate growth is sensitive to both the groups of the initially poorest and richest individuals, independently from their position in the distribution of individual growth.

Further imposing two normative axioms, Population Invariance and Non-Anonymous Pro-Poor Growth, we obtain an aggregate index of non-anonymous pro-poor growth. It is expressed as a weighted average of the individual growth measures whose weights are decreasing with the rank in the initial distribution of income.

It turns out that through our model it is possible to frame the analytical measurement of growth within the logic of social evaluation. The individual measure of growth represents a measure of variation of his/her economic welfare over time. The aggregate measure instead, being defined as a weighted sum of the individual growth measures with weights expressing a concern toward the initially poorest, can be interpreted as a welfare evaluation of growth, that is how aggregate growth contributes to improve (worsen) social welfare, where the social planner endorses monotonic and inequality adverse preferences for income.

We apply our theoretical framework to compare different consecutive growth processes that took place in Italy from 2000 against the growth process 2008-2010. The focus on 2008-2010 stems from the observation that it is the period during which the first wave of the crisis took place. We show that the effect of the recent economic crisis has been worse for the initially poorest individuals than for those initially richest. By comparing the results with those obtained adopting existing frameworks, we also show the applied relevance of our theoretical model.

The paper is organized as follows. In section 2 we introduce the general notation and present our theoretical results. In section 3 we provide the empirical illustration. In section 4 we conclude.

2 The framework

Embracing a non-anonymous approach to the evaluation of growth, in this section we characterize two individual measures of growth and the aggregation of these measures into an societal index of non-anonymous pro-poor growth. We start by defining the notation we will use throughout this paper.

We consider a set of individuals $\{1, ..., n\}$ and the following distributions of income. Let

$$\mathbf{x} = (x_1, ..., x_i, ..., x_n) \in \mathbb{R}^n_{++}$$

be the initial distribution of income. We assume that individual incomes are ordered increasingly: $x_1 \leq ... \leq x_i \leq ... \leq x_n$. Thus, x_1 is the income level of the poorest individual in the initial distribution. Let

$$\mathbf{w} = (w_1, ..., w_i, ..., w_n) \in \mathbb{R}^n_{++}$$

be the final distribution of income, where individuals are ordered according to their position in the initial distribution of income: w_i represents the final income of the individual with position i in the initial distribution of income. Following the perspective of the literature on non-anonymous growth, our aim is to characterize an index $G^n(\mathbf{x}, \mathbf{w}) : D^n \longrightarrow \mathbb{R}$, where

$$D^n = \left\{ (\mathbf{x}, \mathbf{w}) \in \mathbb{R}^n_{++} \text{ such that } x_1 \leq ... \leq x_i \leq ... \leq x_n \right\},$$

and G^n is a non constant function that measures aggregate growth, with special case G^1 measuring the growth experienced by an individual, for which the domain D^1 reduces to $\mathbb{R}^2_{++}$. We first characterize a measure of individual growth. Next we turn to the aggregation of these individual growth measures.

2.1 Individual growth

We propose a relative and an absolute measure of growth. There are good reasons to use either of both measures, a discussion on their pros and cons for a measurement of growth in a non-anonymous context is outside the scope of this work[1].

Three axioms will be used to characterize both a relative and an absolute measure of individual growth. The first is a standard normalization axiom.

N (Normalization): For all $x \in \mathbb{R}_{++} : G^1(x, x) = 0$.

The normalization axiom simply requires that a measure of individual growth should be equal to 0 if the individual does not experience any variation in her level of income.

[1] For a detailed analysis on this issue see Kolm (1976a,b), Atkinson and Brandolini (2010), Ravallion (2004).

The second is a trivial monotonicity axiom: growth is increasing in second period incomes.

M (Monotonicity): For all $x, w, z \in \mathbb{R}_{++} : w > z \Longrightarrow G^1(x, w) > G^1(x, z)$.

The third is an independence condition: for individuals having the same initial level of income, increasing second period incomes changes growth by the same amount, no matter what the original second period level of income is. While **N** and **M** are common axioms in the literature, this axiom is new. It is a relevant axiom since it will be used to characterize both the relative and the absolute measure in a unifying framework.

IND (Independence): For all $x, w, z \in \mathbb{R}_{++}$ and $\theta > 0$:

$$G^1(x, w + \theta) - G^1(x, w) = G^1(x, z + \theta) - G^1(x, z).$$

This axiom will be used to cardinalize the ordering in such a way that it becomes a linear function of second period incomes. Axiom **N** provides further restrictions on the cardinalization.

2.1.1 A measure of relative growth

As is standard, measures of relative growth are scale invariant measures. Formally

SI (Scale Invariance): For all $\lambda > 0$ and all $x, w \in \mathbb{R}_{++}$:

$$G^1(\lambda x, \lambda w) = G^1(x, w).$$

Imposing scale invariance means that the measure of individual growth is not affected by an equiproportional change in the initial and final level of income.

It is easy to obtain the following lemma.

Lemma 1: For all $x, v, w, z \in \mathbb{R}_{++}$ the measure satisfies SI and M if and only if

$$G^1(x, w) > G^1(z, v) \Leftrightarrow \frac{w}{x} > \frac{v}{z}.$$

Proof. The proof is simple: first apply SI, define the function $\hat{G}^1(x) = G^1(1, x)$ and then apply M, to get

$$G^1(x, w) > G^1(z, v) \Leftrightarrow G^1\left(1, \frac{w}{x}\right) > G^1\left(1, \frac{z}{v}\right) \Leftrightarrow \hat{G}^1\left(\frac{w}{x}\right) > \hat{G}^1\left(\frac{z}{v}\right) \Leftrightarrow \frac{w}{x} > \frac{v}{z}.$$

as stated in the lemma. □

The axioms N and IND are used to cardinalize this ordering, yielding the following.

Proposition 1: A growth measure $G^{1R}(x, w)$ satisfies SI, M, N and IND if and only if there exists $\beta > 0$ such that

$$G^{1R}(x, w) = \beta \frac{(w - x)}{x}.$$

Proof. From IND and the definition of the function $\hat{G}^1(x)$, we get

$$\hat{G}^1\left(\frac{w+\theta}{x}\right)-\hat{G}^1\left(\frac{w}{x}\right)=\hat{G}^1\left(\frac{z+\theta}{x}\right)-\hat{G}^1\left(\frac{z}{x}\right).$$

With a trivial redefinition of variables, this becomes that for all a, b and $c \in \mathbb{R}_{++}$,

$$\hat{G}^1(a+c)-\hat{G}^1(a)=\hat{G}^1(b+c)-\hat{G}^1(b),$$

which implies that the function $\hat{G}^1$ must be linear: $\hat{G}^1(x)=\alpha+\beta x$, such that

$$G^1(x,w)=\hat{G}^1\left(\frac{w}{x}\right)=\alpha+\beta\frac{w}{x}.$$

Due to N, we get $\alpha=-\beta$, and from M, $\beta>0$ from which the result follows. □

Proposition 1 characterizes a standard measure of individual growth: the proportionate difference between the final period income and the initial period income.

2.1.2 A measure of absolute growth

As is standard, measures of absolute growth satisfy translation invariance.

TI (Translation Invariance): For all $\theta>0$ and all $x, w \in \mathbb{R}_{++}$:

$$G^1(x+\theta, w+\theta)=G^1(x,w).$$

TI states that the value of the function G^1 does not change if the same amount of income is added to both initial and final income.

It is easy to obtain the following lemma.

Lemma 2: For all $x, v, w, z \in \mathbb{R}_{++}$ the measure satisfies TI and M if and only if

$$G^1(x,w)>G^1(z,v) \Leftrightarrow w-x>v-z.$$

Proof. The proof is simple.

(i) If $w>x$ and $v>z$, apply TI, define the function $\tilde{G}^1(x)=G^1(0,x)$ and then apply M, to get

$$G^1(x,w)>G^1(z,v) \Leftrightarrow G^1(0,w-x)>G^1(0,v-z) \Leftrightarrow \tilde{G}^1(w-x)>\tilde{G}^1(v-z) \Leftrightarrow w-x>v-z.$$

(ii) If $w<x$ and $v<z$, apply TI, define the function $\tilde{\tilde{G}}^1(x)=G^1(x,0)$ and then apply M, to get

$$G^1(x,w)>G^1(z,v) \Leftrightarrow G^1(x-w,0)>G^1(z-v,0) \Leftrightarrow \tilde{\tilde{G}}^1(x-w)>\tilde{\tilde{G}}^1(z-v) \Leftrightarrow w-x>v-z.$$

(iii) If $w>x$ and $v<z$, then $G^1(x,w)>G^1(z,v)$ for every growth measure satisfying TI and M. This follows from M, TI and M again, which yields

$$G^1(x,w) > G^1(x,x) = G^1(z,z) > G^1(z,v).$$

(iv) If $w < x$ and $v > z$, then $G^1(x,w) > G^1(z,v)$ can never hold for any growth measure satisfying TI and M. This follows from M, TI and M again, which yields

$$G^1(x,w) < G^1(x,x) = G^1(z,z) < G^1(z,v).$$

Cases (ii) and (iv) hold automatically for every growth ordering satisfying TI and M, and therefore these cases have no bite. The lemma follows since it holds for all $x, v, w, z \in \mathbb{R}_{++}$. $\square$

The axioms N and IND can be used to cardinalize this ordering. This results in the following.

Proposition 2: A growth measure $G^{1A}(x,w)$ satisfies TI, M, N and IND if and only if there exists $\beta > 0$ such that

$$G^{1A}(x,w) = \beta(w-x).$$

Proof. We only prove the case where ($w > x$ and $z > x$). From IND and the definition of the function $\tilde{G}^1(x)$, we get

$$\tilde{G}^1(w+\theta-x) - \tilde{G}^1(w-x) = \tilde{G}^1(z+\theta-x) - \tilde{G}^1(z-x).$$

With a trivial redefinition of variables, this becomes that for all a, b and $c \in \mathbb{R}_{++}$,

$$\tilde{G}^1(a+b) - \tilde{G}^1(a) = \tilde{G}^1(b+c) - \tilde{G}^1(b),$$

which implies that the function $\tilde{G}^1$ must be linear: $\tilde{G}^1(x) = \alpha + \beta x$, such that

$$G^1(x,w) = \tilde{G}(w-x) = \alpha + \beta(w-x).$$

Due to N, we get $\alpha = 0$, and from M, $\beta > 0$ from which the result follows.

The case where θ is such that both $x > w + \theta$ and $x > z + \theta$ can be developed similarly to show that the function $\tilde{G}^1$ is equal to G^{1A}. $\square$

Proposition 2 characterizes a standard measure of individual growth expressed as the difference in level between the final period income and the initial period income.

The indices of individual growth obtained in Proposition 1 and 2 have been already introduced in the literature and are widely implemented in empirical works. The main contribution of our work in this section is the unifying framework to derive both the relative and the absolute, which has been made possible with the introduction of the new independence axiom.

2.2 From individual to aggregate growth

In this section we characterize a measure of aggregate growth. In order to do so, recall that our framework builds on the assumption that the vector of initial income is sorted increasingly, while in the vector of final incomes individuals are ordered according to their position in the initial distribution of income -see the definition of the domain D^n. It follows that $G^1(x_i, w_i)$, is the measure of growth of the individual ranked i-th in $\mathbf{x}$.

The following axioms will be used the characterize the aggregation procedure in the non anonymous context.

RDM (Rank Dependent Monotonicity): For all $n \in \mathbb{N}$ and all $(\mathbf{x}, \mathbf{w})$ and $(\mathbf{z}, \mathbf{v}) \in D^n$,

$$G^n(\mathbf{x}, \mathbf{w}) > G^n(\mathbf{z}, \mathbf{v}) \text{ if } G^1(x_i, w_i) \geq G^1(z_i, v_i) \text{ for all } i \in \{1, ..., n\},$$

with at least one inequality strictly holding.

This axiom is similar to the Weak Decomposability axiom commonly used in the literature on mobility measure, which allows to express the aggregate measure of growth as a function of each individual measure of growth. However, it is applied to "non-anonymous" distributions of income, that is, distributions where the individuals are ordered according to their rank in the initial period. This implies that this axiom makes it possible to express the aggregate measure of growth as a function of the magnitude of each individual growth, while keeping track of the the identity of individuals, represented by their rank in the initial distribution of income. This axiom states that when comparing two growth processes, the growth process $G^n(\mathbf{x}, \mathbf{w})$ dominates $G^n(\mathbf{z}, \mathbf{v})$ if every individual experiences higher or equal growth in $G^n(\mathbf{x}, \mathbf{w})$ than in $G^n(\mathbf{z}, \mathbf{v})$, with at least one individual experiencing higher growth in $G^n(\mathbf{x}, \mathbf{w})$ than in $G^n(\mathbf{z}, \mathbf{v})$, where the comparisons take place between individuals who sit in the same position in $\mathbf{x}$ and $\mathbf{z}$. In the next two axioms, only one $(\mathbf{x}, \mathbf{w}) \in D^n$ is considered, such that we use the abbreviated notation g_i to denote $G^1(x_i, w_i)$ and $\mathbf{g} \in \mathbb{R}^n$ denotes the corresponding vector of growth rates ordered on the basis of individuals' position in the vector $\mathbf{x}$.

IE (Individual Equivalence): For all $n \in \mathbb{N}$ and all $(\mathbf{x}, \mathbf{w}) \in D^n$,

$$\text{if for all } i \in \{1, ..., n\} : g_1 = ... = g_i = ... = g_n \Longrightarrow G^n(\mathbf{x}, \mathbf{w}) = g_i.$$

IE states that if all individuals have the same level of growth, then aggregate growth can be appropriately represented by that value of individual growth.

RD (Recursive Decomposability): For all $n \in \mathbb{N}$ and all $(\mathbf{x}, \mathbf{w}) \in D^n$ and corresponding $\mathbf{g} \in \mathbb{R}^n$ and for all $\Gamma^n : \mathbb{R}^n \longrightarrow \mathbb{R}$ and for all $\Gamma_a^{n-1}(g_1, ..., g_{n-1})$ and $\Gamma_b^{n-1}(g_2, ..., g_n) : \mathbb{R}^{n-1} \longrightarrow \mathbb{R}$:

$$\Gamma^n(\mathbf{g}) = \Gamma^n\left(\Gamma_a^{n-1}, ..., \Gamma_a^{n-1}, g_n\right) = \Gamma^n\left(g_1, \Gamma_b^{n-1}, ..., \Gamma_b^{n-1}\right).$$

RD represents a new contribution to the literature on growth measurement, even if it can appear a strong requirement. It encompasses an independence property, since it implies the following: letting aggregate growth only depend on the aggregate growth of the $n-1$ poorest individuals in the initial distribution and on the growth of the richest individual is equivalent to letting aggregate growth depend only on the growth of the poorest individual in $\mathbf{x}$ and on the aggregate growth of the

$n-1$ richest individuals. In addition, this property ensures that the income growth distribution of every group of the initially richest (or equivalently, poorest) members of the society can be evaluated without reference to the growth of the remaining individuals. In other words, every group of the richest (or poorest) income recipient is strictly separable from anyone who is poorer (or richer).

This property requires an aggregate growth measure to be strictly recursive (see on this Bossert, 1990). The difference with the decomposition proposed in Demuynck and Van de gaer (2011), Decomposability with respect to highest growth is clear. First, D-HG applies on ordered distributions of growth, thus it depends on the position of individuals in the vector of individual mobilities arranged in ascending order, independently from their position in $\mathbf{x}$. Second, it allows to derive an aggregate measure of growth depending on the lowest individual growth and the aggregate growth of the $n-1$ highest individual growths. Instead, we require aggregate growth to be sensitive to both the groups of the initially poorest and of the initially richest individuals. RD enables to state that the aggregation process is independent of the order in which it is performed. Thus, aggregating from below should give the same overall growth as one would obtain aggregating from above.

The previous axioms allows us to establish the following Proposition.

Proposition 3: For all $n \in \mathbb{N}$, an aggregate index of growth G^n satisfies RDM, IE and RD if and only if there exist coefficients $\gamma_1^n, \gamma_2^n, ..., \gamma_n^n$, such that, for all $(\mathbf{x}, \mathbf{w}) \in D^n$ and corresponding $\mathbf{g} \in \mathbb{R}^n$,

$$G^n(\mathbf{x}, \mathbf{w}) = \sum_{i=1}^{n} \gamma_i^n g_i,$$

where $\sum_{i=1}^{n} \gamma_i = 1$.

Proof. From RDM, there exists an increasing function Γ^n such that

$$G^n(\mathbf{x}, \mathbf{w}) = \Gamma^n(g_1, ..., g_i, ..., g_n).$$

The rest of the proof follows by induction (see Aczèl (1966), pp. 237-230).

Assuming that the proposition holds for $n-1$, yields, after suppressing the superscript $n-1$ of the coefficients,

$$\Gamma_a^{n-1}(g_1, ..., g_{n-1}) = \frac{a_1 g_1 + ... + a_{n-1} g_{n-1}}{a_1 + ... + a_{n-1}}; \; (a_1 + ... + a_{n-1} \neq 0)$$

and

$$\Gamma_b^{n-1}(g_2, ..., g_n) = \frac{b_2 g_2 + ... + b_n g_n}{b_2 + ... + b_n}; \; (b_2 + ... + b_n \neq 0).$$

By RD, we get

$$\Gamma^n(\mathbf{g}) = \Gamma^n\left(\frac{a_1 g_1 + ... + a_{n-1} g_{n-1}}{a_1 + ... + a_{n-1}}, ..., \frac{a_1 g_1 + ... + a_{n-1} g_{n-1}}{a_1 + ... + a_{n-1}}, g_n\right) \equiv \tag{1}$$

$$H^n(a_1 g_1 + ... + a_{n-1} g_{n-1}, g_n),$$

with H^n a real function. By RD, we also have that

$$\Gamma^n(\mathbf{g}) = \Gamma^n\left(g_1, \frac{b_2 g_2 + ... + b_n g_n}{b_2 + ... + b_n}, ..., \frac{b_2 g_2 + ... + b_n g_n}{b_2 + ... + b_n}\right) \equiv \tag{2}$$

$$F^n(g_1, b_2g_2 + ... + b_ng_n),$$

with F^n a real function. Suppose that $g_1 = g_2 = ... = g_{n-2} = 0$, then

$$H^n(a_1g_1 + ... + a_{n-1}g_{n-1}, g_n) = H^n(a_{n-1}g_{n-1}, g_n), \tag{3}$$

$$F^n(g_1, b_2g_2 + ... + b_ng_n) = F^n(0, b_{n-1}g_{n-1} + b_ng_n) \equiv f^n(b_{n-1}g_{n-1} + b_ng_n), \tag{4}$$

such that

$$H^n(a_{n-1}g_{n-1}, g_n) = f^n(b_{n-1}g_{n-1} + b_ng_n). \tag{5}$$

Putting $g_{n-1} = \frac{g}{a_{n-1}}$, the latter equality becomes

$$H^n(g, g_n) = f^n\left(\frac{b_{n-1}}{a_{n-1}}g + b_ng_n\right).$$

From this, it follows that

$$\Gamma^n(\mathbf{g}) = H^n(a_1g_1 + ... + a_{n-1}g_{n-1}, g_n) =$$
$$f^n\left(\frac{b_{n-1}}{a_{n-1}}a_1g_1 + \frac{b_{n-1}}{a_{n-1}}a_2g_2 + ... + b_{n-1}g_{n-1} + b_ng_n\right) =$$
$$f^n(c_1g_1 + ... + c_ng_n),$$

where for $k = 1, .., n-1$, $c_k = \frac{b_{n-1}a_k}{a_{n-1}}$ and $c_n = b_n$. By IE, $g = \Gamma^n(g, g, ..., g)$, which implies that

$$g = \Gamma^n(g, g, ..., g) = f^n[(c_1 + ... + c_n)g]$$

which, with a change in the variables gives $f^n(t) = \frac{t}{c_1+...+c_n}$, such that

$$\Gamma^n(g_1, ..., g_n) = f^n(c_1g_1 + ... + c_ng_n) = \frac{c_1g_1 + ... + c_ng_n}{c_1 + ... + c_n}.$$

Obviously, $\sum_{i=1}^n \left[c_i / \sum_{j=1}^n c_j\right] = 1$. For all $i = 1, .., n$ define $\gamma_i^n = c_i / \sum_{j=1}^n c_j$, and we see that the proposition holds for n, completing the proof. □

Proposition 3 provides a general family of aggregate growth measures, which can be expressed as a weighted average of individual growth measures, with weights based on the relative position in the initial distribution of income. Other than that, no information is available on the functional form of those weights. However, there are a variety of reasons to argue that, from a normative point of view, higher concern can be expressed with respect to the initially poorer individuals than the initially richer[2]. A measure of aggregate growth encompassing this concern will result into a measure of non-ananymous pro-poor growth, since it will be more sensitive to growth experienced by the initially poorest individuals. This final aim can be realized by imposing the following axiom.

NAPPG (Non-Anonymous Pro-poor Growth). *For all* $(\mathbf{x}, \mathbf{w})$ *and* $(\mathbf{x}, \mathbf{z}) \in D^n$ *that are such that (i) for all* $k = 1, ..., n, k \neq i, j : G^1(x_k, w_k) = G^1(x_k, z_k)$ *(ii) there exists a* $\Delta > 0$ *such that* $G^1(x_i, w_i) = G^1(x_i, z_i) + \Delta$ *and* $G^1(x_j, w_j) = G^1(x_j, z_j) - \Delta$ *then*

$$\textit{if } x_i < x_j \Rightarrow G^n(\mathbf{x}, \mathbf{w}) > G^n(\mathbf{x}, \mathbf{z}),$$

[2]One reason is the well established consensus that early stages of life matter more than later-life periods in shaping the individual's lifetime well-being/poverty.

$$and\ if\, x_i = x_j \Rightarrow G^n(\mathbf{x}, \mathbf{w}) = G^n(\mathbf{x}, \mathbf{z}).$$

According to **NAPPG**, we prefer the situation where income growth is redistributed in favor of the initially poorer individual. We can now establish the following lemma.

Lemma 3: All aggregate growth indices G^n satisfying RDM, IE, RD and NAPPG can be written as in proposition 3, with, moreover, $\gamma_1^n \geq ... \geq \gamma_i^n \geq ... \geq \gamma_n^n$, where the inequalities hold strict between consecutive γ_i^n and γ_{i+1}^n unless $x_i = x_{i+1}$ in which case the inequality becomes an equality.

Proof. As these measures satisfy the conditions of proposition 3,

$$G^n(\mathbf{x}, \mathbf{w}) - G^n(\mathbf{x}, \mathbf{z}) = \sum_{i=1}^{n} \gamma_i^n \left[G^1(x_i, w_i) - G^1(x_i, z_i)\right].$$

Now, using the specific vectors $\mathbf{x}, \mathbf{w}$ and $\mathbf{z}$considerd in the definition of NAPPR, we obtain

$$G^n(\mathbf{x}, \mathbf{w}) - G^n(\mathbf{x}, \mathbf{z}) = \gamma_j^n \left[G^1(x_i, z_j) - G^1(x_i, z_i)\right] + \gamma_{j+1}^n \left[G^1(x_j, z_i) - G^1(x_j, z_j)\right] = \left[\gamma_j^n - \gamma_{j+1}^n\right] \Delta,$$

which if $x_i < x_j$ must be strictly positive, such that $\gamma_i^n > \gamma_j^n$, and if $x_i = x_j$ must be zero, such that $\gamma_i^n = \gamma_j^n$. □

The following axiom allows to get a functional form for the weights.

PI (Population Invariance). *For all* $(\mathbf{x}, \mathbf{w}) \in D^n$ *and* $(\mathbf{y}, \mathbf{z}) \in D^{kn}$ *that are such that*

$$\mathbf{y} = \left(\underbrace{x_1, ..., x_1}_{k\ times}, ..., \underbrace{x_n, ..., x_n}_{k\ times}\right)\ and\ \mathbf{z} = \left(\underbrace{w_1, ..., w_1}_{k\ times}, ..., \underbrace{w_n, ..., w_n}_{k\ times}\right),$$

$$G^n(\mathbf{x}, \mathbf{w}) = G^{nk}(\mathbf{y}, \mathbf{z})$$

Population invariance states that the measure of aggregate growth is invariant to a k-fold replication of the same vector of initial and final incomes. This property ensures that we can apply this measure to compare growth processes taking place over distributions with different population size.

Proposition 4. For all $n \in \mathbb{N}$, an aggregate index of growth G^n satisfies RDM, IE, RD, M, NAPPG and PI if and only if there exists a parameter δ, such that, for all $(\mathbf{x}, \mathbf{w}) \in D^n$ with, moreover, $x_1 < ... < x_n$ and corresponding $\mathbf{g} \in \mathbb{R}^n$

$$G^n(\mathbf{x}, \mathbf{w}) = \frac{1}{n^\delta} \sum_{i=1}^{n} \left(i^\delta - (i-1)^\delta\right) g_i, \text{with } 0 < \delta \leq 1.$$

Proof. Use lemma 3 and Donaldson and Weymark (1980), Theorem 1 and 2. □

3 An empirical illustration

In this section we provide an empirical illustration, adopting our framework for analyzing different growth processes. In order to emphasize the relevance of our model, we want to show that the ranking of the growth processes obtained applying our index of pro-poor growth can, indeed, be different from the ranking that would result adopting some of the existing measures, such as the Growth Incidence Curve (GIC), the NaGIC (non-anonymous GIC) and the index of inequality adjusted growth.

3.1 Data

Our empirical illustration is based on the panel component of the last six waves of the Bank of Italy "Survey on Household Income and Wealth" (SHIW), corresponding to survey years 2000–2010. The SHIW is a representative sample of the Italian resident population interviewed every two years.

The unit of observation is the household, defined as all persons sharing the same dwelling. Our individual measure of the living standard is the household net disposable income, which includes all household earnings, transfers, pensions, and capital incomes, net of taxes and social security contributions.

Household income is expressed in constant prices and then adjusted for differences in household size using the OECD equivalence scale (the square root of the household component).

In particular our benchmark is the growth process 2008-2010. We consider this period extremely relevant because the first wave of the crisis took place in Italy in 2008. We compare all previous two-year periods growth processes starting from 2000 against 2008-2010. The comparisons are the following:

- 2000/2002 vs. 2008/2010;
- 2002/2004 vs. 2008/2010;
- 2004/2006 vs. 2008/2010.

The sample size for each comparison is reported in the next section. In line with the literature, for each wave we drop the bottom and top 1% of the sample in order to avoid the effect of the outliers.

The standard errors of our estimates are based on 1000 bootstrap replications. A block bootstrap procedure is implemented to reflect the dependence of the sample over time resulting from the longitudinal nature of the data.

Sample weights are used to compute all estimates. As suggested by the literature (Jenkins and Van Kerm, 2011;2008) we use cross-sectional individual sample weights, at time t. In fact, as shown by Faiella and Gambacorta (2007) for the case of the SHIW, for the production of longitudinal statistics, there is no unambiguous evidence that the use of longitudinal weights always performs better than cross-sectional weighing in terms of efficiency.

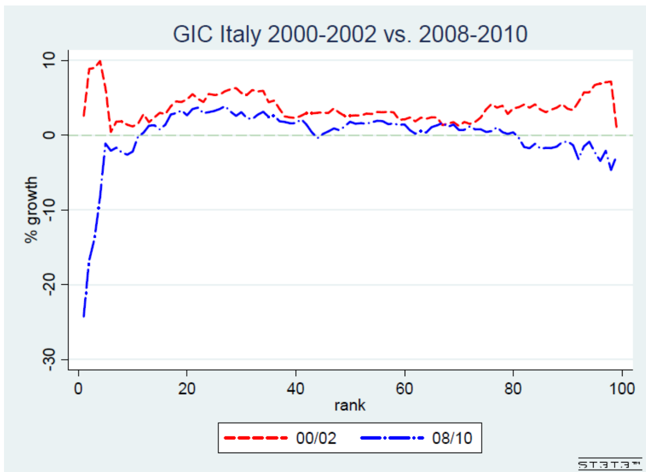
3.2 Results

The first comparison we conduct is between the growth process 2000/2002[3] and the growth process 2008/2010. We partition the distributions into 100 centiles and we start the analysis by applying the standard GIC (Figure 1). As we can see from Figure 1, the first growth process seems

[3] The sample size is 3504 for 2000/02.

to dominate the second one, since the GIC for the 2000/02 lies almost always above the GIC for the 2008/10; the two curves only slightly intersect around the seventieth percentile. Furthermore note that the second process is strongly regressive, as the GIC is increasing, up to the 80th centile, becoming progressive for the upper part of the distribution. The first process, instead, is progressive at the very bottom of the distribution, assuming then different trends over different parts of the distributions.

Figure 1. GICs: Italy 2000/02 vs. 2008/10

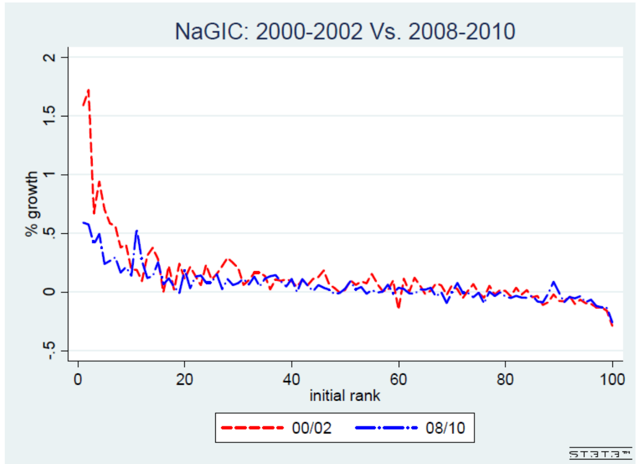


Source: Author's calculation from SHIW.

If we embrace the non-anonymous perspective, the comparison changes dramatically, as shown in Figure 2 where the non-anonymous growth incidence curves are reported[4]. For both periods the non-anonymous GIC (Na-GIC) are strongly different from the respective GICs. Both periods show a progressive path, hence it appears that initially poorest individuals gain more from growth than the initially rich. No dominance can be shown since the two Na-GICs intersect very often, however a particular feature arises: the 2000/02 clearly dominates the 2008/10 for the bottom 10% of the initial distribution, while the two curves overlap for the upper 10%. These results seem to suggest that, despite the progressivity of the Na-GIC, the crisis is affecting more the initially poorest individuals.

Figure 2. NaGICs: Italy 2000/02 vs. 2008/10

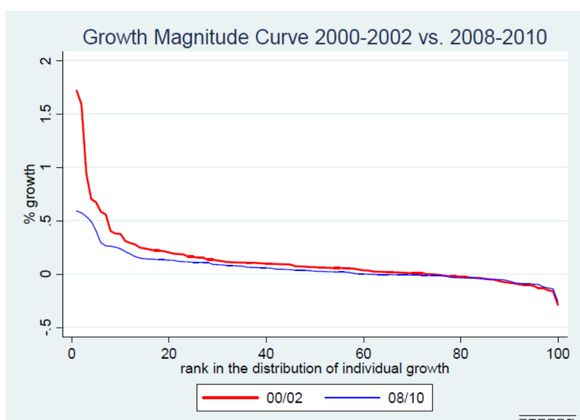
[4] The NaGIC was introduced by Grimm (2007). It plots, against each percentile of the initial distribution of income, the income growth of the individuals belonging to that percentile, no matter their position in the final distribution of income.



Source: Author's calculation from SHIW.

If the focus is on the magnitude of growth and on the inequality in the distribution of the individual growth numbers, as for the index of inequality adjusted growth (Demunynck and Van de gaer, 2011), it could be interesting to compare the previous curves with what we call the Growth Magnitude Curve (GMC). The GMC plots the magnitude of each individual growth, where individuals are ordered decreasingly according to their position in the vector of the individual growth measures, hence starting from the most growing to the least growing individual incomes. The GMCs are reported in Figure 3. As before, the dominance is not clear: the first period seems to dominate the second for the 80% bottom part of the distribution of the individual growth distribution, while the opposite happens for the top 20%.

Figure 3. GMCs: Italy 2000/02 vs. 2008/10



Source: Author's calculation from SHIW.

The curves we have discussed so far show that adopting a non-anonymous approach does matter. In addition, they give very useful information to describe some features of the processes considered, by contrast they do not provide useful information to rank the two growth processes.

We now turn to see what conclusions can, instead, be inferred if we adopt our index of non-anonymous pro-poor growth; we then compare them with those deriving from the application of the inequality adjusted growth index. The results reported in Table 1 show that, although the two indices might appear very similar in their theoretical formulation, they indeed draw a different picture. According to our approach the first growth process dominates the second one, whatever the value given to the weights, and the dominance is always statistically significant, except when $\delta = 0.25$. Hence the 2000/02 is more pro-poor than the 2008/2010. On the contrary, according to the inequality adjusted growth index the reverse happens. The motivation is clear if we look at Figures 2 and 3. While our index is weighting more the initially poorest individuals, the inequality adjusted growth index, is weighting more the initially reach individuals, since their incomes grow the least. Clearly, the two indices give the same result for $\delta = 1$, when they both equal the simple average of the individual growth measures.

Table 1. Growth for Italy: 2000-02 vs. 2008-10.

$I^{PP}_{00/02} > I^{PP}_{08/10}$	δ			
	0.25	0.5	0.75	1
Relative	TRUE ns	TRUE	TRUE	TRUE
Absolute	TRUE ns	TRUE	TRUE	TRUE
$I^{IAG}_{00/02} > I^{IAG}_{08/10}$	δ			
	8	6	4	1
Relative	FALSE	FALSE	FALSE	TRUE
Absolute	FALSE	FALSE	FALSE	TRUE

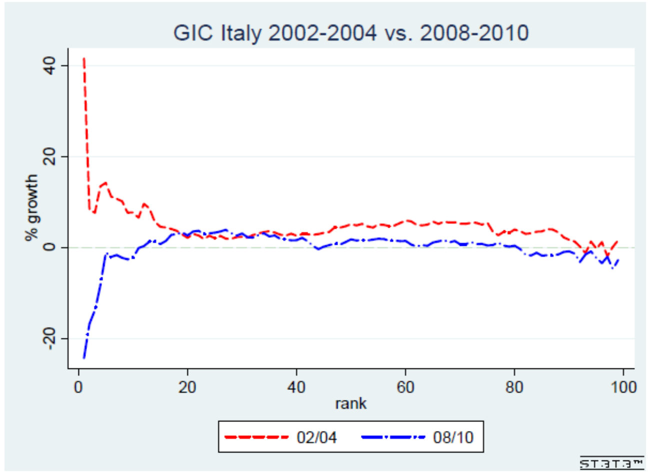
Source: Authors' calculation from SHIW[5].
Note: "ns" not significant at 90%.

We follow the same procedure to compare the growth process 2002/04[6] versus the 2008/10. Hence, we start by computing the GICs over the two distributions divided in 100 centiles. From Figure 4 it is clear that, contrary to the 2008/10, the 2002/04 process is progressive in particular for the lowest part of the distribution. Whether there is dominance is less clear: the first period lies above the second for most of the distribution but the two curves intersect hence one cannot say that a dominance relationship exists.

Figure 4. GICs: Italy 2002/04 vs. 2008/10

[5] We denote by I^{PP} our index of growth and by I^{IAG} the Demuynck and Van de gaer (2011) index of growth. $I^{IAG} = \frac{1}{n^{\sigma}} \sum_{i=1}^{n} \left(i^{\sigma} - (i-1)^{\sigma}\right) g'_i$, with $\sigma \geq 1$, where g'_i is the measure of income growth of the individual ranked i in the distribution of the individual growth measures, where these measures are sorted decreasingly. Hence this index weights more the individuals with lowest growth.

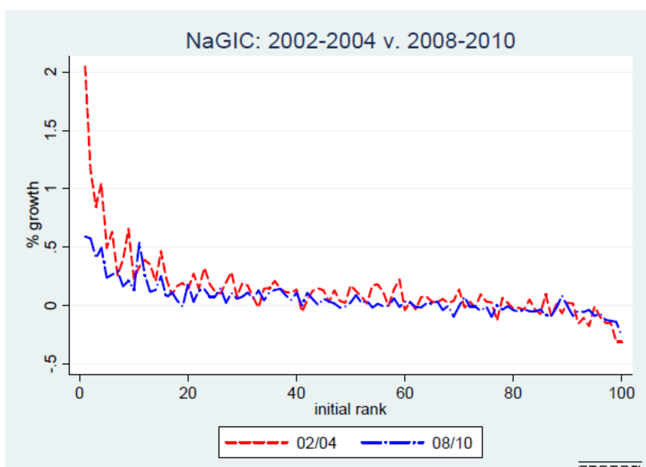
[6] the sample size is 3491 for 2002/04.



Source: Author's calculation from SHIW.

The difficulty in establishing a rank among the two processes is confirmed by the NaGICs. From Figure 5 it comes out that the two processes are very similar in the trend - they are both progressive - and in the extent of growth; they intersect continuously so that we cannot state which process is dominating the other and which process is more non-ananymous pro-poor than the other. However, the feature encountered earlier is confirmed: for the bottom 10% of initial distribution of income the first process dominates the second, while the reverse happen for the up 10%, confirming that, in general, the crisis is hurting the initially poorest individuals.

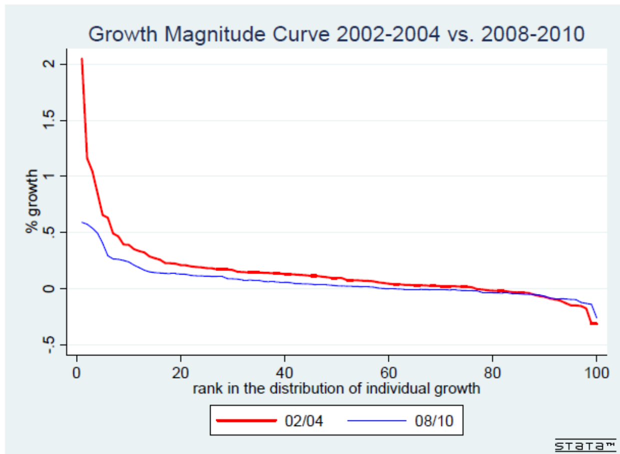
Figure 5. NaGICs: Italy 2002/04 vs. 2008/10



Source: Author's calculation from SHIW.

A similar conclusion can be obtained by focussing on the GMCs, where again the first process is better for the individuals whose incomes grow most, but the situation is reverted for the individuals whose incomes grow least. The two curves intersect around the 90% of the distribution of the individual growth generating inconclusiveness in terms of growth comparisons

Figure 6. GMCs: Italy 2002/04 vs. 2008/10



Source: Author's calculation from SHIW.

This inconclusiveness is solved when we implement the non-anonymous pro-poor growth index, with results reported in Table 2. According to our index, the 2002/04 growth process is more pro-poor than the 2008/10, and the dominance is always statistically significant, except when $\delta = 0.25$. The results in Table 1 also confirm that this rank is different from the one provided by implementing the inequality adjusted growth index. In such case, the second process dominates the second, and the dominance is always statistically significant.

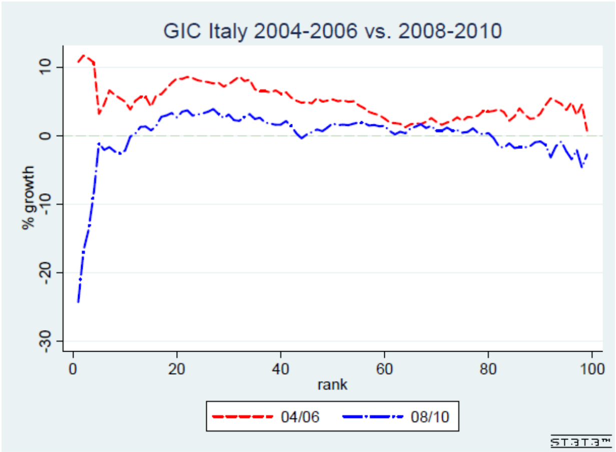
Table 2. Growth for Italy: 2002-04 vs. 2008-10.

$I^{PP}_{02/04} > I^{PP}_{08/10}$	δ			
	0.25	0.5	0.75	1
Relative	TRUE^{ns}	TRUE	TRUE	TRUE
Absolute	TRUE^{ns}	TRUE	TRUE	TRUE
$I^{IAG}_{02/04} > I^{IAG}_{08/10}$	δ			
	8	6	4	1
Relative	FALSE	FALSE	FALSE	TRUE
Absolute	FALSE	FALSE	FALSE	TRUE

Source: Author's calculation from SHIW.

We now turn to the last comparison: the 2004/06[7] growth process versus the 2008/10. Figure 7, which reports the GICs for the two periods, witnesses that there is a dominance between the two. As expected, whatever the shape of the two curves, the 2004/06 curve dominates the 2008/2010 curve.

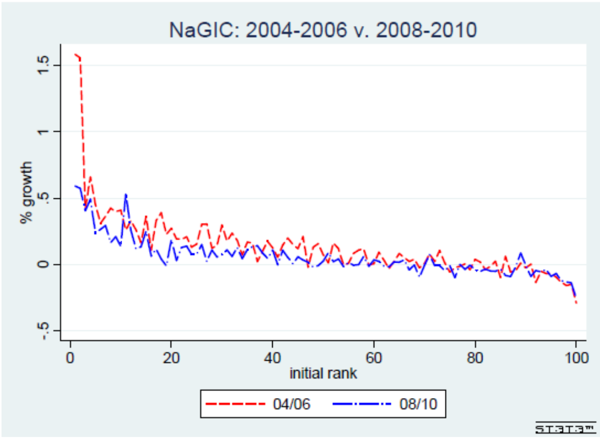
Figure 7. GICs: Italy 2004/06 vs. 2008/10.



Source: Author's calculation from SHIW.

The dominance discussed above cannot be confirmed when anonymity is relaxed. The two processes share the same progressive trend, but the two curves intersect in different parts of the distribution. This feature, not only makes impossible to order the two growth processes in terms of magnitude of growth, but it also makes hard to establish which process is more non-anonymous pro-poor.

Figure 8. NaGICs: Italy 2004/06 vs. 2008/10.

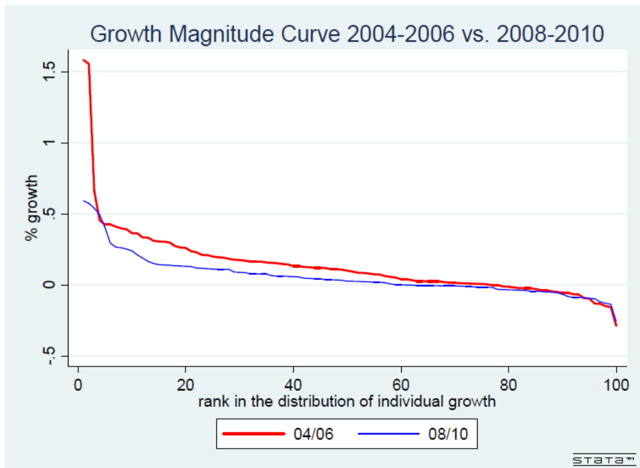


[7] The sample size is 3848 for 2004/06.

Source: Author's calculation from SHIW.

Neither unambiguous is the comparison of the two growth processes if we look at the GMCs reported in Figure 9. The first process is dominating the second, except for last 10% of the distribution of individual growth, which corresponds to the 10% lowest growing individuals. Hence, these individuals seem to have done better under the crises that before, whereas the opposite is true for the rest of the distribution.

Figure 9. GMCs: Italy 2004/06 vs. 2008/10.



Source: Author's calculation from SHIW.

Finally, the results in Table 3 represent the last findings of our measure, according to which the 2004/06 growth process is more pro-poor than the 2008/10. The result is statistically significant for every value of δ, except for $\delta = 0.25$. The dominance would be inverted when we implement the inequality adjusted growth index.

Table 3. Growth for Italy: 2004-06 vs. 2008-10.

$I^{PP}_{04/06} > I^{PP}_{08/10}$	δ			
	0.25	0.5	0.75	1
Relative	TRUE ns	TRUE 90	TRUE	TRUE
Absolute	TRUE ns	TRUE	TRUE	TRUE
$I^{IAG}_{04/06} > I^{IAG}_{08/10}$	δ			
	8	6	4	1
Relative	FALSE	FALSE	FALSE	TRUE
Absolute	FALSE	FALSE	FALSE	TRUE

Source: Author's calculation from SHIW.
Note: "90" dominance statistically significant at 90%.

4 Conclusions

By adopting a non-anonymus perspective, in this paper we have presented a formal characterization of a measure of relative and absolute individual growth; we have realized it using a unifying framework. We then proposed an aggregation of these measures into a measure of societal growth. For, we have introduced the recursive decomposability axiom which enables us to obtain a generalization of measures of growth. Introducing two standard normative axioms we obtained a measure of pro-poor growth, which is more sensitive to the growth experienced by the initially poorest individuals than the initially richest.

We have applied our framework using Italian data. We have shown that, although, the employment of the Na-GIC does not allow to obtain any clear dominance among the processes compared, it is still possible to rank them adopting our measure of non-anonymous pro-poor growth. According to our index it comes out that every growth process considered is more pro-poor than the 2008/10, for a large range of value attached to the parameter δ. These results also confirm that the crisis has affected more the initially poorest individuals.

How standard and non-anonymous measures of growth relate is left for future research.

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