TRUST AND POWER IN A HETEROGENEOUS-AGENT MODEL OF TAX COMPLIANCE WITH PUBLIC EXPENDITURE

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Abstract

This paper presents a model with heterogeneous agents that maximize their individual utility based on income and conjectured level of per capita public expenditure. We formally include in the model psychological drivers that affect the individual behaviour, such as risk-aversion, appreciation for public expenditure, expectations about peers' compliance and natural inclination to comply. Hence, agents' decisions rest on micro-founded rational behaviour, personal characteristics and subjective judgements. The enforcement system, based on random inspections, is standard and only partially known to agents.
The agent-based model is simulated under a variety of settings, representing different "societies". We use the gathered data to estimate the effects of the taxpayers' traits on the personal tax behaviour and to estimate a compliance slippery slope at a societal level. At the individual level, we find a positive dependence of compliance on all variables, with the significant exception of the tax rate that has a negative impact. As far as societies are concerned we show how aggregate tax compliance depends on composite indices of trust and power, with the former being slightly more important than the latter.

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1. Introduction

The problem of tax compliance has always been central in the theory of public finance and its relevance increased over time. The seminal models by Allingham and Sandmo (1972) and Yitzhaki (1974) assume a neat rational framework where individual decisions are taken based on a cost-benefit analysis under uncertainty driven by the probability of auditing and effects of fines. An impressive flow of research has built on this approach to overcome limitations, incorporate other determinants of behaviour and provide more realistic descriptions of observed data. A particularly important issue is the excessive tax compliance observed in the real world with respect to the level that the standard model of tax evasion would predict[1]. Another significant issue is the impact of the level of tax rates on compliance, that in Allingham, Sandmo and Yitzhaki's works is ambiguous or opposite to what is currently supported by common sense[2] or experimental findings. It was influentially pointed out that any tax-related decision is not a purely individual affair but depends on the quantity, efficiency and fairness of public expenditure that ultimately should be financed by tax revenues: with no hope to be exhaustive, see Cowell and Gordon (1988), Cowell (1992), Bordignon (1993) and Rablen (2010) for an up-to-date treatment. Pyle (1991), Andreoni et al. (1998) and, more recently, Sandmo (2005) are well-written summaries of further developments and research themes. After seminal contributions by Schmölders in the 60's, see Frank and Kirchler (2006), in the last decade the recognition that psychological factors can be relevant to understand and model tax behaviours has gained a tremendous momentum, see Kirchler (2007). Compliance is the result of complex interactions between taxpayers and the Government and is situated on a slippery slope, where trust and power both shape the behaviour of a collectivity of agents. Tax morale, see Alm and Torgler (2006), Torgler (2007), and knowledge/perception of the enforcement system are intuitively useful to explain how taxpayers behave and their importance is empirically and experimentally grounded. Fewer works have, in our opinion, investigated how such elements can be introduced in formal models of individual responses. An exception is given by the rich strand of agent-based models that offer some useful insights through numerical simulations of complex behaviour of heterogeneous agents. The seminal contribution in this field can be traced back to Mittone and Patelli (2000) where tax evasion can spread among agents in the form of outbreaks. Agents of different types can copy others actions, if this is myopically deemed

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[1] See Bernasconi (1998).
[2] See Bernasconi and Zanardi (2004).

more convenient and, as a result, in the absence of a sufficient number of audits, evaders multiply at the expenses of more honest taxpayers. Other authors have, since then, taken similar approaches in which a set of distinct types is exogenously assumed to exist, see for instance Hokamp and Pickhardt (2010), who have also agents that make random errors in their reported income. Davis et al (2003) analyze a model with honest and susceptible agents together with evaders and show that if an initially compliant population shifts to evasion it is difficult to reverse the situation and the number of audits must be increased beyond the level that would have kept acceptable compliance in the first place.

Agent based models have the capability to explore several issues that are difficult to deal with in analytical models. Repeated interaction among heterogeneous agents or between the fiscal administration and the taxpayers can be typically simulated. The model presented in Korobow et al. (2007) is an attempt to consider geographical spillovers and “contagion” effects as agents are networked in localized structures and are aware of the actions of their neighbours, facing peer-pressure and conformity reasons. In the paper it is shown that substantial sharing of payoff information and tax practices can lead to less compliant behaviour. While it is not entirely surprising that one bad apple spoils the whole basket, the result also points out that some “impressionability” of taxpayers may support relative high compliance even with modest auditing levels.

The flexibility of the agent-based methodology can, despite the risk of increasing the number of parameters, allow for great sophistication in the depiction of many realistic features of taxpayers and of the data available to the fiscal authority. Bloomquist (2006) is a good introduction to the field and describes the Tax Compliance Simulator where audit efficacy and celerity, together with a host of other parameters, can be changed to test the overall compliance and the success of specific auditing schemes, with regard to both direct and indirect effects (due to additional revenues from fines and increased compliance of other “forewarned” taxpayers, respectively). Bloomquist (2011) builds on some of the previous ideas to present a situated agent-based model with 85000 agents that are calibrated using realistic anonymized public data from the US Internal Revenue Service.

At the other extreme, some recent papers in the econophysics literature simplify the relationships among agents and present terse models of interaction based essentially on the Ising model of ferromagnetism. In Zaklan et al. (2009), 1.000.000 agents with “spin” are in contact on a grid and are subject to local interactions and, possibly, to social forces (due to mass media or cultural biases, say). As it is typical of similar models, there are critical thresholds of the parameters of the model that trigger phase changes. Pickhardt and Seibold

(2011) is a recent paper that shows that Ising-like models can reproduce to some extent situations where different types of taxpayers are present.

This paper presents a model with heterogeneous agents that maximize their individual utility based on (after tax) income and on the conjectured level of per capita public expenditure. Agents have different risk-aversion, distinct relative preference for public expenditure, varying trust in the likelihood that others will pay the due amounts and innate attitude to comply, that can be associated to cultural traits, social constraints or shame. Hence, their final decision rest on micro-founded rational behaviour, personal characteristics and subjective judgements. The enforcement system, based on random inspections, is only partially known to agents who figure out noisy estimates of the auditing probability by meeting other taxpayers and exchanging information about income. In each period, an individual can optimally conceal some income based on conjectures on the perceived probability of being audited, the perceived level of public expenditure and the perceived amount of tax paid by other individuals.

We contribute to the literature in two main ways. First, we formally include in the model psychological drivers that affect the individual behaviour. Some parameters, broadly related to the "trust" and tax morale, capture the appreciation for public expenditure and natural inclination to comply. The inclusion in the model of expectations about peers' reciprocation adds to the "trust" in the Government a novel ingredient of confidence in other agents. Hence, our work suggests ways to link conceptual dimensions to specific personal parameters[3]. Secondly, we use simulated data drawn from a variety of "societies" to estimate a compliance slippery slope and discuss how individual micro-motives aggregate in a variety of social macro-behaviours. Such a link is commonly drawn in agent-based approaches but, as far as we know, this is the first time a full-fledged slippery slope is analyzed. On a more methodological note, the model generates artificial data that are then used to build a surface that nicely matches some stylized facts, even though this is not meant to describe any specific situation.

The paper is structured as follows. Section 2 describes the agents' behaviour leaving some technical details for Section 3 that is devoted to the presentation of our simulations and the parameters that have been used. We stress that data can be examined both at the personal and aggregate level and results are discussed in Sections 4 and 5 keeping into account this duality. Section 6 offers a summary and some final remarks.

[3] In some sense, our agent-based model departs heavily from the econophysics paradigm, in which agents are electrons with +1 or -1 spins and have nearly no interiority, to allow for a good deal of nuanced peculiarities to emerge at the individual level.

2. The model

We consider two types of agents, namely individuals and the Government. Individuals maximize their utility under a fixed monetary income. The utility function depends on the after tax income and on the perceived level of public expenditure, together with an individual attitude to comply that represents agent's tax morale. Agents choose the fraction of the income to declare in order to pay income tax. Hence, based on heterogeneous parameters, they have the option to underreport their income to illegally reduce their tax burden.

The Government decides the income tax rate, collects all tax payments and uses all the revenues to provide public expenditure, so that the Government budget is always in balance.

Government can also contrast tax evasion by controlling a fraction of individuals and by applying fines on evaded taxes.

Agents face the standard problem of deciding how much of their income to declare for income taxation. Each individual $i=1, \ldots, N$ has a cardinal utility function

[1] $$U_i = U\left(y_i, G_i, \theta_i\right)$$

where y_i is the net (after tax) monetary income, G_i is the perceived value of public expenditure and $\theta_i = \left\{\theta_{i1}, \ldots, \theta_{ik}\right\}$ collects individual characteristics. We anticipate that θ_i allows agents to differ in their risk aversion, relative preference for public expenditure, (innate) attitude to comply and expectations about behaviours by others.

We suppose the utility function satisfies the following assumptions:

Assumption 1: positive marginal utility of income and public expenditure:

[2] $$\frac{\partial U}{\partial y_i} = U_y > 0 \text{ and } \frac{\partial U}{\partial G_i} = U_G > 0$$

Assumption 2: concavity of utility function:

[3] $$\frac{\partial U_y}{\partial y_i} = U_{yy} < 0, \ \frac{\partial U_G}{\partial G_i} = U_{GG} < 0 \text{ and } U_{yy}U_{GG} - U_{yG}^2 > 0$$

Assumption 3: minimum level of utility reached when either $y_i = 0$ and $G_i = 0$:

[4] $$U_{\min} = U(y,0) = U(0,G) = U(0,0)$$

The main idea behind our assumptions is that private income and public expenditure are both needed in a society to provide material and immaterial satisfaction to agents. The lack of either source of wealth, hence, results in no utility being enjoyed.

One specification that satisfies all previous assumptions is:

[5] $$U(y_i, G_i, \theta_i) = \phi(y_i, \theta_i)\varphi(G_i, \theta_i)$$

with $\phi_y > 0, \phi_{yy} < 0$, $\varphi_G > 0, \varphi_{GG} < 0$ and $U(0,0,\theta_i) = \phi(0,\theta_i)\varphi(0,\theta_i)$. In particular we assume that:

[6] $$U_i = A_i \left[y_i G_i^{\alpha_i} \right]^{1-\rho_i}$$

where $\phi(y_i, \theta_i) = y_i^{1-\rho_i}$ and $\varphi(G_i, \theta_i) = G_i^{\alpha_i(1-\rho_i)}$, $\rho_i > 0$ is the relative risk aversion parameter, A_i is an individual scale factor and $\alpha_i > 0$ represents the relative intensity of individual preference for public expenditure with respect to net income.[4] This preference parameter encompasses a variety of individual considerations about the Government behaviour in the use the revenue. For instance, the preference can reflect the individual judgement about efficiency, fairness, mix in the provision of public goods and other public expenditures.

To evaluate their utility U_i, agents need some knowledge of the term in square brackets which in turn requires to take into account the likelihood and the effects of audits and to estimate the public expenditure. The income tax is applied at a constant rate τ to the exogenous amount of income I_i, which is only known to the taxpayer. Hence, the amount of taxes paid by individual *i* is:

[7] $$T_i = \tau d_i I_i$$

where $0 \le d_i \le 1$ is the compliance rate, i.e. the share of gross income declared to the Government. Each agent can pick his decision variable d_i, actually opting for full ($d_i = 0$), partial ($d_i \neq 0$) or no ($d_i = 1$) tax evasion.

Given the control activity of the Government, each individual faces a probability q of being controlled. If an individual is audited, her tax evasion is certainly discovered and she has to pay a fine of f times the amount of evaded tax:

[8] $$F_i = f(1-d_i)\tau I_i$$

[4] With this Cobb-Douglas-type utility function, concavity requires $1-\rho_i < 1$, $\alpha_1(1-\rho_i) < 1$ and $(1+\alpha_1)(1-\rho_i) < 1$.

In the simulations of the model that will be presented in Section 3, individuals do not know exactly the probability of control q, but estimate their own subjective p_i at any time period through random interaction with other agents.

If the taxpayer is not controlled, her net income is

[9] $$W(d_i) = I_i - T_i = I_i(1 - d_i\tau)$$

If, instead, the taxpayer is audited, her net income is:

[10] $$Z(d_i) = I_i - T_i - F_i = I_i(1-\tau) - f(1-d_i)\tau I_i$$

Each individual knows that in aggregate the Government budget constraint holds, therefore the value of public expenditures is equal to total tax revenue:

[11] $$G = \sum_i \tau d_i I_i$$

However, no agent has full information on the current G and a conjecture must be used to get a reasonable tentative estimate. Not surprisingly, it turns out that the way G is guessed greatly affect decisions. Public expenditure can be decomposed in two parts: the part paid by individual i and the part paid by all others, so that:

[12] $$G = \tau d_i I_i + \sum_{k=1, k\neq i}^{N} \tau d_k I_k$$

We define the per capita public expenditure financed by all other individuals as:

[13] $$\tilde{g}_i = \frac{1}{N-1} \sum_{\substack{k=1 \\ k\neq i}}^{N} \tau d_k I_k$$

Substituting into [12]:

[14] $$G = \tau d_i I_i + (N-1)\tilde{g}_i$$

Then agent i's perception about public expenditure is then:

[15] $$G_i = \frac{\tau d_i I_i + (N-1)\tilde{g}_i}{r_i(N)}$$

in which $1 \leq r_i(N) \leq N$ is the individual estimate of the degree of rivalness of public expenditure. In a society with a large number of agent we can approximate $1/N$ to 0 and $(N-1)/N$ to 1, then:

[16]
$$G_i = \frac{N}{r_i(N)} \tilde{g}_i$$

It is worthwhile noticing that using the utility function [6] the degree of rivalness and the number of individuals are positive constants that cannot change the optimal choice of the agent:

[17]
$$U_i(y_i, g_i, \theta_i) = A_i \left(\frac{N}{r_i(N)} \right)^{\alpha_i (1-\rho_i)} \left(y_i \tilde{g}_i^{\alpha_i} \right)^{1-\rho_i}$$

and, in particular, we can skip the term $\left(\frac{N}{r_i(N)} \right)^{\alpha_i (1-\rho_i)}$ without altering the results.

When choosing the rate of compliance, each taxpayer has to estimate the reaction of other individuals to variation in her contribution[5]. This expectation is modelled as an individual adjustment on the level of per capita public expenditure of the previous period g_{-1}, that we assume to be publicly observed and known to all taxpayers. Agents anchor their estimate on the last available data and use a conjecture about other people's reaction to her variation of the rate of compliance. Define agent *i*'s expectation about other people's average contribution as:

[18]
$$\tilde{g}_i = \gamma_i \left(\tau d_i \tilde{I}_i \right) + (1 - \gamma_i) g_{-1}$$

where $\tilde{I}_i$ is the agent's estimate of the distribution average income, γ_i $(0 \le \gamma_i \le 1)$ represents a conjecture on how other people react to her tax behaviour and g_{-1} is the observed level of per capita public expenditure in the previous period.

To understand the meaning of the conjecture and of the parameter γ_i, observe that the previous per capita level of public expenditure can be written as $g_{-1} = \tau \bar{d}_{-1} \bar{I}_{-1}$, namely, the product of the tax rate, the average rate of compliance and the average income of the population. As the rate of change of the expected average public expenditure is $\dot{g}_i = \tilde{g}_i / g_{-1} - 1$, then:

[19]
$$\dot{g}_i = \frac{\gamma_i \left(\tau d_i \tilde{I}_i \right) + (1 - \gamma_i) g_{-1} - g_{-1}}{g_{-1}} = \frac{\gamma_i \left(\tau d_i \tilde{I}_i \right) - \gamma_i g_{-1}}{g_{-1}} = \frac{\gamma_i \tau d_i \tilde{I}_i - \gamma_i \tau \bar{d}_{-1} \bar{I}_{-1}}{\tau \bar{d}_{-1} \bar{I}_{-1}} = \gamma_i \left(\frac{d_i}{\bar{d}_{-1}} \frac{\tilde{I}_i}{\bar{I}_{-1}} - 1 \right)$$

[5] See Cornes-Sandler (1986), page 151: "The Nash assumption is that each individual expects no response by the rest of the community to variation in his or her own contribution. However, unless the reaction curve of the rest of the community is in fact horizontal, reflecting an actual response of zero, such a conjecture is not consonant with observable facts."

Therefore, if the estimate of average income is correct, i.e. if $\tilde{I}_i = \bar{I}_{-1}$, then:

[20] $$\dot{g}_i = \gamma_i \left(\frac{d_i}{\bar{d}_{-1}} - 1 \right),$$

and the rate of change of the expected average public expenditure depends on the ratio between the agent's and the population rates of compliance and on the parameter γ_i.

If γ_i approaches 1 the agent believes that others will react in the same direction (*positive conjectural variation*) and if, say, $d_i > \bar{d}_{-1}$ the agent believes that all other individuals will increase their tax payments by the same rate. If, on the contrary, γ_i is close to 0, then the individual believes that other agents will not change their behaviour (*zero conjectural variation*)[6] and therefore assumes that the per capita public expenditure will remain constant regardless of her behaviour.

We expect that the compliance rate will be higher with a higher conjecture parameter γ_i, which can then be interpreted as a "trust" parameter. In more detail, this trust has to do with other citizens and not with the central government, being related to the perceptions of how much the other peer taxpayers will behave given the institutional and social framework.

The scale factor A_i in the utility function incorporates the citizen's attitude towards paying taxes, "trimming" the level of utility by a factor that depends on the rate of compliance d_i and on an individual tax moral parameter $k_i \geq 0$. Letting

[21] $$A_i = d_i^{k_i}$$

from [17] and [19] we get the full specification that follows:

[22] $$U_i(y_i, g_i) = d_i^{k_i} y_i^{\;1-\rho_i} \left[\gamma_i \left(\tau d_i \tilde{I}_i \right) + (1 - \gamma_i) g_{-1} \right]^{\alpha_i (1-\rho_i)}$$

If $k_i = 0$, then $d_i^{k_i} = 1$ for any individual compliance rate d_i and the individual utility is not affected. In particular, it's not reduced in the presence of tax evasion. As k_i increases, the discount factor $d_i^{k_i}$ shrinks utility whenever underreporting is chosen and, as a consequence, agents are urged to more compliant behaviour. Equivalently, k_i can be thought as to measure the "shame" arising with of tax evasion. Some positive degree of shame, *ceteris paribus*, shifts to the right the value of d_i, i.e., increases compliance as expected.

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[6] See Cornes, Sandler (1986)

To summarize the model, each taxpayer chooses at each time the share of income to declare, d_i, in order to maximise the expected utility:

[23] $$EU(d_i) = p_i U_i(Z_i, g_i) + (1 - p_i) U_i(W_i, g_i)$$

While there are many ways to compute p_i and g_i, we assume that updates of both quantities will be based on the results of a series of random meetings with other agents , in which the needed sample information about the actual auditing frequency and income are obtained. We defer a more formal description to the next section and notice that our work generalizes a number of other models. If $\alpha_i = k_i = 0$ in [22], then the standard results of the literature apply[7].

The optimal value of d_i that maximises expected utility determines the amount of income tax, $T_i = \tau d_i I_i$, which is actually paid and, implicitly, the effective tax rate of individual *I*, namely $\tau_i = \tau d_i$. The effective tax rate τ_i can be interpreted as the tax rate that the individual would accept to pay without engaging in tax evasion, i.e. τ_i is the preferred tax rate given all exogenous variables, her preferences and conjectures and the tax enforcement policy of the Government.

3. Compliance in a variety of societies with heterogeneous agents

This section is devoted to describe how the previous model can be simulated in an agent-based framework with many heterogeneous agents. To the best of our knowledge, in fact, it is impossible to derive clear implications from the standard comparative statics exercise based on first order conditions and, in particular, it is extremely hard to characterize the aggregate behaviour of a society of diverse taxpayers with respect to several parameters. Hence, numerical simulation appears to be the methodological tool that better can provide insights on the main features of the model.

We assume that agents are able to decide rationally their best action in terms of compliance rate and, indeed, can optimize their utility given their personal traits, but do not know exactly the probability of auditing and the amount of taxes actually paid by other people.

Each agent collects the necessary information by meeting n other individuals in each period. Encounters are random and independent across agents and along time, i.e., each taxpayer will meet the same number of newly sampled agents at each time. This matching mechanism is

[7] Allinghan and Sandmo (1972), Yithzaki (1974).

perhaps oversimplified but retains the ability to describe settings in which agents have volatile estimates of some important variables. From each of her n meetings, the agent can learn:

- whether the other agent has been controlled;
- the other agent's level of income.

Instead, agents do not exchange information about the due amount of taxes or the degree of compliance, that seem more sensitive pieces of information. As a whole, these pairings allow the agent to compute a noisy estimate p_i of the true audit probability q and of the population average income. In realistic situations, we expect the number of meetings n to be much smaller than the number of agents N, producing quite volatile (but unbiased) estimates. Letting n_i^c (out of n) be the count of the met ones that have been controlled, the subjective audit probability for agent i at time t is n_i^c / n. As the Government can change its audit policy at any time, the agent re-estimates the subjective probability in each period, and we assume that the value p_i used in the individual utility maximization is a weighted average of his previous estimate (with weight π) and the new estimate n_i^c / n (with weight $1-\pi$). So the subjective probability of control of the *i*-th agent is:

[24] $$p_{it} = \pi p_{i,t-1} + (1-\pi)\frac{n_i^c}{n}.$$

The very same mechanism is used to estimate the average income. Hence, we assume that the sample average income $\tilde{I}_i$ observed in the meetings is

[25] $$\tilde{I}_i = \frac{1}{n}\sum_{j\in J(i)} I_j ,$$

where $J(i)$ is the set of agents that i meets at time t (which is omitted for notational simplicity).

The Government is in charge of:

- selecting the tax policy by choosing the level of income tax rate τ;
- providing public goods and services to be financed by the tax revenues;
- picking an adequate auditing policy through the choice the fraction q of individuals to control and collect the proceeds of the penalties applied at the rate f to tax evaders.

With respect to the evasion control policy, we simply assume a purely random auditing policy, so that in each period a share q of all agents is randomly controlled. Furthermore, we assume that Government has chosen a fine rate by comparing the punishment for tax evasion to those for other offenses, so that f, as well as q, are exogenous parameters in the model.

One period of the simulation evolves according to the following steps:

- agents inherit previous period's parameters, maximize utility to decide their tax compliance and pay taxes;
- Government provides public expenditure, performs random control of taxpayers and punishes tax evaders;
- agents gather information meeting other peers and revise estimates of the probability of control and of the average income of the population.

Therefore, we simulate a framework in which agents interact and dynamically reach an equilibrium, see Pellizzari and Rizzi (2011), in which the level of public expenditure and the rate of compliance are stochastic and eventually stabilize to values driven by the distribution of the personal features of the taxpayers and by the policy of the Government (on the top of a handful of other "technical" parameters, like n and π).

In each simulation we sample agents from distributions characterized by different parameters in order to represent different "societies" characterized by specific institutional settings, defined by the tax rate, probability of control, penalty level, and by distinct individual attributes, like risk aversion, relative preference for public expenditures, trust and tax morale parameters.

In other words, we can interpret a society as a specific set of values for some key parameters that shape the features of the inhabitants and the institutional aspects of the tax collecting system. Clearly, different societies are likely to exhibit sundry behaviours with respect to aggregate tax compliance and public expenditure and one of the aims of this work is to clarify which societal dimensions are the most relevant to develop an insightful description of possibly dissimilar tax related conducts. Our model allows for diversity both *across* and *within* societies and it is likely, say, that some free riders coexist with a population of taxpayers that, as a whole, is very compliant. Obviously, our approach does not allow to justify why parameters have specific values or how they evolved to reach those values and, in this sense, we still have a "black box" issue to address. To partially overcome this problem, we try to represent a variety of outcomes and explore possible causal effects by sampling many static societies.

We perform 250 simulations in which we pick a vector of random institutional parameters and a vector of minimum and maximum values that characterize the distributions of individual parameters. To avoid transient effects we run 200 periods in each simulation, keeping only the data obtained in the last period.

Table 1 – Values of parameters used in the simulations

Number of societies	S	250		
Number of agents	N	1000		
Number of matchings per agent	n	10		
Institutional settings				
		Distribution	*Values*	
Tax rate	τ	Uniform	$\tau \in [0.2, 0.5]$	
Probability of control	q	Uniform	$q \in [0.01, 0.1]$	
Penalty per € of tax evaded	f	Uniform	$f \in [1.0, 5.0]$	
Individual characteristics				
		Distribution	*Values*	
Income	I_i	Lognormal	mean: 30000 €	Standard deviation of log: 2
Relative risk aversion	ρ_i	Uniform $[\rho_{\min}, \rho_{\max}]$	$\rho_{\min} \in [0.0,0.5]$	$\rho_{\max} \in [0.5,1.0]$
Preference for public expenditure	α_i	Uniform $[\alpha_{\min}, \alpha_{\max}]$	$\alpha_{\min} \in [0.0,0.5]$	$\alpha_{\max} \in [0.5,1.0]$
Conjecture on other people response	γ_i	Uniform $[\gamma_{\min}, \gamma_{\max}]$	$\gamma_{\min} \in [0.0,0.5]$	$\gamma_{\max} \in [0.5,1.0]$
Attitude to comply	k_i	Uniform $[k_{\min}, k_{\max}]$	$k_{\min} = 0.0$	$k_{\max} \in [0.0,0.2]$
Starting values				
weight π in p_i	π	constant	0.5	
Previous year public expenditure	g_{-1}	constant	0 €	
Initial subjective probability of control	p_i	constant	0	
Estimate of average income	$\tilde{I}_i$	I_i		

Table 1 show the values of parameters used in the simulations. Agent-based models are attractive for their capability to allow for interaction and massive heterogeneity of agents but typically this results in a large number of parameters. This is sometimes referred as "the wilderness of bounded rationality"[8] and is one on the main reasons why it is difficult to solve analytically agent-based models. One way to overcome the problem and reduce the number of effective degrees of freedom is to focus on *meta-parameters* that describe the distribution from which individual traits are sampled. Equivalently, meta-parameters can be thought as descriptions of the range of individual parameters and each society is indeed fully described by some values for its meta-parameters. In what follows, we will consistently use the word

[8] See Hommes (2006) and Sims (1980) for the original idea.

“parameter” when individual quantities, like α_i or γ_i, are referred whereas with “meta-parameters” we denote societal parameters, like $\alpha_{\min}$, $\gamma_{\min}$ or $k_{\min}$, described in Table 1.

The results of the simulations are basically the agents’ rates of compliance, which is usefully summarized by average compliance, and the levels of per capita public expenditure. We expect that societies with low (high) values of $\alpha_{\min}$, $\alpha_{\max}$, $\gamma_{\min}$, $\gamma_{\max}$, $k_{\min}$, $k_{\max}$, namely having low (high) values of preference for public expenditure, trust in peers and attitude to comply, will show low (high) aggregate compliance. In a related fashion, societies where low (high) auditing probabilities and penalty levels are in place would display a tendency to show lower (higher) compliance.

The next Sections present the results of our simulations. In particular, Section 4 specializes on individuals and provides a descriptive analysis of a cross-section of taxpayers’ compliance, together with a regression analysis meant to show how parameters affect the actions of single agents. We also comment on three fictitious taxpayers, dubbed “free rider”, “average” and “trustful”, to exemplify different (but somewhat typical) behaviours. Section 5 leaves aside individuals and focuses on the study of tax compliance in different societies. We identify and discuss the main drivers of aggregate behaviour through regressions and develop indices of power and trust in terms of the meta-parameters that are able to explain to a large extent how societies position themselves in a slippery slope of compliance or level of public expenditure.

4 Individual compliance

This section discusses the individual data produced in our 250 simulations. Some descriptive statistics are reported in Table 2 that shows the mean, standard deviation, minimum and maximum values for each parameter. Most of the values in the “Min” and “Max” columns are the extremes of the sample ranges of the meta-parameters described in Table 1. Notice that our taxpayers exhibit a compliance that spans the whole [0-100%] interval, with an average of 82.6%.

The distribution of individual compliances is depicted in Figure 1*a*, where it is apparent that a large fraction (50.9%) of agents are nearly fully compliant, but there are many agents that massively underreport their income, producing a remarkably left-tailed distribution.

Per capita public expenditure shows a wide range of values too, with some concentration between 7000 and 10000 (see part *b* of Figure 1). To give the flavor of the variability in our results, parts *c* and *d* of Figure 1 show individual compliance against two individual characteristics (attitude to comply and conjecture about other people behaviour), for a random sample of 5000 individuals. There is a remarkable dispersion in the scatter plots, even though some mild correlation between the two variables and the individual compliance can be recognized.

Table 2 – Values of variables used in the regression model

Variable	*Mean*	*Std. Dev.*	*Min*	*Max*
I_i	29968.39	23686.14	876.64	889916.50
ρ_i	0.50156	0.18086	0.00355	0.99850
α_i	0.49427	0.18143	0.00549	0.99753
γ_i	0.50895	0.19133	0.00251	0.99553
k_i	0.05255	0.04576	0.00000	0.19898
τ	0.35676	0.08710	0.20008	0.49992
q	0.05784	0.02633	0.01020	0.09974
f	2.94745	1.14990	1.00638	4.99756
d	0.82559	0.20848	0.00028	0.99996
G	8713.72	1926.24	4494.79	13330.71

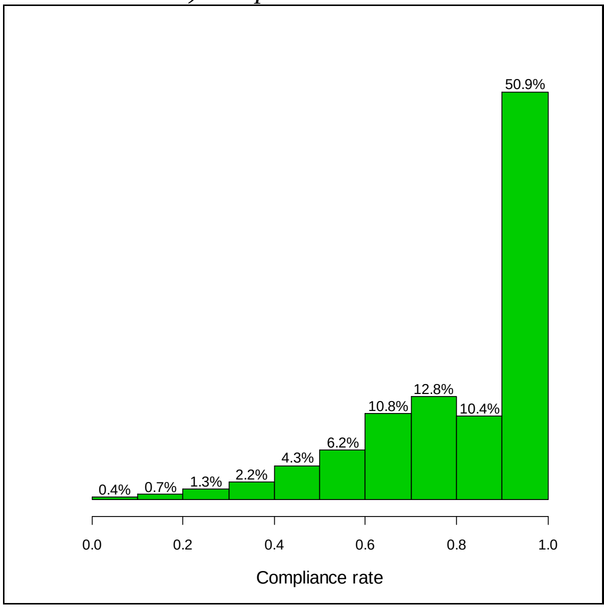
To understand the relative importance of individual parameters, we treat the results of the simulations as if they were individual observations of a cross-section of 250.000 agents.
Hence, we estimate a log-log model on the data where (the log of) the compliance linearly depends on (the log of) individual parameters, according to equation:

[26]
$$\begin{gathered}\ln d_{i,s} = \beta_0 + \beta_1 \ln I_{i,s} + \beta_2 \ln \rho_{i,s} + \beta_3 \ln \alpha_{i,s} + \beta_4 \ln \gamma_{i,s} + \beta_5 \ln k_{i,s} + \\ + \beta_6 \ln \tau_s + \beta_7 \ln q_s + \beta_8 \ln f_s + \varepsilon_{i,s}\end{gathered}$$

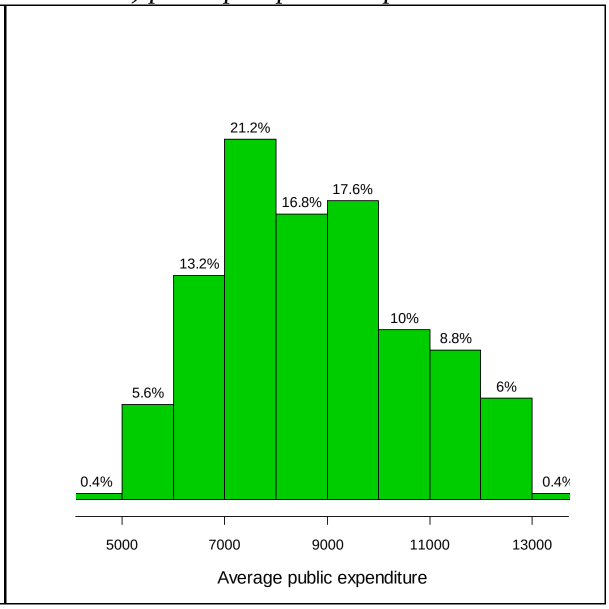
where i denotes the agent, s the simulation, $\varepsilon_{i,s}$ is an error term and estimates are obtained from the cross-section of all agents. The dependent variable d is truncated at the value 1, so we use a tobit estimation to handle the large number of fully compliant individuals. The main reason to use a double log specification is to hugely simplify the interpretation of the results of the regression as it is well known that, in this case, marginal effects can be regarded as elasticities of compliance with respect to independent variables. Therefore, marginal effects can be regarded as percent increments in compliance given a unit percent increment in the given variable. The estimates of the tobit regression are shown in Table 3. All coefficients are extremely significant and there is a positive dependence of individual compliance on income level, risk aversion, relative preference for public expenditure, trust, attitude to comply, auditing probability and penalty rate. In agreement with intuition, instead, the tax rate τ has a negative impact on the declared income.

Figure 1 – Distribution of individual results (all societies)

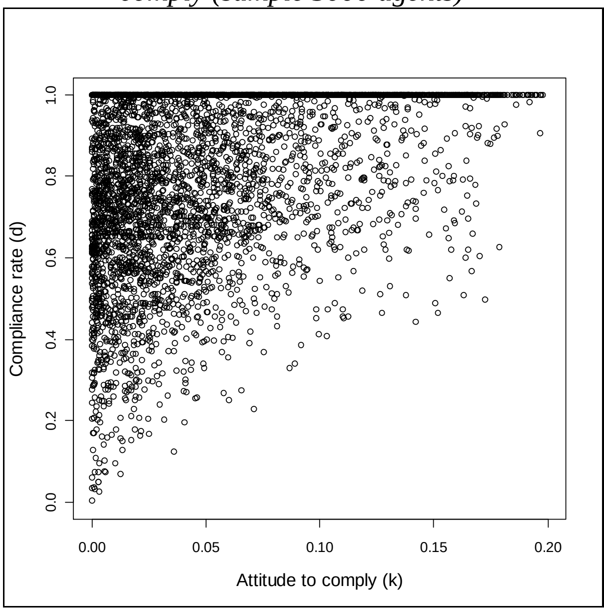
a) compliance rates



b) per capita public expenditure



c) individual compliance rates and attitude to comply (sample 5000 agents)



d) individual compliance rates and conjecture (sample 5000 agents)

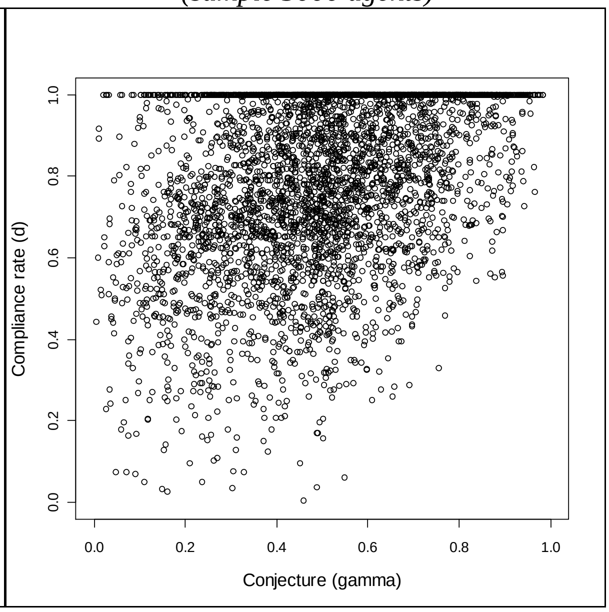


Table 3 – Tobit regression of individual compliance rates

Dependent variable: *d*				Number of obs = 250000		
LR chi2(8) = 176983.32				Prob > chi2 = 0.0000		
Log likelihood = -110411.15				Pseudo R2 = 0.4449		
	Coef.	*Std. Err.*	*t*	*P>|t|*	*[95% Conf. Interval]*	
ln I_i	0.02549	0.00122	20.86	0.0000	0.023	0.028
ln ρ_i	0.19012	0.00181	104.85	0.0000	0.187	0.194
ln α_i	0.38130	0.00173	220.23	0.0000	0.378	0.385
ln γ_i	0.42269	0.00164	257.13	0.0000	0.419	0.426
ln k_i	0.13904	0.00055	250.92	0.0000	0.138	0.140
ln τ	-0.71227	0.00366	-194.86	0.0000	-0.719	-0.705
ln q	0.15178	0.00144	105.28	0.0000	0.149	0.155
ln f	0.31893	0.00186	171.50	0.0000	0.315	0.323
constant	0.33988	0.01397	24.32	0.0000	0.312	0.367
sigma	0.37662	0.00039			0.376	0.377

In a tobit regression, marginal effects are not simply the estimated coefficients and must be computed. Moreover, they do depend on the chosen values of dependent variables. In order to get some insight in the behaviour of agents, we investigate three typical agents characterized by parameters that encouraged us to name them as *free rider*, *average* and *trustful* agents. A free rider has parameters α, γ and k in the lower range of possible values and, hence, has relatively low preference for public expenditure, little trust and attitude to comply. We expect from such dude a greater amount of underreporting, all other variables being equal. Similarly, we consider average agents whose parameters are set as the average of the possible ranges, and trustful individuals with relatively large values for α, γ and k. Such individuals are likely to be very compliant in a variety of situations.

To assess the effects in different institutional frameworks, it is convenient to evaluate the reactions of the illustrative agents previously defined when they are embedded in three societies distinguished by low, average and high enforcing power on the part of the Government. Hence, we consider a *low* (respectively, *average* or *high) power* society where q and f, namely the auditing probability and penalty rate, take low (average or high) values. All in all, we consider 9 cases (3 types times 3 societies) in our analysis and Table 4 summarizes the relevant values of the parameters. As an example, the average taxpayer in an average power society, corresponding to the middle column of the second panel in Table 4 (boldfaced) faces an audit probability of 5% per year and fines equal to three times the evaded amounts.

Table 4 – Simulated typical taxpayers

	free rider			*average*			*trustful*		
	low power	*average power*	*high power*	*low power*	*average power*	*high power*	*low power*	*average power*	*high power*
α_i	.25	.25	.25	.50	**.50**	.50	.75	.75	.75
γ_i	.25	.25	.25	.50	**.50**	.50	.75	.75	.75
k_i	.025	.025	.025	.05	**.05**	.05	.075	.075	.075
q	0.01	.05	.10	0.01	**.05**	.10	0.01	.05	.10
f	1	3	5	1	**3**	5	1	3	5

Note: other variables on average

The marginal effects are reported in Table 5. Take again the average agent in an average power society (figures are boldfaced for clarity). We see that the compliance is almost 92% and all parameters positively affect the contribution with the exception of the tax rate τ. A one point percent increase, say, in the risk aversion parameter ρ results on average in additional compliance of 0.0646%. The same relative increment in α (+1%) produces an effect on compliance that is about the double.

Table 5 – Marginal effects: elasticities for some typical taxpayers

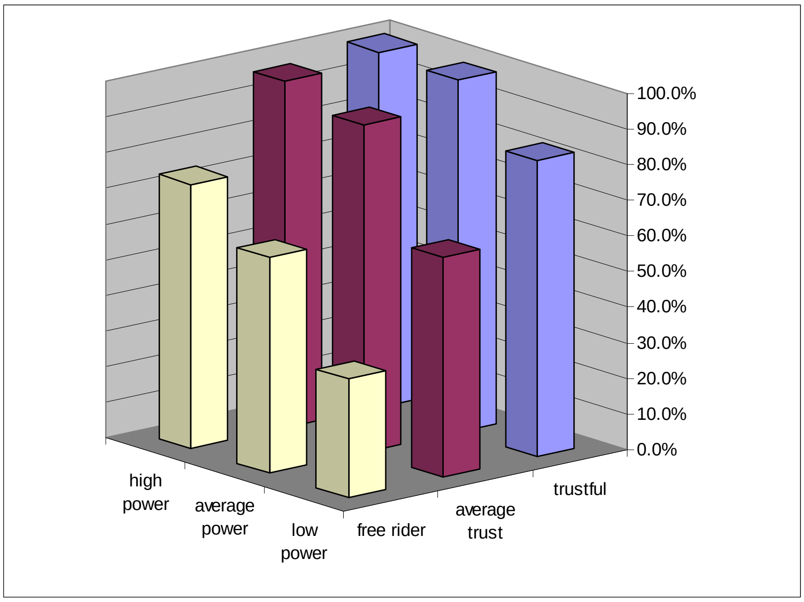
	free rider			*average trust*			*trustful*		
	low power	*average power*	*high power*	*low power*	*average power*	*high power*	*low power*	*average power*	*high power*
compliance rate	33.2%	60.6%	74.2%	61.7%	**91.8%**	97.2%	83.1%	98.8%	99.8%
I_i	0.0255	0.0229	0.0188	0.0227	**0.0087**	0.0037	0.0145	0.0018	0.0004
ρ_i	0.1898	0.1711	0.1400	0.1693	**0.0646**	0.0275	0.1081	0.0134	0.0032
α_i	0.3806	0.3431	0.2809	0.3396	**0.1296**	0.0551	0.2168	0.0270	0.0065
γ_i	0.4220	0.3803	0.3114	0.3765	**0.1436**	0.0610	0.2404	0.0299	0.0072
k_i	0.1388	0.1251	0.1024	0.1238	**0.0472**	0.0201	0.0791	0.0098	0.0024
τ	-0.7110	-0.6409	-0.5247	-0.6344	**-0.2420**	-0.1029	-0.4050	-0.0504	-0.0122
q	0.1515	0.1366	0.1118	0.1352	**0.0516**	0.0219	0.0863	0.0107	0.0026
f	0.3184	0.2870	0.2349	0.2841	**0.1084**	0.0461	0.1814	0.0226	0.0055

Finally, the compliance of the average agent decreases by about 0.24% when the tax rate is increased by 1%. This finding is by far the biggest effect observed in our data and the strongly negative sensitivity to hikes in τ is a robust result of our simulations and nicely agrees with run-of-the-mill intuition.

Table 5 shows that free riders are going to massively underreport income when little power is exerted. Their compliance moves from 33.2% to 60.6% and 74.2% under increased probability of audit and penalty rates. To a different degree, the average and trustful agents share this behaviour and they both need the presence of some power to move close to full compliance. Figure 2 represents the average compliance of our typical taxpayers under the three institutional arrangements and some sort of slippery slope can be recognized, where both power and trust appears as necessary ingredients to obtain high compliance. A more thorough investigation of this topic is deferred to the next subsection.

Table 5 offers at least one additional insight related to the strength of the reactions of different agents. Indeed, the marginal effects of the free rider are always larger than the ones of the average taxpayer, which in turn are larger than those of the trustful agent. At the two extremes, while the sensitivities of free riders in a low power society are remarkably pronounced, trustful agents are virtually insensitive to changes in the parameters in a high power situation (last column of Table 5). The tax rate is again one of the parameters that can visibly modify compliance of trustful taxpayers in average or low power societies.

Figure 2 – Estimated compliance rates for some typical taxpayers



5 Societal compliance

We analyze in this section how meta-parameters (i.e., features of the society) affect average compliance and the provision of public expenditure. We will focus here on the different outcomes that are observable in different social contexts and support the finding that estimates based on simulated societies do produce a slippery slope that depends jointly of measures of power and trust. We are aware that there are obvious limitations in mapping societies to sets of values for the parameters, but the final picture is coherent and, as far as we know, this is the first case in which a full slippery slope is estimated on simulated data arising from “artificial” societies.

From each of our 250 simulated societies, we obtained the average compliance and public expenditure that are regressed against the mean of the parameters’ distributions, which can be derived from the meta-parameters listed in Table 1. As both the dependent variables are endogenously generated in each society, there will be a reciprocal influence on their determination. Therefore we estimate a reduced form in which the compliance rate and the public expenditure depend on all other variables in a context of seemingly unrelated regressions[9]. Tables 6 shows the estimated coefficients of the following model for the average compliance:

[27]
$$\begin{aligned} \ln d &= \left[a_1 \ln \mu_\rho + a_2 \ln \tau + a\right] + \\ &+ \left[t_1 \ln \mu_\alpha + t_2 \ln \mu_\gamma + t_3 \ln \mu_k\right] + \left[p_1 \ln q + p_2 \ln f\right] + \varepsilon \end{aligned}$$

where μ_X denotes the sample mean of individual parameter X (the index s=1,…,250 is omitted for simplicity). We indicate with $a_j, j = 1,2$, the estimates that factor out as a constant term, with $t_j, j = 1,...,3$, the coefficients that form a trust index, and with $p_j, j = 1,2$, the estimates used to build a power index.

Along the very same lines, Table 6 also reports the estimated coefficients relative to a model where the dependent variable is the per capita public expenditure.

[9] As the rate of compliance is now an average across individuals in each society, the data are no longer truncated and lies all inside the range [0,1], therefore the tobit regression is not required to obtain efficient estimations of the regression parameters.

Table 6 – Seemingly unrelated regression results: Average compliance

	Dependent variable: average compliance rate				*Dependent variable: per capita public expenditure*			
	Coef.	*Std. Err.*	*Z*	*P>|z|*	*Coef.*	*Std. Err.*	*Z*	*P>|z|*
$\ln \mu_\alpha$	0.17499	0.02123	8.24	0.00	0.18562	0.02215	8.38	0.00
$\ln \mu_\rho$	0.12640	0.02037	6.20	0.00	0.12872	0.02126	6.06	0.00
$\ln \mu_\gamma$	0.22546	0.01921	11.74	0.00	0.22326	0.02004	11.14	0.00
$\ln \mu_k$	0.06031	0.00454	13.29	0.00	0.05836	0.00474	12.32	0.00
$\ln q$	0.06969	0.00729	9.57	0.00	0.06796	0.00760	8.94	0.00
$\ln f$	0.14590	0.00947	15.40	0.00	0.14169	0.00988	14.34	0.00
$\ln \tau$	-0.27271	0.01666	-16.37	0.00	0.73084	0.01739	42.04	0.00
constant	0.14121	0.04278	3.30	0.00	10.45942	0.04464	234.31	0.00
Equation			*Obs*	*Parms*	*RMSE*	*"R-sq"*	*chi2*	*P*
average compliance rate			250	7	0.06704	0.8041	1026.37	0.00
per capita public expenditure			250	7	0.06995	0.9011	2278.34	0.00

All coefficients are highly significant (with the exception of the constant term in the model of average compliance). The regressors that have a stronger effect on tax compliance and public expenditure are μ_α, μ_γ, f and τ. The effects are quite similar for the two models, apart from the tax rate τ. Estimates can be interpreted as elasticities: consider, as an example, a 1% increment in the value of μ_α, the average of the individual α's in the population: compliance increases by 0.175% and accordingly public expenditure raises by 0.186%. On the contrary, varying τ has opposite effects on the two: a 1% increment *reduces* compliance by 0.273% but *inflates* revenues by 0.731%. Relationships among coefficients in the two regressions are expected as $G = \tau dY$, where Y is the total income of the society. Hence, because of the *log-log* formulation of the models, the coefficient of the tax rate for G approximately equals one plus the corresponding coefficient in the compliance model and all the other coefficients should be the same.

Compliance and public expenditure both depend on variables that can be broadly related to the “power” of the Government and to “trust”. Indeed, there is a unique way to rewrite [27] as:

[28] $$d = A\delta_T{}^b \delta_P{}^c ,$$

in which $\delta_T \in [0,1]$, $\delta_P \in [0,1]$ are trust and power indices and A,b,c embody the coefficients estimated using our simulated data. In particular, the two indices are:

[29] $$\delta_T = \left(\frac{\mu_\alpha}{\overline{\mu}_\alpha}\right)^{t_1/b} \left(\frac{\mu_\gamma}{\overline{\mu}_\gamma}\right)^{t_2/b} \left(\frac{\mu_k}{\overline{\mu}_k}\right)^{t_3/b}$$

[30] $$\delta_P = \left(\frac{q}{\overline{q}}\right)^{p_1/c} \left(\frac{f}{\overline{f}}\right)^{p_2/c}$$

where $b = t_1 + t_2 + t_3$, $c = p_1 + p_2$ and $\overline{\mu}_\alpha, \overline{\mu}_\gamma, \overline{\mu}_k, \overline{q}, \overline{f}$ are the maximum attainable values of the means given the meta-parameters (see Table 1).

The constant *A* collects all the variables that have not been used to build the indices and other normalizing factors.[10] Because of the functional form used in [27], the trust and power indices prove to be a weighted geometric averages of their components, normalized with their maxima, whose weights are dependent on estimated coefficients.

Equation [28] is an intuitive expression relating trust and power indices to aggregate compliance which can be analyzed in different frameworks where attributes of power and trust change and all the other relevant parameters are taken as fixed. Based on the estimates of Table 6, we obtain the values for A,b,c given in Table 7 for different levels of the tax rate. Exponents b and c are different and trust appears to contribute more than power to raise compliance and public expenditure.

Table 7 – Coefficients of simulated slippery slopes at some tax rates

	Average compliance $d = A\delta_T{}^b\delta_P{}^c$	*Public expenditure per capita* $G = A\delta_T{}^b\delta_P{}^c$
Constant A, τ =20%	1.63699	9841.1
Constant A, τ =30%	1.46563	13235.4
Constant A, τ =40%	1.35504	16332.3
Constant A, τ =50%	1.27504	19225.3
Trust: *b*	0.46076	0.46724
Power: *c*	0.21559	0.20965

Therefore, in our model that incorporates preferences for public expenditure, *voluntary compliance* seems more relevant than *enforced compliance* in determining the average rate of evasion in a society. Observe that our trust index encompasses more than the confidence in the ability of the Government to provide valuable public services (mainly captured by the preference α for public expenditure vis-à-vis private income) as it depends also on the belief

[10] The constant can be shown to be $A = \exp(a + a_1 \ln \overline{\mu}_\rho + a_2 \ln \tau + t_1 \ln \overline{\mu}_\alpha + t_2 \ln \overline{\mu}_\gamma + t_3 \ln \overline{\mu}_k + p_1 \ln \overline{q} + p_2 \ln \overline{f})$.

that other taxpayers will react in the same way (basically due to γ), together with an innate attitude to comply (the role of k). In other words, our trust index blends an implicit evaluation of the public expenditure and personal inclination to comply *with* perceptions related to other agents' behaviour. While the first two components are not new and often referred to in the literature, we feel that "γ-trust" in fellow taxpayers is an addition to the standard picture. High values for γ would imply, for example, the expectation that more honest behaviour on the part of the single will lead to more honest behaviour of others and, conversely, would suggest that an increase in free riding may spread out and be adopted widely.

Our power index is more typical and based on the two most natural attributes of enforcement, namely audit probability and penalty rate.

Figure 3 shows the slippery slope of compliance in societies where different degrees of trust and enforcement are in place.[11] The surface is shown for some levels of the tax rate, which influences the constant term A in [28]. Because of the negative sign of its estimated coefficient, a higher tax rate leads to a lower constant A and therefore to a lower average compliance. In Figure 3 this shift can be seen by inspection noticing that the upper plateau of the surface (that shows the area of full compliance) shrinks as the tax rate increases (from part *a* to part *d* of the figure).

We stress that the surface is the by-product of the estimation of statistical models on data gathered by simulating agent-based societies of heterogeneous utility maximizing taxpayers, whereas other researchers, Kirchler et al. (2008), have discussed in depth the merits of the slippery slope as a conceptual model of tax behaviour and have found some empirical support for its plausibility, Muehlbacher et al. (2011). Similarly, our average compliance depends on both the power and trust dimensions and, interestingly, high values of both indices are needed to get full compliance. By contrast, neither maximal trust nor maximal power alone would be sufficient to reach, under all circumstances, the full compliance visible in the upper plateau of the surface. This outcome nicely squares with the compliance exhibited by the illustrative types of taxpayers portrayed in the previous section and, indeed, Figure 2 can be thought as a caricature of a detailed slippery slope.

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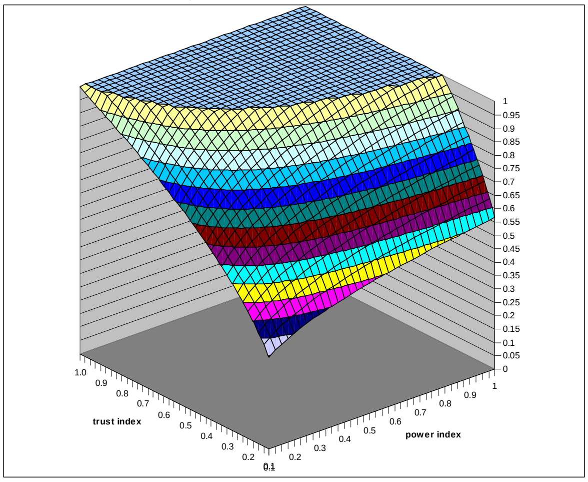
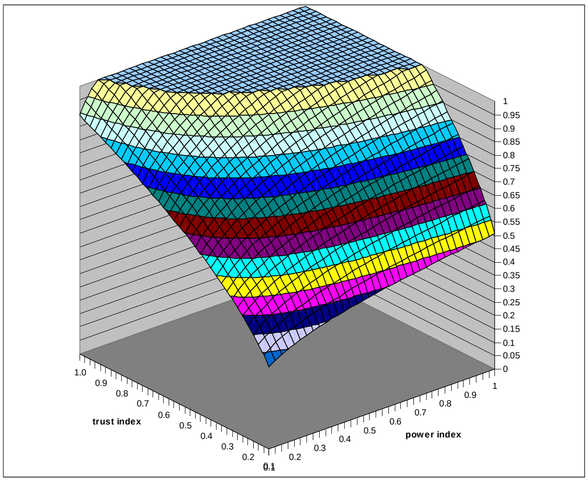
[11] As the value of A reported in Table 7 exceeds 100%, we have truncated compliance in the graph when needed.

Figure 3 – Simulated slippery slope: societal compliance rate

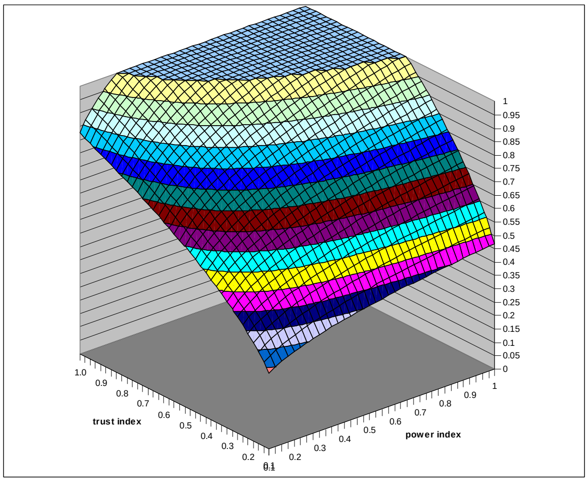
a) tax rate = 20%



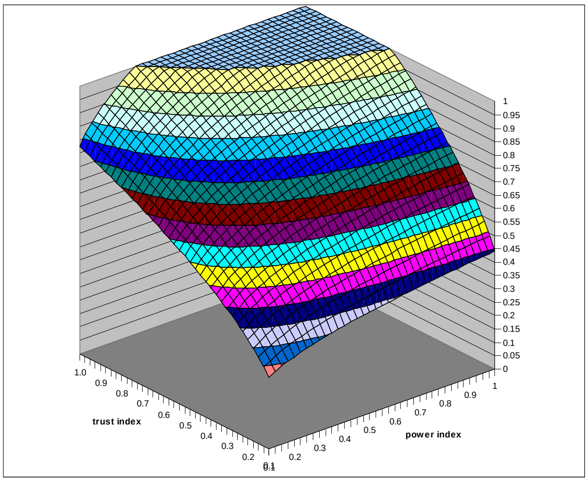
b) tax rate = 30%



b) with tax rate = 40%



c) with tax rate = 50%



6. Conclusion

We have described an agent-based model of individual tax compliance where heterogeneous agents maximize utility under risk based on several individual features, like their preference for public expenditure, personal attitude to comply, consideration of reaction by other taxpayers and risk aversion. Decisions obviously depend on individual traits but also on perceived quality and quantity of the per capita public expenditure and on the recognition that the bulk of revenues comes from other taxpayers. Simulations show a wide range of conducts, which stretch from full honesty to massive underreporting, as expected because of the large heterogeneity of agents. As a whole, however, each society of taxpayers has an aggregate behaviour shaped by the meta-parameters that describe individual features' distributions. We estimate from our artificial cross-section data a model that shows how individual compliance positively depends on the relative preference for public expenditure, other trust-related quantities and on the strength of the enforcement system. The tax rate, instead, negatively affects compliance other things being fixed.

We then describe and build indexes of “trust” and “power” based on meta-parameters in several societies, and estimate a slippery slope fitting the artificial data we have generated. This exercise produces a surface that is remarkably similar to what other scholars have qualitatively described and confirms the insights of empirical studies. Estimates show that both trust and power are needed to achieve high compliance, with trust-related variables having a larger impact than power-related quantities. While this may be caused to some extent by our exploratory choice of meta-parameters’ values, which are not calibrated to any specific situation, still the predicted rates of compliance in many diverse societies are plausible.

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References

Allingham, M. G., & Sandmo, A. (1972). Income Taxation: A Theoretical Analysis, *Journal of Public Economics*, 1, 323-338.

Alm, J., & Torgler, B. (2006). Culture differences and tax morale in the United States and in Europe, *Journal of Economic Psychology*, 27(2), 224-246.

Andreoni, J., Erard, B., & Feinstein, J. (1998). Tax Compliance, *Journal of Economic Literature*, 36(2), 818-860.

Bloomquist, K. M. (2006). A Comparison of Agent-Based Models of Income Tax Evasion, *Social Science Computer Review*, 24(4), 411-425.

Bloomquist, K. M. (2011). *Agent-Based Simulation of Tax Reporting Compliance*, mimeo, Shadow 2011, Muenster.

Bernasconi, M. (1998). Tax Evasion and Orders of Risk Aversion, *Journal of Public Economics*, 67, 123–134.

Bernasconi, M., & Zanardi A. (2004). Tax Evasion, Tax Rates, and Reference Dependence, *FinanzArchiv*, 60(3), 422-45.

Bordignon, M. (1993). A fairness approach to income tax evasion, *Journal of Public Economics*, 52(3), 345-362.

Cornes, R., & Sandler, T., (1986) *The Theory of Externalities, Public Goods and Club goods*, Cambridge University Press, Cambridge.

Cowell, F. A. (1992). Tax evasion and inequity., *Journal of Economic Psychology*, 13(4), 521-543.

Cowell, F. A., & Gordon J.P.F. (1988). “Unwillingness to Pay”, *Journal of Public Economics*, 36, pp. 305-321.

Davis, Jon S., Hecht, Gary & Perkins, Jon D. (2003), “Social Behaviors, Enforcement and Tax Compliance Dynamics”, *Accounting Review*, vol. 78(1), pp. 39-69.

Epstein, Joshua M. (2002) *Modeling civil violence: An agent-based computational approach*, PNAS vol. 99, May 14, 2002, p. 7243-7250.
Frank, B., & Kirchler E. (2006). (selected and introduced) *Günter Schmölders. The Psychology of Money and Public Finance*. New York: Palgrave.
Hokamp, S., & Pickhardt M. (2010), Income Tax Evasion in a Society of Heterogeneous Agents – Evidence from an Agent-Based Model, *International Economic Journal*, 24(4), 541-553
Hommes, C. H. (2006). Heterogeneous Agent Models in Economics and Finance, in Tesfatsion, L., & Judd K. L. (eds) *Handbook of Computational Economics*, Volume 2, 1109-1186, Elsevier B.V.
Kirchler, E. (2007). *The Economic Psychology of Tax Behaviour*, Cambridge, UK: Cambridge University Press.
Kirchler, E., Hoelzl E., & Wahl I. (2008). Enforced versus voluntary tax compliance: The slippery slope framework, *Journal of Economic Psychology* , 29, 210-225.
Korobow, A., Johnson, C., & Axtell, R. (2007). An Agent-based Model of Tax Compliance with Social Networks, *National Tax Journal*, vol. 60(3), 589-610.
Mittone, L., & Patelli P. (2000). "Imitative behaviour in tax evasion", in Stefansson, B., & Luna, F., (eds.), *Economic simulations in swarm: Agent-based modelling and object oriented programming*, 133-158, Amsterdam: Kluwer.
Muehlbacher, S., Kirchler, E., & Schwarzenberger, H. (2011). Voluntary versus enforced tax compliance: empirical evidence for the ''slippery slope'' framework, *European Journal of Law and Economics*, 32(1), 89-97.
Pellizzari, P., & Rizzi, D. (2011). *A Multi-Agent Model of Tax Evasion with Public Expenditure*, Working Paper 15/2011, Department of Economics, Ca´ Foscari University of Venice, Italy.
Pickhardt, M., & Seibold, G. (2011). *Income tax evasion dynamics: Evidence from an agent-based econophysics model*, CAWM Discussion Papers 53, Center of Applied Economic Research Münster (CAWM), University of Münster.
Pyle, D.J. (1991). The Economics of Taxpayer Compliance, *Journal of Economic Surveys*, 5(2), 163-198.
Rablen, M. D. (2010). Tax Evasion and Exchange Equity: A Reference-Dependent Approach, *Public Finance Review*, Vol. 38, pp. 282-305.
Sandmo, Agnar (2005). The Theory of Tax Evasion: A Retrospective View, *National Tax Journal*, 58(4), 643-663.
Sims, C. A. (1980). Macroeconomics and Reality, *Econometrica*,48(1), 1-48.
Torgler, B. (2007). Tax compliance and tax morale: A theoretical and empirical analysis. Cheltenham, UK: Edward Elgar.
Yitzhaki, S. (1974). A Note on Income Tax Evasion, *Public Finance Quarterly*, 15(2), 123-137.
Zaklan, G., Westerhoff F., & Stauffer, D. (2009). Analysing Tax Evasion Dynamics via the Ising Model, *Journal of Economic Interaction and Coordination*, 4(1), 1-14.