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THE WELFARE COST OF UNPRICED HETEROGENEITY IN INSURANCE MARKETS

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We consider the welfare loss of unpriced heterogeneity in insurance markets, which results when private information or regulatory constraints prevent insurance companies to set premiums reflecting expected costs. We propose a methodology which uses survey data to measure this welfare loss. After identifying some 'deep types' which determine expected risk and insurance demand, we use these deep types to derive the demand and cost functions for each unobservable type, quantifying the efficiency costs of unpriced heterogeneity. We apply our methods to the US Long-Term Care and Medigap insurance markets, where we find that unpriced heterogeneity causes substantial inefficiency.

JEL Classification Numbers D82, G22, I11.

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PRELIMINARY DRAFT: COMMENTS ARE WELCOMED

1. INTRODUCTION

In many insurance markets private information or regulatory constraints prevent insurers to set premiums reflecting individuals' costs. In this context, *unpriced heterogeneity* refers to all characteristics which affect insurance demand and expected claims, but are not priced by insurance companies. Under unpriced heterogeneity, individuals pay insurance premiums which do not reflect expected costs. Since efficiency requires that individuals should purchase insurance if and only if their willingness to pay for the contract is greater than their expected costs, the existence of unpriced heterogeneity implies that some individuals overbuy and some underbuy. How large is the resulting welfare loss? If contracts were priced conditional on individual expected risks, how large would the welfare gain be? In this paper, we propose a methodology to address this question.

The methodology combines *survey data* on costs and coverage with price elasticity estimates from the literature to quantify the inefficiency caused by unpriced heterogeneity, allowing for a potential wide applicability of our methods. To measure welfare loss, we recover willingness-to-pay and expected costs for each partecipant in the market by segmenting the market into 'deep types' that have costs independent of coverages, using a discrete multivariate finite mixture model. If these types were contractible, they could be segregated into separate 'synthetic' markets, each with its own insurance contract. Within each such synthetic market individuals differ only by idiosyncratic preference and realized costs, and (expected) marginal costs equal average costs; there would be no welfare loss from charging the same price to all customers. To estimate the demand function in each market we calculate willingness-to-pay combining semiparameteric estimates of coverage and cost decisions with parametric assumptions on the shape of the demand function and external information on the aggregate price elasticity. The estimates of willingness-to-pay and expected costs allow us to calculate counterfactual price changes conditional on the features

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of the unobserved markets, allowing prices to vary with expetcted costs, as in Bundorf, Levin and Mahoney (2012). The magnitude of the welfare gain that could be achieved pricing these types depends on the slope of the demand curve in each market through the usual deadweight loss triangle.

We apply our methods to two different markets: the US Long-Term Care and Medigap insurance markets. In both markets we find that unpriced heterogeneity causes substantial inefficiency. In LTC insurance we find that, conditional on insurers' risk classification, there are two underlying unobserved synthetic markets: one for the 'cautious' types, who value highly the insurance but have low expected costs; and one for the 'reckless' types, with low valuation of the insurance contract but high expected costs. High-risk individuals are almost 2.5 times more likely to use a nursing home than low-risk ones. Since conditional on insurers' risk classification these individuals face the same premium in both unobserved markets, this implies an implicit cross-subsidization of high risks at the expense of low ones: we find that a one-dollar subsidy to the high-risk types costs the low-risks between 2.8 and 5.5 dollars, a very large number which is due to the low valuation that high-risks give to the insurance contract. In our sample we also find that the welfare loss of current pricing –compared to pricing at expected costs in each synthetic market– ranges from 7 to 10 percent of total coverage cost, depending on the assumed elasticity of aggregate demand. We notice that the inefficiency of current contracts stems almost entirely from the lack of price discrimination between high and low risks, rather than from the non socially-optimal level at which the (uniform) price is set: an optimally set uniform price would reduce welfare loss by less than one percent relative to the current uniform (non optimally set) price.

In the Medigap market, where premiums are heavily regulated, estimation suggests the existence of heterogenous types, with low and high expected costs and low and high tolerance for risk, which can be cross-classified into types with striking differences in estimated insurance and medical care choices. In our sample, the welfare loss of current pricing –compared to pricing at expected costs in each synthetic market– ranges between 11 and 40 percent of total coverage costs. A one-dollar subsidy to the high risks costs between 1.4 and 3.6 dollars to the low risks, which suggests that in Medigap the efficiency cost of subsidizing high risks through uniform prices could be rather substantial. We also find that optimally set uniform (non-discriminatory) prices reduce the estimated welfare loss only marginally (less than 3 percent), suggesting that the loss of allocative efficiency of current pricing is for the most part due to the lack of price discrimination.[1]

The literature on the empirical appraisal of the welfare implications of pricing insurance contracts is recent (see Einav, Finkelstein and Levin (2010) for a survey). Our paper is mostly related to the papers of Einav, Finkelstein and Cullen (2010) and Bundorf, Levin and Mahoney (2012).[2] The seminal paper of Einav, Finkelstein and Cullen (2010) studies the welfare cost of private information in a simple and intuitive framework where the researcher describes preferences for insurance and expected utilization estimating the aggregate demand, marginal cost and average cost curves. This allows a rich counterfactual policy analysis with a minimum of maintained assumptions on the underlying microfoundation of the insurance market. Bundorf, Levin and Mahoney (2012) consider a more structural model of health plan choice taking into account both observable and privately known dimensions of health risk, which interact in the estimation of the demand and expected cost functions, allowing also for heterogenous consumer tastes.

Our paper differs form current literature in several aspects. First, compared with Einav, Finkelstein and Cullen (2010), we decompose the observed market into separate structural markets which

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[1]It is worth noting that our modeling approach ignores a potential benefit of unpriced heterogeneity: the cross-subsidy generated by the lack of price discrimination could transfer resources to those with a relatively higher marginal utility of consumption. In fact, our measure of the welfare cost of the cross-subsidization of high risks at the expense of low risk types is upwardly biased if high risk types have a higher marginal utility of consumption. Results of Finkelstein, Luttmer and Notowidigdo (2009) suggests that those with higher medical utilization do not in fact have a higher marginal utility of consumption.

[2]Recent papers closely related with ours are also Lustig (2011) on the efficiency of Medigap insurance and Geruso (2012) on unpriced heterogeneity in health plan choice. These studies analyze how heterogeneous preferences over insurance –uncorrelated with individual insurable risk– can induce, under a uniform price setting, inefficient self-sorting into plans.

extract key information on the nature of the distortions created by unobserved heterogeneity. In comparison with Bundorf, Levin and Mahoney (2012), who model private risk as a univariate normally distributed unobserved variable and insurance preference heterogeneity as an idiosyncratic demand shifter, we allow -besides idiosyncratic consumer taste differences- for structural heterogeneity in any variable which jointly affects claims and coverages, in a nonparametric fashion. The other major difference between the current paper and existing ones is that we appraise the efficiency of unpriced heterogeneity in insurance markets using survey data, rather than administrative or firm data requiring exogenous variation in prices. This allows to explore the efficiency in many markets with large representative samples, and given the wealth of internationally available survey data, allows exploring many markets where the researcher does not have access to firm data. The most relevant limitations of our approach however are due precisely to the use of survey data, which typically do not contain detailed information on individual insurance premiums. To sidestep this issue, we use external information on the price elasticity of demand in the observed market. In both of our applications we use elasticity estimates from recent literature; in general the researcher may leverage a set of credible estimates of the policy effects of interest using reasonable ranges for this parameter.

The paper is organized as follows: in the next section we describe a simple model of insurance markets under unpriced heterogeneity. In section 3 we discuss the empirical setting. In section 4 we apply a parametric specification to insurance demand and show how the welfare loss measures can be calculated from estimated claims and coverage probabilities. Section 5 contains our discrete multivariate finite mixture model which allows estimation of the types' claims and coverage probabilities. Sections 6 and 7 present our two applications. Section 8 concludes.

2. The insurance market under unpriced heterogeneity

Consider an insurance market where individuals can buy a given insurance contract which protects from a probabilistic loss. Firms offer the contract at a price p which depends on some observable characteristics $\boldsymbol{x}$, and individuals make a binary choice of whether to buy the given contract.[3] To describe the market, we adapt the simple theoretical model of Einav, Finkelstein and Cullen (2010), where consumers make a discrete choice to buy the contract, and risk neutral providers set prices in a Nash equilibrium. To keep notation simple, we consider initially the analysis conditional on a given value of the observable characteristics $\boldsymbol{x}$ used by insurance companies to set prices. Thus, all individuals face the same price and are undistinguished by insurers.

In this context, residual *unpriced heterogeneity* refers to all variables which affect claims and coverage after conditioning on $\boldsymbol{x}$. Such variables include not only unobserved private information considered in standard insurance models, but possibly any other characteristic (observable or not) which cannot be used either by regulatory laws or political economy concerns. Thus, individuals in the market may differ in several dimensions with respect to both risk factors and risk preferences. We let $w(\boldsymbol{\zeta}_i)$ and $c(\boldsymbol{\zeta}_i)$ denote respectively the willingness to pay for the contract and the expected monetary cost of the insurable risk for individual i, with $\boldsymbol{\zeta}_i$ being a vector collecting all the components of residual heterogeneity after conditioning on $\boldsymbol{x}$.

We assume that the population is made of a discrete number of "types" $t \in \{1, 2, \dots, M\}$. Willingness to pay and expected monetary costs for individual i who belongs to type t are

$$w(\boldsymbol{\zeta}_i) = w(t) + \theta_i, \;\; c(\boldsymbol{\zeta}_i) = c(t) \tag{1}$$

where $w(t)$ and $c(t)$ are discrete "structural" components which are common for all individuals who belong to a given type, and θ captures idiosyncratic taste for insurance which is orthogonal to expected costs. Notice that the label of the types is arbitrary, and no order is assumed on the types. In this context, different types are simply meant to capture significant residual heterogeneity

[3] Compare with most of the theoretical literature which follows the seminal Rothschild and Stiglitz (1976) analysis, which endogenizes not only the pricing of insurance contracts but also the level of coverage.

without any assumption on its underlying structure; what matters here is that a sufficient number of types is used to capture all relevant structural heterogeneity.[4]

For each type t there is an unobserved "synthetic" market where the demand for insurance is

$$D_t(p) = P(w(t) + \theta \geq p) \tag{2}$$

and (expected) average and marginal costs are constant

$$AC_t(p) = MC_t(p) = c(t). \tag{3}$$

The observed aggregate demand for insurance is

$$D(p) = \sum\nolimits_t P(T = t) D_t(p), \tag{4}$$

where T denotes the discrete random variable in $\{1,2,\ldots,M\}$ which classifies the different types. If identical risk neutral insurance providers set prices in a Nash equilibrium, the observed aggregate (expected) total cost curve is

$$TC(p) = \sum\nolimits_t P(T = t) D_t(p) c(t) \tag{5}$$

where, as stressed by Einav, Finkelstein and Cullen, aggregate costs are determined by the costs of the sample of individuals who endogenously choose to buy the contract.[5]

Following Einav and Finkelstein (2011), a description of the observed aggregate market can be obtained by plotting the demand and cost curves as in Figure 1, where the horizontal axis indicates the proportion of individual who buy the contract. The key observation of Einav and Finkelstein

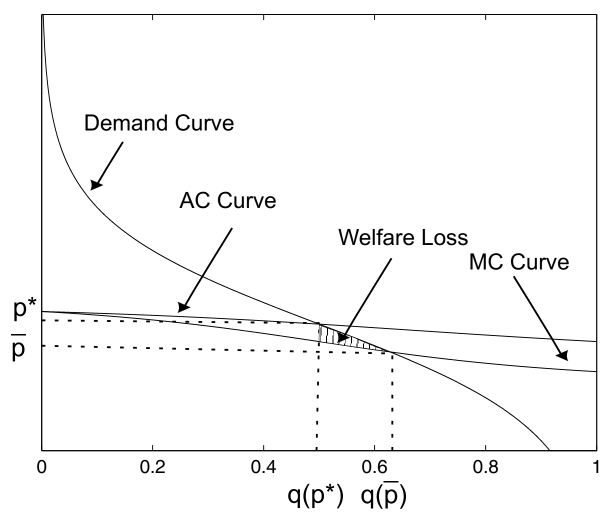

FIGURE 1. Demand, AC and MC in the aggregate insurance market

is that, under adverse selection, individuals who have the highest willingness to pay for insurance are those who, on average, have the highest expected costs. Therefore, under adverse selection the

[4]Our formulation is meant to model individuals' willingness to pay without a precise specification of the nature of unpriced heterogeneity. However, to fix ideas, consider individuals with CARA preferences who face a loss with fixed monetary value. Suppose that, in the population, the structural parameters (risk aversion and loss probability) are binary variables; after cross-classification, in the population there are four types with different valuations of the contract. The actual valuation of the contract by a given individual then depends on his type, and on an error term capturing his heterogeneous taste for insurance.

[5]The (expected) aggregate AC and MC curves can be derived from (5) as

$$AC(p) = \frac{\sum_t P(T = t)\big[1 - F_\theta\big(p - w(t)\big)\big] c(t)}{\sum_t P(T = t)\big[1 - F_\theta\big(p - w(t)\big)\big]}; \quad MC(p) = \frac{\sum_t P(T = t) f_\theta\big(p - w(t)\big) c(t)}{\sum_t P(T = t) f_\theta\big(p - w(t)\big)};$$

where F_θ and f_θ are respectively the c.d.f. and p.d.f. of θ.

MC curve is downward-sloping. There is no selection in the market if and only if the MC curve is flat.[6] This link between the demand and cost curves is a key distinction between insurance and other markets.

If the MC curve is downward sloping, the AC curve will lie above it. The zero-profit equilibrium price, denoted p^* in the figure, is found where the AC curve crosses the demand curve. The socially efficient uniform price $\bar{p}$ equates demand with the MC curve. If a social planner could set (say by appropriate subsidies) the price at $\bar{p}$, the resulting quantity $q(\bar{p})$ will be greater than the equilibrium quantity $q(p^*)$, which is the standard Akerlof (1970) result of underinsurance in adverse selection markets. The welfare loss caused by average cost price p^* compared with marginal cost price $\bar{p}$ is described by the shadowed area in the figure.

Suppose that the aggregate insurance market in Figure 1 is actually generated by two underlying heterogeneous types $t = 1, 2$, which can be segmented into two unobserved synthetic markets such that within each market the MC curve is flat and equals the AC curve. Individuals in the first market have on average *both* a higher probability of buying the contract and to experience the loss compared with the individuals of the second market. Assume there is an equal number of individuals in the two markets. These two markets are depicted in Figure 2, which decomposes the observed population of the aggregate market illustrated in Figure 1 above. By assumption

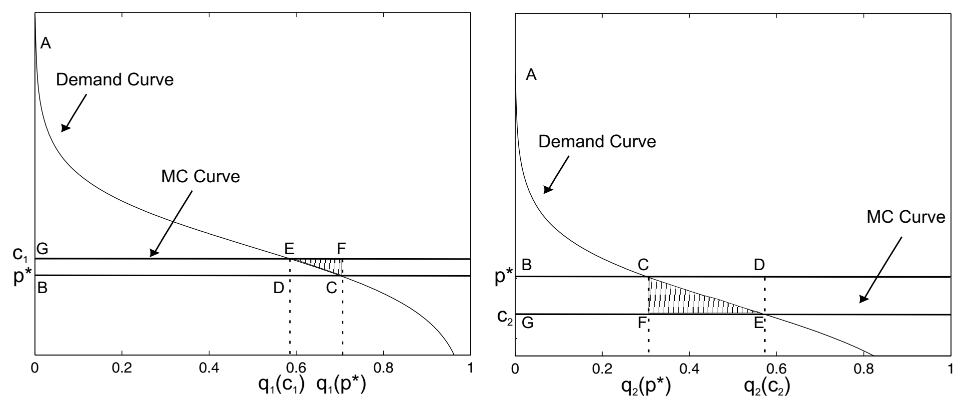

FIGURE 2. Synthetic markets for types 1 and 2

individuals in both markets have the same characteristics $\boldsymbol{x}$, so they all face the same price p^*. Since social efficiency requires that individuals buy the contract if and only if their willingness to pay exceeds their expected cost, the resulting welfare loss is given by the (weighted) average of the usual deadweight loss triangles (the shadowed area in the two figures). The welfare loss is the result of the implicit cross-subsidization of high risks at the expense of low ones caused by the uniform price.

If the two groups were contractible, there would be two independent markets with different (flat) MC curves, and price discrimination according to expected costs would generate social efficiency. Thus, the welfare loss triangles capture the inefficiency of actual pricing compared to optimal discriminatory prices. The welfare loss from unpriced heterogeneity is equal to the weighted sum of the deadweight loss triangles CEF in the two figures. To get an idea of its relative importance we can scale it, for example, by the total maximum consumer surplus under the efficient allocation (which equals the weighted sum of the areas AGE), or to the total coverage costs (the value of current trade, the area BC$q_i(p^*)$0 in the figures). Alternatively, notice that while by construction the financial gain of cross subsidization for high risk types 1 equals the financial loss for low risk

[6]Contrary to the theoretical prediction of the standard adverse selection model, a number of empirical studies (e.g. Finkelstein and McGarry (2006) and Fang, Keane and Silverman (2008)), have found *negative* risk-coverage correlation, a phenomenon which has been named advantageous selection. Under advantageous selection, the MC curve is upward-sloping, indicating that individuals with the highest willingness to pay have the lowest expected risk.

types 2 (the areas GFCB are equal), the welfare gain of types 1 is smaller than than the welfare loss of types 2 (compare the areas BCEG in the two figures). Thus, we can also appraise the (in)efficiency of cross-subsidization as the ratio between the welfare loss of low risks and the welfare gain of high risks (the weighted ratio of the areas BCEG in the two figures). This gives a measure of the welfare cost of transferring one dollar from the low risk to the high risk types. An interesting exercise is to measure the proportion of the measured inefficiency which is attributable to non-optimal uniform pricing. In other words, we can measure the inefficiency of actual prices compared to a situation where the social planner could not price-discriminate, but could choose a uniform price so as to minimize welfare loss (i.e. the price $\bar{p}$ in Figure 1). Comparing the weighted sum of the two welfare loss triangles in Figure 2 with the constrained-efficient welfare loss given in Figure 1 gives a measure of the proportion of the total inefficiency due to lack of price-discrimination.

In this example, riskier types have both higher expected claims and higher demand for insurance, so that claims and coverages are *comonotone*. In general, selection in this setting can be detected by testing the comonotonicity of claims and coverages across types, which is the counterpart of Einav, Finkelstein and Cullen's MC test for selection which is performed on the aggregate market. When types differ only for their riskiness, claims and coverages are comonotone. Recent empirical literature (e.g. Cutler, Finkelstein and McGarry (2008)) has however emphasized the relevance of multidimensionality of private information. When unpriced heterogeneity is multidimensional, not only the definition of selection becomes blurred, but also looking at the aggregate market to detect selection can be very misleading. In particular, since aggregate expected costs come from a mixture of high and low risk individuals, it may happen that the aggregate MC curve is very close to the AC curve even in the presence of substantial heterogeneity in expected costs. In this case, which we actually document below in our analysis of the market for Medigap insurance, looking at the aggregate demand and cost curves we might conclude that there is no significant selection and welfare loss in the market, while in fact unpriced heterogeneity may cause a substantial welfare loss compared to optimally set discriminatory prices.

3. The empirical setting

We describe the insurance market by a binary variable I which takes value 1 if an individual has bought an insurance contract which protects him from a fixed loss with monetary value D, and a binary variable L which takes value 1 if the individual incurs the loss. This is a good setting when using survey data, where typically the researcher can only observe whether the individual occurred the risk and whether she is covered by an insurance plan. To ease notation, unless explicitly mentioned we set $D = 1$, so that the loss probability equals expected costs, but we relax this assumption whenever appropriate.

Conditional on a given value of the pricing variables $\boldsymbol{x}$, if T represents the residual 'deep' types which systematically differ in their expected costs and their willingness to buy the contract, the observed joint distribution of (I, L) is the aggregation of a finite mixture of M unobserved joint distributions such that, conditional on each type t, I and L are *independent*:

$$P(I, L) = \sum\nolimits_{t=1}^{M} P(T = t)P(I \mid t)P(L \mid t). \tag{6}$$

In this sense, types extract all systematic variation in cost and insurance demand, and all residual heterogeneity can be considered as idiosyncratic. Thus, in each market t realized costs and taste for insurance differ individually, but expected cost are constant and unrelated to individuals willingness to buy insurance (since L and I are conditionally independent). In other words, within each type, there is no selection.

To perform our policy analysis, the key statistics needed are the demand for insurance $P(I = 1 \mid T = t)$ and expected costs $P(L = 1 \mid T = t)$ in each synthetic market t, jointly with an an estimate on how many types compose the observed aggregate market, and what is their proportion. The key challenge here is clearly that, conditionally on $\boldsymbol{x}$, all individuals in the market face the same price p.

In the next two sections, we discuss how survey data can be used to obtain, conditional on $\boldsymbol{x}$, a nonparametric estimate of the claims and coverage probabilities for each heterogeneous type.[7] Having obtained an estimate of of these probabilities in each synthetic market, and an estimate of the proportion of individuals in each type, we discuss how parametric assumptions and appropriate external information can lead to recover the parameters of interest for welfare analysis in each unobserved synthetic market. In doing so, we first discuss the latter issue assuming that we have already obtained the probability estimates of interest. For notational simplicity, in the next section we will still conduct our discussion conditional on the pricing variables $\boldsymbol{x}$.

4. A Parametric Specification of Heterogeneous Demand and Welfare Loss

Suppose we have an estimate of the proportion of types $\pi_t = P(T = t)$, the demand for insurance $P(I = 1 \mid t)$ and expected costs $P(L = 1 \mid t)$ for all unobserved synthetic markets $t = 1, \ldots, M$. In this section we show how this information, coupled with functional form assumptions on the demand for insurance and appropriate external information, can give a quantitative appraisal of the efficiency implications of the unpriced heterogeneity.

To model insurance demand, we assume that idiosyncratic taste for insurance θ in equation (1) is logistic. Individuals buy the contract if and only if their willingness to pay is greater than the price p. Insurance demand thus follows a discrete choice model

$$I = 1 \leftrightarrow w(t) + \sigma\epsilon_i \geq p \tag{7}$$

where ϵ denotes a standard logistic error and σ is a scale parameter. Notice that, as usual in the discrete choice literature we assume that the scale parameter σ does not change across heterogeneous types.[8] In section 5 below we discuss how this assumption can be tested when one introduces the pricing variables $\boldsymbol{x}$.

From equation (7) the demand function in market t is given by

$$q_t(p) = \frac{\exp\left(-\frac{p-w(t)}{\sigma}\right)}{1+\exp\left(-\frac{p-w(t)}{\sigma}\right)} \tag{8}$$

where $q_t(p)$ denotes the proportion of t-type individuals who buy the contract. Letting p^* denote the actual price of the insurance contract, $q_t(p^*) = Pr(I = 1 \mid t)$. When p^* is the average cost (e.g. in a perfectly competitive or contestable market with no administrative costs), using (5)

$$p^* = P(L = 1 \mid I = 1) = \frac{\sum_t \pi_t q_t(p^*) c(t)}{\sum_t \pi_t q_t(p^*)}. \tag{9}$$

Clearly without further information we cannot identify $w(t)$ and σ in equation (8), and thus we cannot identify the location and scale of the demand functions in each unobserved market. However it is immediate to see from equations (7) and (8) that, if we had an estimate of σ, from p^* and $q_t(p^*)$ we could recover the demand function in each market. Our strategy is to use external information, namely the price elasticity of the aggregate demand for insurance, to identify σ.

Differentiate then (8) w.r.t. p

$$\frac{\partial q_t(p)}{\partial p} = -\frac{1}{\sigma}(1 - q_t(p))q_t(p), \tag{10}$$

so that, given aggregate demand $q(p) = \sum_t \pi_t q_t(p)$,

$$\frac{\partial q(p)}{\partial p} = -\frac{1}{\sigma}\sum_t \pi_t(1 - q_t(p))q_t(p).$$

[7] As it will be explained in section 5, when taking into account $\boldsymbol{x}$ our estimation is semiparametric.

[8] See e.g. Bundorf *et al.* ((2012), equation 3.

Given an external estimate of the elasticity of the aggregate demand η at the aggregate equilibrium $(p^*, q(p^*))$, we have

$$\eta = -\frac{\big(\sum_t \pi_t(1-q_t(p^*))q_t(p^*)\big)p^*}{\sigma q(p^*)}$$

and so

$$\sigma = -\frac{\big(\sum_t \pi_t(1-q_t(p^*))q_t(p^*)\big)p^*}{\eta q(p^*)}. \tag{11}$$

From estimated p^* and σ and equation (8) we then derive w_t for each market t, and thus obtain an estimate of the demand function (8) and of the inverse demand function

$$p_t(q) = w_t + \sigma \log\big(\frac{1-q}{q}\big) \tag{12}$$

for each synthetic market. Integrating (8), the consumer surplus for t-type individuals is equal to

$$CS_t(p) = \sigma \log\big[1 + \exp\big(-\frac{p-w_t}{\sigma}\big)\big] \tag{13}$$

which can be compared with the Rosen and Small (1981) multinomial logit logsum measure.

Summing up, from estimated $\big(\pi_t, P(I=1 \mid t), P(L=1 \mid t)\big)$ for each synthetic market, the average cost assumption allows an estimate of p^*, and external information on η gives an estimate of σ.[9] This gives all the required parameters to allow quantitative welfare analysis and efficiency evaluations of counterfactual policies.

Before giving an example of application of these measures, we offer a few remarks. Notice that when D is different from 1, without further information on p^* or D welfare calculations cannot be given a monetary value. However, since varying D both consumer surplus and costs are multiplied by the same factor, we can still calculate welfare loss *relative* to other measures which use the same estimated demand and cost functions. Notice also that when estimating p^* from conditional claims and coverage probabilities (see equation (9)), one needs to assume that p^* equals average cost excluding administrative costs. In the presence of administrative costs (in general the costs of running an insurance company), $p^* = \lambda \cdot \frac{\sum_t \pi_t q_t(p^*)c(t)}{\sum_t \pi_t q_t^*} \cdot D$ for some $\lambda > 1$. In this case we can assume that the marginal expected costs which are relevant for efficiency calculations are inclusive of running costs, that is, the relevant marginal expected cost of selling an insurance contract to t-types is $EC(T=t) = \lambda \cdot c(t) \cdot D$, so that zero-profits are defined to include running costs, but, for the purpose of calculating relative welfare loss nothing of substance is changed. Finally, recall that these calculations are computed for a given value of the conditioning variables $\boldsymbol{x}$. In practice, one can either calculate these measures at a given value of $\boldsymbol{x}$ (such as the average or the median value), or calculate them for all combinations of $\boldsymbol{x}$ in the sample and average them out.

4.1. **An example.** We end this section with an example. Suppose an insurance market is composed of two synthetic markets, and we estimate the following statistics:

$$\begin{aligned} &P(T=1) = P(T=2) = .5;\\ &P(I=1 \mid T=1) = 0.7;\ P(I=1 \mid T=2) = 0.3;\\ &P(L=1 \mid T=1) = 0.7;\ P(L=1 \mid T=2) = 0.3. \end{aligned} \tag{14}$$

From these, we directly calculate the average cost price $p^* = 0.58$ and the aggregate quantity $q(p^*) = 0.5$. Using an external estimate of the equilibrium elasticity of the aggregate market, say $\eta = -1$, we get $\sigma = 0.24$, $w_1 = 0.79$ and $w_2 = 0.37$. Using (12) we find the efficient allocation in the two synthetic markets $(p_1 = 0.70, q_1(p_1) = 0.59)$ and $(p_2 = 0.30, q_2(p_2) = 0.57)$, which can be compared with the actual allocation $(p^* = 0.57, q_1(p^*) = 0.70)$ and $(p^* = 0.59, q_2(p^*) = 0.30)$. 50% of individuals are actually insured in this market, compared to the efficient quantity

[9] Since there is no guarantee that the chosen external η captures precisely the true average elasticity of the aggregate demand derived from the estimated synthetic markets, it is a good practice, in line with this literature, to compute the welfare measures robustly under different values of η.

(58%). This example actually generates the market discussed in section 2 above and illustrated in figures 1 and 2.

Using the consumer surplus formula (13), we can calculate the welfare cost measures under counterfactual efficient prices in the two markets. The total welfare loss from unpriced heterogeneity is equal to $0.022 \times D$, which is is about 7.6% of total costs, and about 10.4% of maximum consumer surplus. One dollar gain for the riskier types costs about 1.57 dollars to the low risks. The socially efficient uniform price which minimizes welfare loss (from Figure 1) is equal to $\bar{p} = 0.46$, which is lower than the average cost price $p^* = 0.58$. The constrained-efficient allocation implies a welfare loss of $0.0041 \times D$, which is only 18.4% of the total welfare loss (the ratio between the welfare loss in figures 1 and 2).

To check the robustness of the estimated welfare loss, it is a good practice to compare the estimates obtained under logit demand functions with the standard formula for the excess burden of taxes and subsidies,

$$WL_t = -\frac{1}{2}\eta_t \frac{q_t}{p^*}\Delta_t(p)^2, \tag{15}$$

where, for each market t, $\Delta_t(p)$ equals $p_t - p^*$ and η_t is the elasticity of demand at (p^*, q_t). Under the assumption that in each market t the demand functions at the current price have approximately the same slope, η_t is equal to $\eta \frac{q}{q_t}$ and approximate welfare loss is

$$WL = -\frac{1}{2}\eta \frac{q}{p^*} \sum\nolimits_t \pi_t \Delta_t(p)^2, \tag{16}$$

which can be calculated from estimated prices and quantities in each synthetic market and external information on the elasticity of the aggregate demand η. In the example above, approximate welfare loss is 91% of the true welfare loss so that, for example, WL is about 6.9% of total cost compared to 7.6% under the true model with logit demand functions. Since we do not know the exact functional form of the insurance demand functions, equation (16) is a useful tool for checking the robustness of measured welfare loss.

5. The estimation of the types markets: A discrete multivariate finite mixture model

To estimate insurance purchase and loss probabilities for each vector $\boldsymbol{x}$ of insurance used controls, we assume that $P(I = 1 \mid \boldsymbol{x}, t)$ and $P(L = 1 \mid \boldsymbol{x}, t)$ are linearly additively separable in $\boldsymbol{x}$ and T:

$$P(I = 1 \mid \boldsymbol{x}, t) = F(\alpha_I(t) + \boldsymbol{x}'\boldsymbol{\beta}_I), \;\; P(L = 1 \mid \boldsymbol{x}, t) = F(\alpha_L(t) + \boldsymbol{x}'\boldsymbol{\beta}_L) \tag{17}$$

for some appropriate link function F (we use the logit link).

Before showing how to estimate α's and β's in (17), we pause to comment on the assumptions behind equation (17). As discussed in section 4 above, in line with this literature we have assumed that the scale parameter σ in the willingness to pay equation (7) does not change across types, which may be seen as a significant restriction. Furthermore, in equation (17) for parsimony we assume that the coefficients of $\boldsymbol{x}$ do not change across types.

Consider a general extension of (7) to include the pricing variables $\boldsymbol{x}$:

$$I = 1 \leftrightarrow w(t) + \boldsymbol{x}'\boldsymbol{\gamma}(t) + \sigma(t)\epsilon_i \geq p, \tag{18}$$

so that both the scale parameter and the coefficients of $\boldsymbol{x}$ are allowed to vary across types. To test the restrictions implied by (17), we can estimate the unrestricted demand for insurance

$$P(I = 1 \mid \boldsymbol{x}, t) = F(\alpha_I(t) + \boldsymbol{x}'\boldsymbol{\beta}_I(t)), \tag{19}$$

so that $\alpha_I(t) = \frac{w(t)-p}{\sigma(t)}$ and $\boldsymbol{\beta}_I(t) = \frac{1}{\sigma(t)}\boldsymbol{\gamma}(t)$. The null of homogeneous scale and pricing coefficients across types can be tested by a standard test statistic for the linear restrictions $\boldsymbol{\beta}_I(1) = \cdots = \boldsymbol{\beta}_I(M)$.

To estimate the parameters of interest for each synthetic market t we use of a *discrete multivariate finite mixture model* which is composed of four parts:

(1) The demand for insurance equation

$$P(I = 1 \mid \boldsymbol{x}, t) = F(\alpha_I(t) + \boldsymbol{x}'\boldsymbol{\beta}_I). \tag{20}$$

(2) The expected cost equation

$$P(L = 1 \mid \boldsymbol{x}, t) = F(\alpha_L(t) + \boldsymbol{x}'\boldsymbol{\beta}_L). \tag{21}$$

(3) An auxiliary system of equations which is instrumental to achieve identification of the heterogeneous types. The source of external variation is provided by a set of observable *indicators*, that is, observable manifestations of the unobserved heterogeneity which affects insurance choice and expected costs. Following the logic of Finkelstein and Poterba (2006) and Cutler, Finkelstein and McGarry *et al.* (2008), appropriate indicators can be chosen from the set of variables which are observable by the econometrician but not used by insurance companies. Examples of such variables are wealth, cognitive abilities, occupational risk, risk reducing or increasing behavior such as preventive care, seat belt use, smoking and drinking or, if panel data are available, past insurance choices and claims.[10] Having found a set $Z_1, \ldots, Z_H$ of suitable indicators, we form an auxiliary system of conditional probabilities for the indicators[11]

$$\begin{aligned} P(Z_1 = 1 \mid t) &= F(\alpha_{Z_1}(t)), \\ &\vdots \\ P(Z_H = 1 \mid t) &= F(\alpha_{Z_H}(t)). \end{aligned} \tag{22}$$

(4) The types membership probabilities. To force the types probabilities to lie between zero and one and sum to one, it is convenient to use a multinomial logit parameterization:

$$P(T = t) = \frac{\exp(\alpha_T(t))}{\sum_{t=1}^{M} \exp(\alpha_T(t))}, \; \alpha_T(M) = 0. \tag{23}$$

The $M-1$ logit parameters α_T are simply reparameterizations of the membership probabilities, and do not impose any parametric restriction on the distribution of T.

The discrete multivariate finite mixture model is defined by equations (20-21-22-23), with α's and β's being the model parameters. Model (20-21-22-23) can be seen as an instance of a discrete multivariate MIMIC model (Joreskog and Goldberger (1975)). Contrary to the MIMIC model, the unobserved heterogeneity T is not a continuous univariate variable on the real line, but an unstructured nonparametric variable. The unstructured nature of the random effects $\alpha_I(t)$ and $\alpha_L(t)$ in equations 20-21 is well suited to capture the possibly multidimensional residual heterogeneity. Our model can be seen as a multivariate extension of the single equation *semiparametric finite mixture model* (see e.g. Follmann and Lambert (1989) for a binary logit example) which has become popular in economics after the seminal paper by Heckman and Singer (1984).[12] The multivariate nature of our model can be seen as key for linking the unobservable determinants of

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[10]For simplicity, we assume that chosen indicators are actually binary variables. This restriction aims to simplify the discussion, but our econometric analysis can be performed as long as these indicators are discrete.

[11]In the auxiliary system (22) one could also condition the probability of each indicator to the controls $\boldsymbol{x}$, in which case the unobservable types are defined relatively to $\boldsymbol{x}$. For example, if Z_h is the choice of wearing seat belts which acts as an indicator of risk preference, conditioning say on age, helps identifying risk attitudes relatively to age, while if no conditioning is made, one tends to identify unadjusted risk preferences. In our experience, for the purpose of estimating the parameters of the main system of interest, in practice there is little difference in the results obtained under the two approaches.

[12]In fact, it can be easily seen that, since the α's are the only model parameters after conditioning to $\boldsymbol{x}$, and since the α's are one-to-one and differentiable functions (i.e. reparameterizations) of the probabilities of interest, model (20-21-22-23) is actually *nonparametric* conditional to a given value of $\boldsymbol{x}$ (c.f. equation (26) in Appendix A). When the variables in $\boldsymbol{x}$ are discrete, take a limited number of distinct configurations (strata), and sufficient observations are available for each stratum, estimation is nonparametric. When $\boldsymbol{x}$ is continuous or each strata contains too few observations, each conditional model in equations in 20-21 is linked by F and thus the analysis is semiparametric. In the LTC insurance market application in section 6 the analysis is semiparametric; the Medigap application in section 7 is nonparametric.

demand and the unobserved determinants of costs, using identifying variation from the auxiliary set of indicators to uncover residual heterogeneity.[13]

Suppose we have independent observations $(I_i, L_i, Z_{1,i}, \ldots, Z_{H,i}, \boldsymbol{x}_i)$ for a sample of N units, and let the binary variable $U_{t,i}$ indicate whether subject i belongs to type t. If $U_{t,i}$ were observable, the complete-data log likelihood for model (20-21-22-23) could be written as

$$\Lambda = \sum_{i=1}^{N}\sum_{t=1}^{M} U_{t,i}\log P(T=t) + \sum_{i=1}^{N}\sum_{t=1}^{M}\sum_{j=1}^{S} U_{t,i}Y_{j,i}\log P(Y_{j,i}=1 \mid T=t, \boldsymbol{x}_i)$$

where to simplify notation we have renamed the response variables $I, L, Z_1, \ldots, Z_H$ as $Y_1, \ldots, Y_S$ with $S = H + 2$, and the dependence of Λ on the model parameters is specified in the model equations (20-21-22-23). Estimation of the model parameters by maximization of Λ is a problem of incomplete data which may be tackled by the EM algorithm, which is the standard approach for maximum likelihood estimation of finite mixture models, and has been shown (Dempster, Laird and Rubin (1977)) to converge to the maximum of the true likelihood. Given the binary nature of the observed variables, the E-step is equivalent to compute, for each subject, the posterior probability of belonging to each unobservable type. The M-step requires maximization of a multinomial likelihood with individual covariates.[14] It is well known that the EM algorithm may converge even if the model is not identified, a crucial issue for finite mixture models. Identification of model (20-21-22-23) is discussed in Appendix A.

Despite the usefulness of finite mixture models to detect underlying residual heterogeneity, one unresolved issue in their application is how to determine the number of unobserved types M. The currently preferred approach suggests the use of Schwartz's Bayesian Information Criterion (BIC) to guide this choice, which in certain conditions is known to be consistent and generally helps preventing overparameterization (see McLachlan and Peel (2000) for a thorough introduction to finite mixture models and a review of existing criteria for the choice of the number of types). BIC is calculated from the maximized log-likelihood $L(\boldsymbol{\psi})$ by penalizing parameters' proliferation, $BIC(\boldsymbol{\psi}) = -2L(\boldsymbol{\psi}) + \upsilon log(n)$, where n denotes sample size and υ the number of parameters; the model with the lowest BIC is preferred.

5.1. **Interpretation of the types.** An output of our procedure is an estimate of the joint distribution of the set of indicators $\boldsymbol{Z} = [Z_1, \ldots, Z_H]$ and the residual heterogeneity T. From the estimated joint distribution of ($\boldsymbol{Z}$,T) we can get an estimate of the so called *posterior type probabilities* for some focal observable individual behaviour. Given a vector $\tilde{\boldsymbol{Z}}$ of observable indicators of focal interest, from $P(T, \tilde{\boldsymbol{Z}})$ posterior type probabilities are obtained using

$$P(T = t \mid \tilde{\boldsymbol{Z}} = \tilde{\boldsymbol{z}}) = \frac{P(T = t, \tilde{\boldsymbol{Z}} = \tilde{\boldsymbol{z}})}{P(\tilde{\boldsymbol{Z}} = \tilde{\boldsymbol{z}})}.$$

Posterior type probabilities help understanding the underlying structure of residual heterogeneity, since they may give useful information on the nature of the estimated types.

5.2. **Multiple losses.** In many circumstances the insurance contract protects against multiple losses. For example, Medigap protects against high out of pocket expenses for several health care services, such as inpatient, outpatient and specialist visits. The framework above can then be extended by simultaneously considering say J binary outcomes L_j, $j = 1, \ldots, J$, which take value one if the individual experiences the loss of type j for which he is insured. The demand and

[13]For comparison, the standard approach of estimating demand and cost with two finite mixture logit models, using the auxiliary set of indicators as controls in the types' membership probabilities, has some major shortcomings: it does not allow a direct link between the unobservable determinants of demand and expected costs; it cannot be used in the nonparametric case; and in our experience even with a rich set of covariates $\boldsymbol{x}$ generally implies a much more fragile identification of the unobserved types.

[14]We are grateful to Antonio Forcina for kindly providing the Matlab code for the EM estimation. Our estimation has been done with a code rewritten in Stata, which is available upon request.

cost functions of intererest are

$$\begin{aligned} P(I=1 \mid \boldsymbol{x}, t) &= F(\alpha_I(t) + \boldsymbol{x}^{'}\boldsymbol{\beta}_I) \\ P(L_j=1 \mid \boldsymbol{x}, t) &= F(\alpha_{L_j}(t) + \boldsymbol{x}^{'}\boldsymbol{\beta}_{L_j}),\ j=1,\dots,J \end{aligned} \tag{24}$$

For sharper identification of the mixture components, this system of equations of main interest is integrated by the auxiliary system, and the complete model is (22)-(23)-(24).

5.3. **Conditional independence.** The decomposition of the observed market into synthetic markets with given demand and constant MC curves is key for our policy analysis. The decomposition is based on the assumption that the unpriced heterogeneity variable T extracts all systematic differences in individuals' valuation of the insurance contract and expected costs; from this assumption it follows that if the 'deep' types were contractible and heterogeneity was accordingly priced, welfare loss would disappear.[15]

It is important to realize that for efficiency what counts is that types with the same expected costs face the same price; residual heterogeneity in willingness to pay within types with the same expected cost is not important. For example, if there are two types with the same expected costs but different expected valuation of the contract, but we lump them together into a single type, pricing at the common marginal cost would still eliminate welfare loss. On the other hand, if two types have different costs and demands but we lump them together, it is theoretically possible that pricing the lumped type at their expected costs does not increase efficiency (see e.g. the example in footnote 3 in Einav and Finkelstein (2011)). This underscores the importance of uncovering all deep types with expected cost differences. In practice, depending on the nature of the sample, the econometric procedure extracts types which give a statistically sufficient fine partition of the unpriced heterogeneity.

6. Application to the US long-term care insurance market

In a seminal paper Finkelstein and McGarry (2006) (henceforth FM) study the long-term care (LTC) insurance market in the USA. LTC expenditure risk is one the greatest financial risks faced by the elderly in the US; to get a quantitative feeling of its importance, the amount of expenditure in nursing home care in 2004 was about 1.2% of the US GDP. FM notice that in the sample there is *negative* correlation between insurance purchase and nursing home use. They show that individuals who exhibit more cautious behavior - as measured either by their investment in preventive health care or by seat belt use - are both more likely to have long-term care insurance coverage and less likely to use long-term care, so they conclude that the market is advantageously selected.

6.1. **Data and variables definition.** We use FM dataset as reported in table 4 of their paper (FM (2006) pg. 948). This is a subsample of individuals in the top quartile of the wealth and income distribution without any health characteristics that might make them ineligible for long-term care insurance.[16] We use as insurance purchase and risk occurrence two binary variables, namely *LTC Insurance* which takes value one if the individual has long-term care insurance, and *Nursing Home* which takes value one if the individual enters a nursing home in the following 5 years. As observed characteristics used by insurance company ($\boldsymbol{x}$) we use the individual probability of entering a nursing home, which is calculated by FM from a standard actuarial model, with a cubic specification.

As indicators for the residual unobserved heterogeneity we use the following binary variables: *Seat Belt* which takes value 1 if the subject always wears seat belts; *No Smoking and Drinking* which takes the value 1 if the subject either currently does not smoke or has less than three drinks

[15] The assumption that, within each synthetic market, individuals' valuation of the contract and expected costs are independent can be tested by estimating M association parameters of the joint distribution of (I, L) for each t. In particular, we can estimate the *log-odds ratio* between I and L for each type $t = 1, \dots, M$, and test their significance by a LRT statistic which is asymptotically distributed as χ^2_M.

[16] FM's dataset is available in the AER website. We thank FM, and the AEA for their policy of providing data for published articles.

per day; *Subjective Riskiness* which takes value 1 if the individual self-reported probability of nursing home utilization is higher than the insurance company estimated probability; *Preventive Care* which takes value 1 if the subject has taken more gender appropriate preventive care procedures in the past year than the median value. In this sample there are 1491 individuals; about 17% have long-term care insurance in 1995 and 10% enter a nursing home in the following 5 years period.

6.2. **Results.** The finite mixture model (20-21-22-23) is estimated using the 4 indicators above to set the auxiliary system (22). To choose the number of mixture types we use Schwartz's BIC, which achieves the minimum value with two mixture types. For completeness, estimates of α's and β's and their standard errors are reported in Appendix B, but for economy of space estimated coefficients are not discussed in the main text. About 30% of individuals are of type 1, and 70% of type 2. We first test for the homogeneity of $\sigma(t)$ and $\boldsymbol{\beta}(t)$ by estimating the unrestricted model as explained in section 5 above. The LR is equal to 2.977, which is distributed as chi-square with 3 d.f. and has a p-value of .395. Thus, the homogeneity null in this sample cannot be rejected.

Table 1 reports the conditional probabilities by types for the six observed variables.[17] Conditionally on $\boldsymbol{x}$ there seems to be a substantial difference in insurance purchase and nursing home use between the two types; type 1 individuals are four times less likely to buy LTC insurance, but more than twice as likely to use a nursing home as types 2. The table confirms the presence of unpriced heterogeneity, and in particular of advantageous selection, since claims and coverages are anticomonotone across types. There seems to be a natural ordering of the types in terms of their cautiousness, such that, going from types 1 to types 2, there is a significant increase in the probability of using seat belts and preventive care, of refraining from smoking and drinking, and believing that one may need a nursing home in the near future with a higher probability than that predicted by insurance companies.

TABLE 1. Estimated conditional probabilities by types

	T=1	T=2
Seat Belt	.611	.932
Subjective Riskiness	.355	.491
Preventive Care	.254	.449
No Smoking and Drinking	.790	.949
Long-Term Care Insurance	.072	.276
Nursing Home	.213	.095

From the estimated joint distribution of the indicators and the unobserved types we can get an estimate of the posterior type probabilities for some focal observable behaviour: a 'cautious' individual who always wears a seat belt, does not smoke and drink, has a cautious estimate of his probability of needing a nursing home in the future and engages in preventive care, and a 'reckless' individual with the opposite attitudes. Table 2 reports the estimated posterior type probabilities for these two individuals. A glance at the table confirms the presumption that types 2 are predominantly the 'cautious' types, while types 1 are the 'reckless' ones.

TABLE 2. Estimated posterior type probabilities

	T=1	T=2
Cautious	.087	.912
Reckless	.946	.054

Our analysis suggests that in LTC insurance there are two unobserved synthetic markets: one for the cautious types, with a high demand curve and a low expected MC curve; and one for

[17] The conditional probability for nursing home use and LTC insurance are averaged across insurance risk classification.

the reckless types, with low demand curve and a high MC curve.[18] The existence of these two synthetic markets with large differences in cost structures (recall that reckless types are 2.2 times more likely to use LTC) implies potentially serious inefficiency in actual contractual practices. We recover the parameters needed for our policy analysis using estimated proportion of types and claims and ceverage probabilitites from Table 1, coupled with external information on the average elasticity of the demand for LTC insurance. In two recent papers, Courtemanche and He (2009) and Goda (2011) find a substantial price responsiveness of the demand for LTC insurance at the current low ownership rate, with the average price elasticity of LTC insurance η estimated at at -3.9 and -3.3 respectively. We use $\eta = -3.5$ for our calculations.

Using the consumer surplus formula (13), we calculate the welfare cost measures under counterfactual efficient prices in the two markets. The welfare loss from unpriced heterogeneity is 10% of total insurers' costs, and 28.6% of maximum consumer surplus.[19] Comparing the welfare loss for low risk types $T = 1$ with the welfare gain for the high risk types $T = 2$, a one dollar gain for the riskier types costs a whopping 5.5 dollars to the low risks. This latest huge number can be explained by the very low value that 'reckless' type in this sample give to the insurance contract compared to the 'cautious' ones.

For robustness, we redo these calculations using a lower value for the unusually high external estimate of the elasticity η. This is particularly relevant since, in line with Finkelstein and McGarry (2006), for homogeneity we have used the subsample of healthy and wealthy individuals, and there is no guarantee that the true price elasticity η in our estimated market reflects accurately those calculated using a more representative sample of LTC insurance. We use a conservative estimate by halving η at = -1.75. When $\eta = -1.75$, welfare loss is equal to 7% of total costs and 12.4% of maximum consumer surplus. The relative dollar cost of the cross subsidization is 2.76.

The aggregate market is depicted in Figure 3. The constrained-efficient uniform price ($\bar{p}$ where the MC curve crosses the demand curve) is only marginally higher than the average cost price p^*. We calculate that less than 1% of the current welfare loss is attributable to the difference in welfare

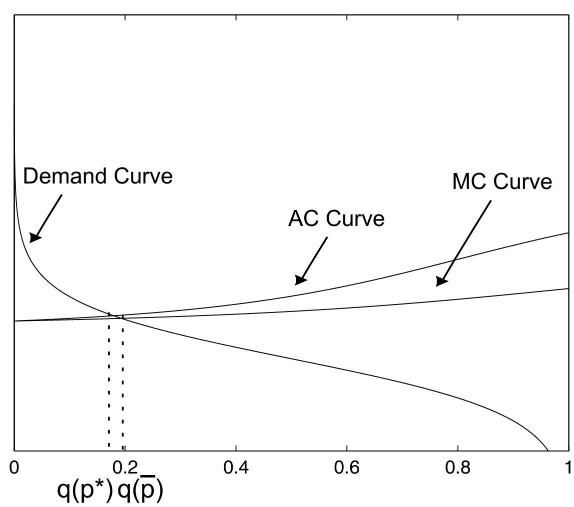

FIGURE 3. LTC: Aggregate market

maximizing uniform price. Furthermore, while advantageous selection implies that the optimally

[18]As discussed in Section 5.3 above, a test of the absence of residual loss/coverage correlation in each synthetic market can be performed by estimating the log-odds ratios for the association between *LTC Insurance* and *Nursing Home* in each market. The LRT statistic of their significance is equal to 0.036 with p-value .98, so we can safely conclude that, within each synthetic market, there is no significant residual correlation between individuals' valuation of the contract and expected costs.

[19]We also calculate approximate welfare loss using linear demands as in (16). At $\eta = -3.5$, welfare loss with linear demand is about 12% higher than that calculated using logit demands.

set uniform price $\bar{p}$ would actually *decrease* the aggregate quantity of LTC insurance, optimal MC prices in each synthetic market would *increase* LTC insurance purchase by almost 40% (if types were priced at their MC, the total quantity of insurance purchase would increase from 17% to 24%, since cautious individuals would substantially increase their demand for insurance, while reckless ones would decrease their demand by a much smaller quantity). Since the small size of the LTC insurance market is currently an important policy concern (e.g. Brown and Finkelstein (2011)), our results suggest that targeting the heterogeneous types is an important element of policy intervention. In sum, our estimates suggest that exploring appropriate policies of price discrimination may have relevant efficiency returns, since lack of price-discrimination is responsible for most of the inefficiency of current prices.

7. Application to the US Medigap insurance market

A Medigap insurance plan is a health insurance contract sold by a private company to fill 'gaps' in coverage of the basic Medicare plan. Medigap plans offer additional services and help beneficiaries pay health care costs (deductibles and co-payment) that the original Medicare plan does not cover. As a relevant example for our application, Medicare's coinsurance or copayments for hospital stays, physician visits or outpatient care are covered by the Medigap plan.

The Medigap insurance market is quite interesting to study as a further application of our methods since, contrary to the LTC insurance market, it is highly regulated.[20] In a recent influential paper, Fang, Keane and Silverman (2008) consider private information in the Medigap market using data obtained by imputing HRS observations for year 2002 in the Medicare Current Beneficiary Survey. They find a negative correlation between Medigap supplemental coverage and ex post medical expenditure, and argue that individual cognitive ability is the main source driving advantageous selection in the Medigap market.

7.1. **Data and variables definition.** The HRS is a biennial survey targeting elderly Americans over the age of 50 sponsored by the National Institute on Aging. Although the survey is not conducted on an yearly basis, from 1992 it provides longitudinal data for an array of information, consistently administrated, on several different fields such as health and health care utilization, type of insurance coverage.

We use data from waves 2-9 of the HRS, covering from 1994 to 2010. Since the interview is conducted every two years, we focus on the coverage decision for the individuals who are either 65 or 66 years old at the time they were interviewed, and analyze two separate subsamples for males and females. Thus, within each subsample individuals are homogeneous in terms of the structure of the contract they face, and, in terms of our finite mixture model, the pricing vector $\boldsymbol{x}$ is empty. Therefore, in contrast to the LTC insurance application above, the analysis is nonparametric.

Following Fang, Keane and Silverman (2008) we define Medigap status (*Medigap* to be equal to one if an individual is covered by Medicare and has deliberately purchased a supplemental plan additional to Medicare. We excluded from the dataset individuals who were younger than 65 years and older then 67 in each wave, or are also enrolled in any other public program or receive Medigap insurance coverage by his/her or spouse's former employer. Claims are measured by the following binary variables which take 1 if an individual: i) had any hospital stay (*Hospital*); ii) had more than five doctor visits (*Doctor*); iii) used any outpatient service such as surgery or home care facilities in the twelve months prior to the interview (*Out. Services*). As additional indicators to identify unobserved types we also use *Subjective Health*, which equals 1 if the individual reports good or very good health; *Income* which equals 1 if the individual is in the top wealth quartile; (*Hospital-1*), (*Doctor-1*), (*Out. Services-1*) which indicate whether the individual used medical care in the previous wave; *Insurance-1* which indicates if individual was covered by a health insurance in the

[20] Federal Law affects the Medigap market at least in three ways. First, Medigap plans are standardized into ten plans,'A' through 'J', and the basic plan 'A' must be offered if any other more generous plan is also offered. Second, there is a six-month enrollment period when people turn 65 years old where Medigap cannot refuse any person even if there are pre-existing conditions. Third, pricing criteria are mainly based on individual's age and sex. Therefore, insurers are not free to offer any insurance contract at any price they choose.

previous wave; *Life Insurance* which equals 1 if the individual has a life insurance; *Annuity* which indicates if the individual purchased a life annuity; *Safe Assets* which equals 1 if the individual has a share of portfolios invested in Treasury bills or savings accounts greater than those invested in stock. There are 5437 observations in the sample and there are about 56% of females.[21]

7.2. **Results.** We estimate the finite mixture model (22)-(23)-(24) using the 9 indicators discussed above to set the auxiliary system (22). Table 3 reports the maximized log-likelihood and the BIC for different numbers of unobserved types for the males and females subsample.

TABLE 3. Log-likelihood and BIC

	2LC	3LC	4LC	5LC	6LC
			Males		
Log-Likelihood	-17416.93	-17301.13	-17237.40	-17186.78	-17148.26
BIC	35044.15	34921.58	34903.15	34910.95	34942.95
			Females		
Log-Likelihood	-22351.59	-22186.81	-22078.96	-22011.85	-21971.63
BIC	44919.58	44702.23	44598.72	44576.70	44608.49
# of parameters	27	41	55	69	83

The BIC seems to indicate that four types for males and five types for females are adequate to represent any residual unobserved heterogeneity. We report only the models with the better fit.[22] Calculating the types membership probabilities, for male about 21% of individuals are of type 1, 25% of type 2, 33% of type 3 and 21% of type 4, and for females about 25% of individuals are of type 1, 21% of type 2, 27% of type 3, 12% of type 4, and 15% of type 5.

7.3. **The males subsample.** Table 4 reports the estimated conditional probabilities by types for the four variables of interest and the nine auxiliary indicators for the males subsample.

TABLE 4. Estimated conditional probabilities for males

	T=1	T=2	T=3	T=4
Panel A: Main Equations				
Hospital	0.0728	0.0642	0.3111	0.4143
Doctor	0.1902	0.1342	0.8166	0.7951
Out. Services	0.0882	0.1661	0.3639	0.3278
Medigap	0.1364	0.3294	0.1865	0.1797
Panel B: Auxiliary Indicators				
Hospital-1	0.0877	0.0833	0.2121	0.3847
Doctor-1	0.1150	0.1540	0.6143	0.7526
Out. Services-1	0.0779	0.1572	0.2831	0.2092
Insurance-1	0.0932	0.2104	0.1077	0.0943
Sub. Health	0.4306	0.7380	0.4260	0.0835
Life Insurance	0.6656	0.8078	0.8171	0.7218
Annuity	0.2210	0.7891	0.7596	0.2588
Income	0.0946	0.3426	0.4143	0.0405
Safe Assets	0.1352	0.4186	0.3431	0.1447
Panel C: Types Proportion				
	0.21	0.25	0.33	0.21

[21]Following Fang, Keane and Silverman (2008), we have also considered cognitive skills as indicator. However, variables mesuring cognitive skills are available only for a single wave, so including it in the analysis would drastically reduce the sample size.

[22]Estimated parameters for the other cases are available from the authors. The general picture emerging in other cases is similar to the results discussed below. For completeness we report estimates of α's and their standard errors in Appendix B, but for economy of space estimated coefficients are not discussed in the main text.

To calculate the expected marginal costs for the four types, since there are multiple losses, we use an estimation of the relative proportion of the cost of claims in the three medical care use variables. According to a study conducted in 2006 for America Health Insurance Plans (AHIP) (PriceWaterhouseCoopers (2006)), about 18% of health care insurance premiums cover inpatient care, 24% physician services and 22% outpatient care.[23] After weighting the three medical care use variables by their relative weight, we find that expected marginal costs for the four types is equal to $c_1 = 0.122$, $c_2 = 0.125$, $c_3 = 0.519$ and $c_4 = 0.527$; and estimated expected average cost is 0.312.[24]

Expected cost estimates indicate a striking heterogeneity in claims across different types. Types 3 and 4 are high risk, and are on average 4 times more likely to use medical resources than low risk types 1 and 2. Looking at coverage probabilities across types we see that claims and coverages are not comonotone; for example, low risk types 1 have a much lower probability of buying insurance compared to high risk types 3 and 4, but the opposite is true for low risk types 2. Lack of comonotonicty between claims and coverages suggests multidimensional heterogeneity. This is confirmed also by looking at Panel B in Tables 4, which reports conditional probabilities for the auxiliary indicators, where it emerges that there is no clear unique common underlying order of the types in terms of wealth, subjective health, past use of medical resources and insurance coverage, and risk preferences.

We recover the parameters needed for our policy analysis from the estimated parameters in Table 4, coupled with external information on the average elasticity of the demand for Medigap insurance, which has been recently estimated by Starc (2010) at -1.11. Using the consumer surplus formula (13), we can calculate the welfare cost measures under counterfactual efficient prices in the four markets.[25]

Before calculating the inefficiency of cross-subsidization in the four synthetic markets, we use our estimates to examine the characteristics of the *aggregate* Medigap market implied by our estimates. Figure 4 illustrates the aggregate market. In the aggregate market, the difference between

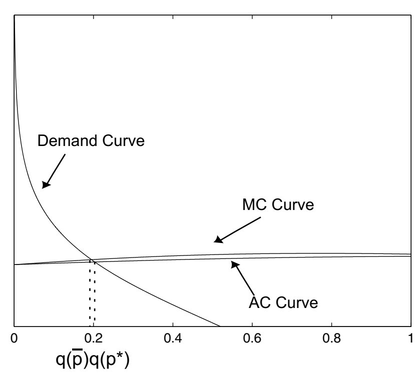

FIGURE 4. Medigap: Male aggregate market

the AC and MC curves is very small, and thus the estimated welfare loss is negligible. This is due to the fact that aggregate willingness to pay does not map into a unique value of individuals' riskiness, and average realized costs come from a finite mixture of high and low risk individuals,

[23]The remaining portion going to prescription drugs, home and nursing home care, government payments and administrative costs, consumer services and profits.

[24]Since what matters in our approach is to capture differences in expected costs between types rather than between different claims, it is unlikely that this approach introduces significant distortions in the results.

[25]Absence of residual loss/coverage correlation within each synthetic market can be tested by estimating 12 log-odds ratios for the association of *Medigap* with each of the three medical care variables (three log-odds ratios for each of the four markets). The LRT statistic is equal to 11.47 with p-value 0.4884. Thus, also in this case we can safely conclude that, within each unobserved market, there is no significant residual correlation between individuals' valuation of the contract and expected costs, which is a necessary condition for valid counterfactual analysis.

so that the aggregate MC curve is almost flat. Therefore, looking at the aggregate demand and cost functions we might conclude that in the Medigap market there is insignificant residual heterogeneity and that the market is constrained-efficient. On the other hand, the four synthetic markets show that there is a large difference in expected costs between high and low risk individuals, and current contracts cause substantial inefficiencies due to the implicit cross-subsidization of high risks at the expense of low risks. In fact, the estimated welfare loss from unpriced heterogeneity is $0.014 \cdot D$ per individual per year, which is about 86 times greater than the welfare loss estimated in the aggregate market. The estimated welfare loss can be compared to the total coverage costs (21.5%) and to the total surplus from the efficient allocation (21.4%). One dollar subsidy to the high risk types 3 and 4 costs two dollars to the low risk types 1 and 2.

For robustness, we again redo these calculations, halving and doubling Starc's (2010) external elasticity estimate. When $\eta = -0.555$, welfare loss scaled to total costs is 11% and to maximum consumer surplus is 6.6%. The relative dollar cost of the cross subsidization is 1.43. When $\eta = -2.22$, welfare loss scaled to total costs is 40.5% and to maximum consumer surplus is 50.7%. The relative dollar cost of the cross subsidization is 3.67.

7.4. **The females subsample.** Table 5 reports the estimated conditional probabilities by types for the females subsample.

TABLE 5. Estimated conditional probabilities for females

	T=1	T=2	T=3	T=4	T=5
Panel A: Main Equations					
Hospital	0.1702	0.1672	0.0423	0.2405	0.6238
Doctor	0.7289	0.5481	0.1184	0.7413	0.9312
Out. Services	0.2889	0.1380	0.1094	0.2611	0.5097
Medigap	0.1293	0.1153	0.3359	0.6944	0.2678
Panel B: Auxiliary Indicators					
Hospital-1	0.1524	0.1407	0.0322	0.1489	0.5036
Doctor-1	0.7261	0.4796	0.0855	0.5646	0.9073
Out. Services-1	0.2769	0.1091	0.0788	0.1844	0.4261
Insurance-1	0.0326	0.0545	0.2237	0.5313	0.1021
Sub. Health	0.5966	0.2784	0.7751	0.3910	0.0828
Life Insurance	0.7179	0.6786	0.6365	0.5289	0.6576
Annuity	0.7709	0.1424	0.6325	0.7057	0.3713
Income	0.4899	0.0133	0.2790	0.2323	0.1346
Safe Assets	0.4316	0.0778	0.3458	0.3989	0.1728
Panel C: Types Proportion					
	0.25	0.21	0.27	0.12	0.15

Expected marginal cost for the five types are equal to $c_1 = 0.421$, $c_2 = 0.300$, $c_3 = 0.094$, $c_4 = 0.435$ and $c_5 = 0.700$; and estimated expected average cost is 0.346. Expected cost estimates indicate striking heterogeneity in claims also in the females subsample, with the highest risk type having more than seven times greater expected claims than the lowest risk type. Also in this subsample, claims and coverages are not comonotone across types, and multidimensional heterogeneity is also confirmed by looking at the lack of a common underlying order of the types in terms of auxiliary indicators.[26]

Figure 5 illustrates the aggregate market, which reveals a very similar picture to the males subsample. The difference between the AC and MC curves is very small and estimated welfare loss is negligible; this is in strack contrast with the five synthetic markets which show large differences

[26]Absence of residual loss/coverage correlation within each synthetic market can be tested by estimating 15 log-odds ratios for the association of *Medigap* with each of the three medical care variables (three log-odds ratios for each of the four markets). The LRT statistic is equal to 23.12 with p-value 0.0815. Thus, also in this case we can conclude that, within each unobserved market, there is no significant residual correlation between individuals' valuation of the contract and expected costs.

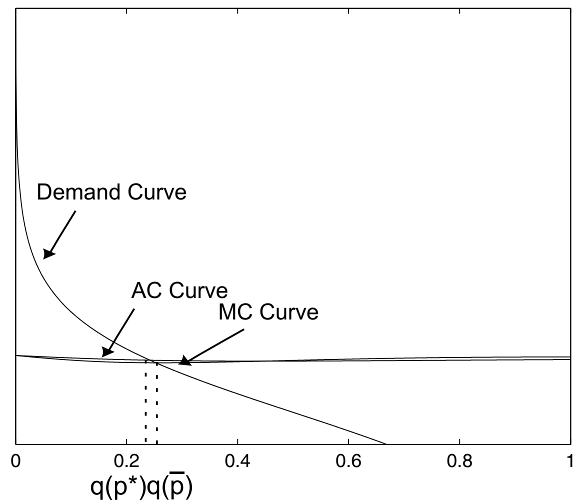

FIGURE 5. Medigap: Female aggregate market

in expected costs. The estimated welfare loss is 20% of the total coverage costs and 21.5% of total surplus from the efficient allocation. One dollar subsidy to types 1, 4 an 5, who are riskier than the average female population, costs 2.16 dollars to types 2 and 3. When $\eta = -0.555$, welfare loss scaled to total costs is 10.5% and to maximum consumer surplus is 6.8%. The relative dollar cost of the cross subsidization is 1.51. When $\eta = -2.22$, welfare loss scaled to total costs is 34.5% and to maximum consumer surplus is 48.7%. The relative dollar cost of the cross subsidization is 3.62.

Summing up, in both the male and female subsamples we show the multidimensional nature of unpriced heterogeneity, which imply the existence of both adverse and advantageous selection in different segments of the population. From the policy perspective, we find that imposing uniform prices through regulation preventing categorical discrimination is a rather inefficient way to subsidize individuals with high health risk. While the magnitude of the possible efficiency gains of price discrimination seems to be quite dependent on the true elasticity of the demand for Medigap insurance, all plausible scenarios suggest that these efficiency gains can be large, suggesting that reforming the Medigap market may give substantial returns. The design of an efficient pricing scheme to uncover differences in expected costs, however, involves subtle dynamic issues and is clearly outside the scope of the present paper.

8. CONCLUSION

In this paper we considered the welfare loss of unpriced heterogeneity in insurance markets, which results when private information or regulatory constraints prevent insurance companies from setting premiums reflecting expected costs.[27] Using commonly available survey data on claims and coverage probabilities, we uncover some 'deep types' which determine expected costs and insurance demand given actual pricing, and create a finite number of 'synthetic' markets. This allows counterfactual policy analyses quantifying the efficiency costs of unpriced heterogeneity.

We apply our methods to the US Long-Term Care and Medigap insurance markets, where we find that unpriced heterogeneity causes substantial inefficiency, due to the implicit cross-subsidization of high risks at the expense of low ones. If these deep types were contractible, such inefficiency would disappear and there would be no selection. Our quantitative welfare loss measures are generally higher than existing ones (e.g. Carlin and Town (2010), Einav, Finkelstein and Cullen (2010) and Bundorf, Levin and Mahoney (2012)). Notice however that the welfare loss

[27]There may be other reasons which prevent setting premiums equal to expected costs. For example, political economy considerations may prevent setting premiums according to, say, sexual orientation, even if expected costs depended on it.

measure in Einav, Finkelstein and Cullen (2010) and in Carlin and Town (2010) (the former reports an estimate of relative welfare loss equal to 2%; the latter lower than 1%) refer to the inefficiency of current pricing compared to optimally set uniform prices which obtain if a social planner minimized welfare loss, could not price discriminate, but is not constrained by the zero-profit condition since it could tax or subsidy the market. On the other hand, Bundorf, Levin and Mahoney (2012) estimated a welfare loss of 1-13% of total coverage costs, comparing current prices with risk-rated premiums, which is more in line with our approach. Our larger estimated welfare loss can probably be explained in the LTC insurance market by the large elasticity of the demand for insurance, and in the Medigap market by the rather extreme regulation, particularly in the open enrollment period. In doing these comparisons, it should be however kept in mind that our results refer to specific markets, which differ from the market for employers-sponsored health plans considered by these papers.

Our methodology allows using large rapresentative samples, and given the wealth of commonly available and well known survey data, allows exploring the efficiency of many insurance markets where the researcher does not have access to firm data. The most relevant limitation of our approach is probably the need to use external information on the price elasticity of the demand for insurance. In common with most of this literature, our model is static, and appraises the efficiency of the pricing of existing contracts ignoring the inefficiency that unpriced heterogeneity may create by distorting the set of contracts offered.

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Appendix A. Identification

In this section we discuss the identification of the finite mixture model (20-21-22-23). Suppose we want to identify demand for insurance and expected costs for the different types conditionally on a given value of $\boldsymbol{x}$, which we omit for simplicity:

$$P(L = 1 \mid T = t),\ P(I = 1 \mid T = t),\ P(T = t),\ t = 1, \ldots, M, \tag{25}$$

so that we want to identify $2 \times M + M - 1$ unobservable parameters. Since we observe only I and L (the joint distribution of two binary variables with 3 free parameters), unless covariates' variation and parametric restrictions are imposed, it is impossible to identify (25) even with $M = 2$.

Our strategy is to look for external sources of variation by means of the set of indicators of the unobservable variable T, $Z_h = 1, \ldots, H$, which allow to increase the number of observable parameters; and at the same time to impose appropriate conditional independence assumptions which reduce the number of parameters to be identified.[28] Let $S = H + 2$ denote the number of observed *responses*, that is, I and L which are of main interest and the set of indicators which are instrumental for identification. To simplify notation let us remane the response variables $I, L, Z_1, \ldots, Z_H$ as $Y_1, \ldots, Y_S$. The joint distribution of the observed responses is a function of the unobserved types' marginal probabilities and responses' conditional probabilities:

$$P(Y_1 = i_1, \ldots, Y_S = i_S) = \sum\nolimits_{t=1}^{M} P(T = t) \prod\nolimits_{j=1}^{S} P(Y_j = i_j \mid T = t),\ i_j \in \{0, 1\}. \tag{26}$$

[28]The use of conditional independence assumptions as a possible strategy for achieving nonparametric identification is discussed, among others, in the survey by Matzkin (2007).

Equation (26) implies that observed parameters are linked to unobserved ones by a nonlinear system of $2^S - 1$ equations into $(S+1) \times M - 1$ unknowns. If a sufficient number of indicators are available, a large number of types can be identified. For example, in the LTC insurance application $S = 6$, and in principle up to 9 different types can be identified. In the Medigap application $S = 13$, so in principle a very large number of types can be identified.

It is well known that counting the number of observable parameters is only a necessary condition for identification of model parameters; there are pathological examples in the literature on finite mixture models which show that this counting condition is not sufficient. More formally, if we write compactly equation (26) above as $\boldsymbol{q} = \boldsymbol{f}(\boldsymbol{p})$, where $\boldsymbol{p}$ denotes the vector which arrays the unobservable parameters of the mixture model and $\boldsymbol{q}$ the vector which arrays the observable joint distribution of the responses, model (20-21-22-23) is identified if, for every $\boldsymbol{p}'$ and $\boldsymbol{p}''$,

$$\boldsymbol{f}(\boldsymbol{p}') = \boldsymbol{f}(\boldsymbol{p}'') \rightarrow \boldsymbol{p}' = \boldsymbol{p}'' \tag{27}$$

that is, the model is identified if the mapping between the parameters of the observed variables and the mixture model parameters is one-to-one; in other words, the model is identified if the equation $\boldsymbol{q} = \boldsymbol{f}(\boldsymbol{p})$ has at most one solution for each value of $\boldsymbol{q}$.

While the state of the mathematical art is such that there are no general conditions for the uniqueness of the solution of nonlinear systems of equation, Allman, Matias and Rhodes (2009) provide sufficient conditions for the *generic identifiability* of finite mixture models. In particular, in Corollary 5 they show generic identifiability of finite mixtures of binary variables whenever the number of observed variables S and the number of mixing types M satisfies

$$S \geq 2\lceil \log_2(M) \rceil + 1 \tag{28}$$

where $\lceil x \rceil$ denotes the smallest integer at least as large as x. Since generic identifiability implies that the set of nonidentifiable parameters has measure zero, generic identifiability of a model is sufficient for statistical inference, since observed data has probability one of coming from an identifiable distribution. Equation (28) implies that, for all values of $\boldsymbol{x}$, in the LTC insurance market application at least 4 types are identified, while in the Medigap market application at least 64 types are identified.

Appendix B. Tables

TABLE 6. Long-Term Care Insurance: Estimated α parameters

	$T = 1$	$T = 2$
Panel A: eq. 20-21		
Long-term Care Insurance	-2.867	-1.231
	(0.728)	(0.139)
Nursing Home	-1.853	-2.894
	(0.263)	(0.247)
Panel B: eq. (22)		
Seat Belt	0.453	2.623
	(0.356)	(0.478)
Subjective Riskness	-0.599	-0.0354
	(0.203)	(0.0977)
Preventive Care	-1.077	-0.204
	(0.273)	(0.113)
No Smoking and Drinking	1.327	2.929
	(0.273)	(0.361)
Panel C: eq. (23)		
	0.842	-
	(0.565)	-

Standard errors in brackets.

TABLE 7. Long-Term Care Insurance: Estimated β Parameters

	LTCI	NH
Risk Classification	0.926	1.645
	(0.510)	(0.627)
Risk Classification^2	-0.0531	-0.0197
	(0.0251)	(0.0274)
Risk Classification^3	0.0647	-0.000171
	(0.0309)	(0.0326)

Standard errors in brackets.

TABLE 8. Medigap Males: Estimated α parameters

	$T = 1$	$T = 2$	$T = 3$	$T = 4$
Panel A: eq. (24)				
Hospital	-2.545	-2.679	-0.795	-0.346
	(0.287)	(0.283)	(0.114)	(0.129)
Doctor	-1.449	-1.864	1.493	1.356
	(0.215)	(0.319)	(0.215)	(0.184)
Out. Services	-2.335	-1.614	-0.559	-0.718
	(0.268)	(0.160)	(0.103)	(0.130)
Insurance	-1.846	-0.711	-1.473	-1.518
	(0.219)	(0.127)	(0.129)	(0.157)
Panel B: eq. (22)				
Hospital-1	-2.342	-2.399	-1.312	-0.470
	(0.249)	(0.219)	(0.130)	(0.134)
Doctor-1	-2.041	-1.703	0.465	1.112
	(0.291)	(0.207)	(0.128)	(0.185)
Out. Services-1	-2.472	-1.679	-0.929	-1.330
	(0.275)	(0.158)	(0.107)	(0.152)
Insurance-1	-2.276	-1.322	-2.115	-2.262
	(0.242)	(0.137)	(0.161)	(0.209)
Sub. Health	-0.279	1.036	-0.298	-2.396
	(0.153)	(0.174)	(0.133)	(0.399)
Life Insurance	0.688	1.436	1.497	0.954
	(0.135)	(0.151)	(0.131)	(0.135)
Annuity	-1.260	1.319	1.151	-1.052
	(0.310)	(0.244)	(0.186)	(0.248)
Income	-2.258	-0.652	-0.346	-3.165
	(0.299)	(0.133)	(0.135)	(0.711)
Safe Assets	-1.856	-0.329	-0.649	-1.777
	(0.250)	(0.132)	(0.112)	(0.208)
Panel C: eq. (23)				
	-0.187	-0.259	0.430	-
	(0.261)	(0.198)	(0.240)	-

Standard errors in brackets.

TABLE 9. Medigap Females: Estimated α parameters

	$T=1$	$T=2$	$T=3$	$T=4$	$T=5$
Panel A: eq. (24)					
Hospital	-1.584	-1.606	-3.121	-1.150	0.506
	(0.146)	(0.162)	(0.274)	(0.193)	(0.164)
Doctor	0.989	0.193	-2.007	1.053	2.605
	(0.139)	(0.123)	(0.272)	(0.231)	(0.344)
Out. Services	-0.901	-1.832	-2.097	-1.040	0.0387
	(0.111)	(0.181)	(0.152)	(0.180)	(0.136)
Insurance	-1.907	-2.038	-0.682	0.821	-1.006
	(0.264)	(0.222)	(0.0954)	(0.316)	(0.148)
Panel B: eq. (22)					
Hospital-1	-1.716	-1.809	-3.402	-1.743	0.0146
	(0.145)	(0.168)	(0.308)	(0.231)	(0.143)
Doctor-1	0.975	-0.0818	-2.370	0.260	2.281
	(0.148)	(0.124)	(0.313)	(0.186)	(0.301)
Out. Services-1	-0.960	-2.100	-2.459	-1.487	-0.298
	(0.112)	(0.200)	(0.181)	(0.210)	(0.132)
Insurance-1	-3.391	-2.854	-1.244	0.126	-2.174
	(0.644)	(0.319)	(0.107)	(0.280)	(0.228)
Sub. Health	0.391	-0.953	1.238	-0.443	-2.405
	(0.122)	(0.148)	(0.130)	(0.184)	(0.337)
Life Insurance	0.934	0.747	0.560	0.116	0.653
	(0.114)	(0.115)	(0.0891)	(0.164)	((0.131)
Annuity	1.214	-1.796	0.543	0.875	-0.526
	(0.159)	(0.306)	(0.0976)	(0.195)	(0.143)
Income	-0.0404	-4.305	-0.950	-1.195	-1.861
	(0.124)	(1.438)	(0.0990)	(0.204)	(0.206)
Safe Assets	-0.275	-2.473	-0.638	-0.410	-1.566
	(0.107)	(0.299)	(0.0920)	(0.161)	(0.179)
Panel C: eq. (23)					
	-0.191	0.258	-0.775	0.166	-
	(0.158)	(0.141)	(0.225)	(0.235)	-

Standard errors in brackets.