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Deprivation and Social Exclusion*

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Abstract

This paper is concerned with the definition and the measurement of social exclusion, both at the individual and at the aggregate level. Social exclusion is characterized by "disintegration and fragmentation of social relations and hence a loss of social cohesion. For individuals in particular groups, social exclusion represents a progressive process of marginalization leading to economic deprivation and various forms of social and cultural disadvantage" (European commission's programme specification for targeted socioeconomic research). Measures of social exclusion have been proposed and used in empirical studies (Bradshaw et al., 2000, Tsakloglou and Papadopoulos, 2001). However, these measures capture only some aspects of the phenomenon and, often, the theoretical presuppositions are unclear. In this paper, we focus on the idea of social exclusion as having two basic determinants: the lack of identification with other members of society and the aggregate alienation experienced by an agent with respect to those who are better off. We adopt an axiomatic approach to develop a theoretical framework for the measurement of social exclusion. Finally, we apply some of the measures suggested in the theoretical part of the paper to the relevant groups in EU member states from 1994 to 1999. *Journal of Economic Literature* Classification No.: D63.

Keywords: Social Exclusion, Deprivation, Equity.

1 Introduction

This paper is concerned with the definition and the measurement of social exclusion. When is an individual socially excluded? What is the level of social exclusion in a given society? Can we say that in the UK there is more social exclusion than in Italy? These are the kind of questions that motivate our study.

Social exclusion has gained a primary role in official documents and in the political debate, and it has received considerable attention by social scientists. Under the heading of social exclusion are included concerns for social phenomena as diverse as poverty, deprivation, low educational attainment, unemployment and other labour market disadvantages, poor housing, lack of participation to social and political institutions. While this might be part of its political appeal, it can undermine its value in a proper scientific context. Moreover, as a result of this vagueness in the conceptualization, there is not an established and theoretically rigorous method for its measurement; this, in turn, renders a difficult exercise to make cross country comparisons or to evaluate social inclusion policies.

In this paper we propose an analytical framework in order to model social exclusion and we derive a class of measures which could be used to make social exclusion comparisons or to evaluate social inclusion policies.

Broadly speaking, a person is said to be socially excluded if she/he is unable to participate in the basic economic and social activities of the society in which she/he lives. In the European Commission's Programme specification for 'targeted socioeconomic research', social exclusion is described as "disintegration and fragmentation of social relation and hence a loss of social cohesion. For individual in particular groups, social exclusion represents a progressive process of marginalization leading to economic deprivation and various forms of social and cultural disadvantage". Starting from this general ideas, a number of different conceptualizations of social exclusion have been proposed in the literature (see, most notably, the contributions by Barry, 1998, Burchard et al., 2002, and Atkinson et al., 2002). We will not review all the many attempts at definition present in the literature, nor shall we choose one among the available menu of definitions. Rather, we shall focus on the properties of this concept which recur in all the existing definitions, in order to identify a common ground about the ideas generally included under the heading of social exclusion. From this minimal conceptualization, we shall construct an analytical framework in order to model and to measure social exclusion.

According to the existing definitions of social exclusion, the basic elements characterizing this phenomenon are: multiple deprivation, relativity, dynamics, social participation.

It is a multi-dimensional concept that includes economic, social and political aspects: social exclusion deals with the failure to reach different valuable functionings, where the functionings can be either quantitative (as it is generally the case for the economic aspects) or qualitative (as for the political and social aspects). It is a relative concept, in that an individual can be socially excluded only in comparison with a particular social group at a given place and time; hence there is no "absolute" social exclusion, and an individual can be declared as socially excluded only with respect to the society he belongs to. It is a dynamic concept, in that an individual can become socially excluded if his condition of deprivation is persistent or worsens over time. Finally, it has a strong relational dimension.

On the basis of these properties it is easy to detect the similarities and the differences with the related concepts of inequality and poverty. It is a multidimensional phenomenom, hence it is fundamentally different from income (or consumption) inequality and income (or consumption) poverty. Unlike multidimensional inequality, which is a measure of the dispersion in a multidimensional distribution of quantities (consumption or functionings) for different individuals (Tsui, 1999), social exclusion focus on the sub-group of individuals who do not have access to the set of relevant functionings, and it is not necessarily dependent on the dispersion in the overall distribution. A measure of multidimensional poverty typically specifies a poverty threshold for each functioning, then looks at the shortfalls of different individuals from the threshold levels of each functioning, and finally aggregates these shortfalls into an overall index of poverty (Bourguignon and Chakravarty, 2002). Thus, both multidimensional poverty and social exclusion deal with functionings failures; however, while in the former the different dimensions are of quantitative nature, so that one can measure the shortfalls from the selected threshold for each functioning, in the latter many dimensions refer to political or social functionings which are typically qualitative. Hence a metric of social exclusion requires an analytical framework able to handle both quantitative and qualitative variables. Finally, social exclusion is characterized by the persistence of the situation of deprivation over time, thus its measurement requires the inclusion of time as an important variable.

The paper is organized as follows. The next section introduces the formal framework for measuring the social exclusion of an individual, followed by an extension to aggregate exclusion measures in section three and to dynamic measures in section four. In the fifth section we describe the data source and the variables used in the analysis. The sixth section presents the results of the estimation. The last section summarizes the main results.

2 Deprivation and exclusion

We use $\mathbb{N}$ to denote the set of all positive integers and $\mathbb{R}$ denotes the set of all real numbers. $\mathbb{Q}_{++}$ (resp. $\mathbb{Q}_+$) is the set of all positive (resp. non-negative) rational numbers. For $n \in \mathbb{N}$, $\mathbb{Q}^n_+$ is the n-fold Cartesian product of $\mathbb{Q}_+$, and $\mathbf{1}_n$ is the vector consisting of n ones. The set of individuals in a society is $N = \{1, \ldots, n\}$ where $n \in \mathbb{N} \setminus \{1\}$ is population size, assumed to be fixed. For a non-empty set $M \subset N$, the set $\mathbb{Q}^M_+$ is the set of $|M|$-dimensional vectors of non-negative rational numbers whose components are numbered by the elements in M.

We assume that, for each individual, there exists a measure of functioning failure which indicates the degree to which functionings that are considered relevant are not available to the agent. The individual functioning failures constitute the primary inputs for our analysis. A plausible possibility for such a measure is the number of functioning failures, which is the measure used in our empirical application. However, we use a more general approach that assumes the set of possible values of a measure of functioning failure to be the rational numbers. This is plausible because functioning failures could be partial or the measures could incorporate weights that reflect the relative importance of these functioning failures. Our characterization results are true if all real numbers are allowed as functioning-failure values but, because the irrational reals are not required, we work with the set of rational numbers.

Our axiomatization proceeds in three steps. First, we characterize a class of measures of individual deprivation based on the individual functioning failures. In a second stage, we move from individual deprivation to exclusion. An important aspect that distinguishes exclusion from deprivation is the intertemporal aspect of exclusion; see, for example, Atkinson (1998). Consequently, our notion of social exclusion is obtained as an aggregate of the levels of deprivation experienced by an individual in each of a given number of periods. In a final step, these individual indicators of social exclusion are aggregated across individuals to arrive at a class of measures of exclusion for the society as a whole.

2.1 Individual deprivation

For each individual $j \in N$, $e_j \in \mathbb{Q}_+$ is the functioning failure suffered by j in a given period. In this subsection, we consider individual deprivation in a single period which, in order to simplify notation, is not identified explicitly. The vector $e = (e_1, \ldots, e_n) \in \mathbb{Q}^n_+$ is a functioning-failure profile. Let $e, \bar{e} \in \mathbb{Q}^n_+$ and suppose $M \subset N$ is non-empty. The vector $e_M \in \mathbb{Q}^M_+$ is defined by $e_M = (e_j)_{j \in M}$ and, analogously, $e_{-M} \in \mathbb{Q}^{N \setminus M}_+$ is $e_{-M} = (e_j)_{j \in N \setminus M}$.

Finally, $(e_{-M}, \bar{e}_M) \in \mathbb{Q}^n_+$ is given by $(e_{-M}, \bar{e}_M)_j = e_j$ if $j \in N \setminus M$ and $(e_{-M}, \bar{e}_M)_j = \bar{e}_j$ if $j \in M$.

An individual deprivation index for individual $i \in N$ is a function $E_i\colon \mathbb{Q}^n_+ \to \mathbb{R}_+$. For each $e \in \mathbb{Q}^n_+$, $E_i(e)$ is the degree of deprivation suffered by individual i in profile e. The set of individuals whose deprivation level is lower than that of i in a profile e is $\mathcal{J}_i(e) = \{j \in N \mid e_j < e_i\}$.

We now formulate some desirable properties of E_i. The first of these is a normalization axiom. We use zero as the minimal value of the deprivation index and, due to the relative nature of the notion of deprivation, we assume that this minimal value of E_i is obtained whenever no-one in society has fewer functioning failures than individual i. That is, $E_i(e)$ is equal to zero whenever the set $\mathcal{J}_i(e)$ is empty. Conversely, we require the degree of individual i's deprivation to be positive whenever there are people who experience fewer functioning failures than i. As a result, our normalization axiom requires that the deprivation of individual i is zero if and only if the set of individuals with fewer functioning failures is empty—that is, if and only if i's functioning failures are minimal within the profile under consideration.

Normalization: For all $e \in \mathbb{Q}^n_+$, $E_i(e) = 0$ if and only if $\mathcal{J}_i(e) = \emptyset$.

The following axiom illustrates an important difference between the notion of inequality and that of deprivation. In determining the degree of deprivation suffered by an individual $i \in N$, it seems plausible to assume that i's deprivation depends only on its own functioning failures and on those of the individuals who are less deprived than i, that is, those in $\mathcal{J}_i(e)$. The idea that a person's feeling of deprivation in a society arises out of comparing his situation with those who are better off has first been formulated by Runciman (1966) and then used by Sen (1976). Sen argues that individual i's level of deprivation is an increasing function of the number of people who are better off than i. We adapt this general idea to our framework and assume that the extent to which an individual considers itself deprived does not depend on the situation of individuals who have a degree of functioning failures equal to or exceeding that of the individual itself. Thus, unlike in the case of inequality, there is an asymmetry between those who are better off (in terms of functionings) than an individual i and those who are at most as well-off as i itself.

Focus: For all $e, \bar{e} \in \mathbb{Q}^n_+$, if $\mathcal{J}_i(\bar{e}) = \mathcal{J}_i(e)$, $\bar{e}_i = e_i$ and $\bar{e}_j = e_j$ for all $j \in \mathcal{J}_i(e)$, then $E_i(\bar{e}) = E_i(e)$.

As usual, anonymity requires that the identities of the individuals are irrelevant in obtaining a social index. For the individual index E_i, however, it is clear that individual i itself may (and usually will) play a special role. Thus, the anonymity axiom we employ is restricted to the set of individuals other than i and we obtain the following conditional version.

Conditional anonymity: For all $e, \bar{e} \in \mathbb{Q}_+^n$ and for all $j, k \in N \setminus \{i\}$, if $\bar{e}_j = e_k$, $\bar{e}_k = e_j$ and $\bar{e}_\ell = e_\ell$ for all $\ell \in N \setminus \{j, k\}$, then $E_i(\bar{e}) = E_i(e)$.

The next axiom is a standard condition in much of economic theory. It is (linear) homogeneity, which requires that if a profile is multiplied by a positive number, the corresponding level of deprivation is multiplied by the same number. This property ensures that a proportional change in the profile of functioning failures leads to an equiproportional change in individual deprivation.

Homogeneity: For all $e \in \mathbb{Q}_+^n$ and for all $\lambda \in \mathbb{Q}_{++}$, $E_i(\lambda e) = \lambda E_i(e)$.

Translation invariance imposes a restriction on the response of an index to equal absolute changes in a profile. If the same real number is added to each functioning failure, the value of the deprivation index is unchanged. We employ a stronger axiom that applies to additions of different numbers, provided that the set of individuals with fewer functioning failures than i is unchanged and, moreover, the value added to the functioning failures of those that are equally well or worse off than i is the arithmetic mean of the values added to those in $\mathcal{J}_i(e)$.

Strong translation invariance: For all $e, \bar{e} \in \mathbb{Q}_+^n$ and for all $\delta \in \mathbb{Q}^{\mathcal{J}_i(e)}$, if $\mathcal{J}_i(\bar{e}) = \mathcal{J}_i(e)$, $\bar{e}_j = e_j + \delta_j$ for all $j \in \mathcal{J}_i(e)$ and $\bar{e}_k = e_k + \frac{1}{|\mathcal{J}_i(e)|} \sum_{j \in \mathcal{J}_i(e)} \delta_j$ for all $k \in N \setminus \mathcal{J}_i(e)$, then $E_i(\bar{e}) = E_i(e)$.

The standard translation-invariance axiom is implied by the above condition; it corresponds to the case where the δ_j are equal for all $j \in \mathcal{J}_i(e)$.

Finally, we introduce a monotonicity property that is intended to capture the response of the index to a change in the number of individuals in $\mathcal{J}_i(e)$. Another source of deprivation (in addition to the shortfall of functionings as compared to those who are better off) is a lack of the capacity to identify with other members of society. In our model, the possibility of group identification can be represented by the size of the group of agents who experience at least as many functioning failures as agent i. More precisely, consider a profile e such that there is an agent j whose functioning failure is equal to that of agent i. Now suppose we move to a profile $\bar{e}$ which differs from e in that the functioning failure of j is diminished by ε and the functioning failure of another agent k, who is a member of $\mathcal{J}_i(e)$, is augmented by ε while preserving the position of k as one of the members of society who are better off than i. That is, j is a member of $\mathcal{J}_i(\bar{e})$ but not a member of $\mathcal{J}_i(e)$, k is a member of both $\mathcal{J}_i(e)$ and $\mathcal{J}_i(\bar{e})$ and the total excess of i's functioning failure over those of the agents who are better off is unchanged. Monotonicity requires that the deprivation of i should increase because the total functioning shortfall of i is unchanged but the number of agents i can identify with has diminished. This intuition is formalized in the following axiom.

Monotonicity: For all $e, \bar{e} \in \mathbb{Q}^n_+$, for all $j, k \in N \setminus \{i\}$ and for all $\varepsilon \in \mathbb{Q}_{++}$, if $e_j = e_i$, $\bar{e}_j = e_j - \varepsilon$, $\bar{e}_k = e_k + \varepsilon < e_i$ and $\bar{e}_\ell = e_\ell$ for all $\ell \in N \setminus \{j, k\}$, then $E_i(\bar{e}) > E_i(e)$.

The class of deprivation measures characterized by the above axioms has the following structure. The degree of deprivation is obtained as the product of two terms with the following interpretation. The first factor is an increasing function of the number of agents who have fewer functioning failures than i. As mentioned earlier, this number is an inverse indicator of agent i's capacity to identify with other members of society. The second factor is the sum of the differences between e_i and the functioning failures of all agents in $\mathcal{J}_i(e)$. This part captures the aggregate alienation experienced by i with respect to those who are better off. We obtain:

Theorem 1 *: E_i satisfies normalization, focus, conditional anonymity, homogeneity, strong translation invariance and monotonicity if and only if there exists an increasing function $F_i \colon \mathbb{N} \to \mathbb{R}_{++}$ such that, for all $e \in \mathbb{Q}^n_+$,*

$$E_i(e) = \begin{cases} 0 & \text{if } \mathcal{J}_i(e) = \emptyset \\ F_i(|\mathcal{J}_i(e)|) \sum_{j \in \mathcal{J}_i(e)} (e_i - e_j) & \text{if } \mathcal{J}_i(e) \neq \emptyset. \end{cases}$$

Proof. That the indices defined in the theorem statement possess the required properties is straightforward to verify.

Conversely, suppose E_i satisfies the axioms of the theorem statement. Let $e \in \mathbb{Q}^n_+$. If $\mathcal{J}_i(e)$ is empty, normalization immediately implies $E_i(e) = 0$ as desired. Now suppose that $\mathcal{J}_i(e) \neq \emptyset$. By definition of this set, this implies that e_i is positive.

First, we consider the subclass of profiles e such that all agents in $\mathcal{J}_i(e)$ have a functioning failure of zero. By conditional anonymity, the identities of the individuals in $\mathcal{J}_i(e)$

are irrelevant and, therefore, we can consider the profile $(e_{-\mathcal{J}_i(e)}, 0\mathbf{1}_{|\mathcal{J}_i(e)|})$. By the focus axiom,

$$E_i(e_{-\mathcal{J}_i(e)}, 0\mathbf{1}_{|\mathcal{J}_i(e)|}) = E_i(e_i\mathbf{1}_{|N\setminus\mathcal{J}_i(e)|}, 0\mathbf{1}_{|\mathcal{J}_i(e)|}). \tag{1}$$

Using homogeneity with $\lambda = e_i$, it follows that

$$E_i(e_i\mathbf{1}_{|N\setminus\mathcal{J}_i(e)|}, 0\mathbf{1}_{|\mathcal{J}_i(e)|}) = e_iE_i(\mathbf{1}_{|N\setminus\mathcal{J}_i(e)|}, 0\mathbf{1}_{|\mathcal{J}_i(e)|}). \tag{2}$$

Now define

$$a_i(|\mathcal{J}_i(e)|) = E_i(\mathbf{1}_{|N\setminus\mathcal{J}_i(e)|}, 0\mathbf{1}_{|\mathcal{J}_i(e)|}).$$

Using (1) and substituting into (2), we obtain

$$E_i(e_{-\mathcal{J}_i(e)}, 0\mathbf{1}_{|\mathcal{J}_i(e)|}) = e_ia_i(|\mathcal{J}_i(e)|). \tag{3}$$

Because $\mathcal{J}_i(e)$ is non-empty, $a_i(|\mathcal{J}_i(e)|)$ is positive by normalization.

Now consider an arbitrary profile $e \in \mathbb{Q}^n_+$. By the focus axiom, we can without loss of generality assume that $e_k = e_i$ for all $k \in N \setminus \mathcal{J}_i(e)$. Construct a new profile $\bar{e} \in \mathbb{Q}^n_+$ by letting $\bar{e}_j = e_j - e_j = 0$ for all $j \in \mathcal{J}_i(e)$ and $\bar{e}_k = e_i - \frac{1}{|\mathcal{J}_i(e)|}\sum_{j\in\mathcal{J}_i(e)} e_j$ for all $k \in N\setminus\mathcal{J}_i(e)$. Using strong translation invariance with $\delta_j = -e_j$ for all $j \in \mathcal{J}_i(e) = \mathcal{J}_i(\bar{e})$, it follows that $E_i(\bar{e}) = E_i(e)$. Because all agents in $\mathcal{J}_i(\bar{e})$ have a functioning failure of zero in $\bar{e}$, (3) implies

$$\begin{aligned}
E_i(e) = E_i(\bar{e}) &= \bar{e}_ia_i(|\mathcal{J}_i(\bar{e})|) \\
&= \left(e_i - \frac{1}{|\mathcal{J}_i(e)|}\sum_{j\in\mathcal{J}_i(e)} e_j\right) a_i(|\mathcal{J}_i(e)|) \\
&= \left(|\mathcal{J}_i(e)|e_i - \sum_{j\in\mathcal{J}_i(e)} e_j\right) \frac{a_i(|\mathcal{J}_i(e)|)}{|\mathcal{J}_i(e)|} \\
&= \frac{a_i(|\mathcal{J}_i(e)|)}{|\mathcal{J}_i(e)|}\sum_{j\in\mathcal{J}_i(e)} (e_i - e_j) \\
&= F_i(|\mathcal{J}_i(e)|)\sum_{j\in\mathcal{J}_i(e)} (e_i - e_j),
\end{aligned}$$

where $F_i(|\mathcal{J}_i(e)|) = \frac{a_i(|\mathcal{J}_i(e)|)}{|\mathcal{J}_i(e)|}$. F_i is positive-valued because a_i is, and the increasingness of F_i follows from monotonicity. ■

2.2 Aggregate deprivation

To obtain a measure of aggregate deprivation, we employ an additive approach. Our objective is to define a function $\mathbf{D}: \mathbb{Q}^n_+ \to \mathbb{R}_+$ where, for all $e \in \mathbb{Q}^n_+$, $\mathbf{D}(e)$ is the degree of deprivation in the society under consideration. Deprivation additivity requires that aggregate deprivation is an additive function of the individual deprivations suffered by the members of the society.

Deprivation additivity: There exists an increasing function $\rho: \mathbb{Q}_+ \to \mathbb{R}_+$ such that, for all $e \in \mathbb{Q}^n_+$, $\mathbf{D}(e) = \rho\left(\sum_{i \in N} E(e)\right)$.

Using Theorem 1, it is straightforward to verify that the following class of aggregate deprivation measures results from adding deprivation additivity to our earlier axioms.

Theorem 2 *:* $\mathbf{D}$ *satisfies deprivation additivity with functions* E_i *satisfying normalization, focus, conditional anonymity, homogeneity, strong translation invariance, and monotonicity if and only if there exist increasing functions* $F_i: \mathbb{N} \to \mathbb{R}_{++}$ *for all* $i \in N$ *and* $\rho: \mathbb{Q}_+ \to \mathbb{R}_+$ *such that:*

(a) for all $i \in N$ *and for all* $e \in \mathbb{Q}^n_+$,

$$E_i(e) = \begin{cases} 0 & \text{if } \mathcal{J}_i(e) = \emptyset \\ F_i(|\mathcal{J}_i(e)|) \sum_{j \in \mathcal{J}_i(e)} (e_i - e_j) & \text{if } \mathcal{J}_i(e) \neq \emptyset; \end{cases}$$

(b) for all $e \in \mathbb{Q}^n_+$,

$$\mathbf{D}(e) = \rho\left(\sum_{i \in N} E_i(e)\right). \tag{4}$$

2.3 Individual exclusion measures

To incorporate the dynamic aspect of exclusion, we now consider an intertemporal extension of the deprivation measures introduced in the previous subsection. Suppose the set of time periods is $T = \{1, \ldots, t\}$ with $t \in \mathbb{N} \setminus \{1\}$. An intertemporal functioning-failure profile is a vector $\mathbf{e} = (e^1, \ldots, e^t) = ((e^1_1, \ldots, e^1_n), \ldots, (e^t_1, \ldots, e^t_n)) \in \mathbb{Q}^{tn}_+$ where, for all $i \in N$ and for all $\tau \in T$, e^τ_i is the functioning failure of agent i in period τ. An exclusion measure for individual $i \in N$ is a mapping $\mathbf{E}_i: \mathbb{Q}^{tn}_+ \to \mathbb{R}_+$ that assigns i's level of exclusion to each profile of intertemporal functioning failures.

A natural way to aggregate across different time periods is to use a measure that is additive in the per-period deprivations suffered by individual i. This requirement is formalized in the following intertemporal-additivity axiom. Suppose that, for each $\tau \in T$, $E_i^\tau \colon \mathbb{Q}_+^n \to \mathbb{R}_+$ is a measure of agent i's deprivation in period τ.

Intertemporal additivity: There exists an increasing function $\varphi_i \colon \mathbb{Q}_+ \to \mathbb{R}_+$ such that, for all $\mathbf{e} \in \mathbb{Q}_+^{tn}$, $\mathbf{E}_i(\mathbf{e}) = \varphi_i \left(\sum_{\tau \in T} E_i^\tau(e^\tau) \right)$.

Using Theorem 1, it is straightforward to verify that the following class of individual exclusion measures results from adding intertemporal additivity to our earlier axioms.

Theorem 3 *: $\mathbf{E}_i$ satisfies intertemporal additivity with functions E_i^τ satisfying normalization, focus, conditional anonymity, homogeneity, strong translation invariance, and monotonicity if and only if there exist increasing functions $F_i^\tau \colon \mathbb{N} \to \mathbb{R}_{++}$ for all $\tau \in T$ and $\varphi_i \colon \mathbb{Q}_+ \to \mathbb{R}_+$ such that:*

(a) for all $\tau \in T$ and for all $e^\tau \in \mathbb{Q}_+^n$,

$$E_i^\tau(e^\tau) = \begin{cases} 0 & \textit{if } \mathcal{J}_i(e^\tau) = \emptyset \\ F_i^\tau(|\mathcal{J}_i(e^\tau)|) \sum_{j \in \mathcal{J}_i(e^\tau)} (e_i^\tau - e_j^\tau) & \textit{if } \mathcal{J}_i(e^\tau) \neq \emptyset; \end{cases}$$

(b) for all $\mathbf{e} \in \mathbb{Q}_+^{tn}$,

$$\mathbf{E}_i(\mathbf{e}) = \varphi_i \left(\sum_{\tau \in T} E_i^\tau(e^\tau) \right).$$

Theorem 3 allows for a broad class of individual intertemporal aggregation rules. If the functions F_i^τ are identical for all $\tau \in T$, we obtain a measure that treats all time periods equally. If discounting is considered desirable, one possibility of choosing these functions is, for example, to define $F_i^\tau(J) = d^\tau g_i(J)$ for all $\tau \in T$ and for all $J \in \mathbb{N}$ with an increasing function $g_i \colon \mathbb{N} \to \mathbb{R}_{++}$ and a discount factor $d \in (0,1)$. This corresponds to the case of geometric discounting.

2.4 Social exclusion measures

The final step in our derivation of a class of social-exclusion measures consists of aggregating the individual measures characterized in the previous subsection across individuals. For that purpose, we define an aggregate exclusion measure as a function $\mathbf{E} \colon \mathbb{Q}_+^{tn} \to \mathbb{R}_+$ that assigns an aggregate level of exclusion to each profile of intertemporal functioning failures. We assume that the measure is additive in the individual levels of exclusion as measured by the functions $\mathbf{E}_i$.

Interpersonal additivity: There exists an increasing function $\psi\colon \mathbb{Q}_+ \to \mathbb{R}_+$ such that, for all $\mathbf{e} \in \mathbb{Q}_+^{tn}$, $\mathbf{E}(\mathbf{e}) = \psi\left(\sum_{i\in N} \mathbf{E}_i(\mathbf{e})\right)$.

In addition, we impose an anonymity condition that ensures that individuals are treated symmetrically with respect to their functioning failures.

Anonymity: For all $\mathbf{e}, \bar{\mathbf{e}} \in \mathbb{Q}_+^{tn}$ and for all $j, k \in N$, if $(\bar{e}_j^1, \ldots, \bar{e}_j^t) = (e_k^1, \ldots, e_k^t)$, $(\bar{e}_k^1, \ldots, \bar{e}_k^t) = (e_j^1, \ldots, e_j^t)$ and $(\bar{e}_\ell^1, \ldots, \bar{e}_\ell^t) = (e_\ell^1, \ldots, e_\ell^t)$ for all $\ell \in N \setminus \{j, k\}$, then $\mathbf{E}(\bar{\mathbf{e}}) = \mathbf{E}(\mathbf{e})$.

Together with the result of Theorem 3, these two conditions characterize a class of aggregate exclusion measures with an additive structure. Due to the anonymity axiom, the functions φ_i and F_i^τ can be chosen to be independent of i. We obtain

Theorem 4 *: $\mathbf{E}$ satisfies intertemporal additivity with functions $\mathbf{E}_i$ satisfying the axioms of Theorem 3 and anonymity if and only if there exist increasing functions $F^\tau\colon \mathbb{N} \to \mathbb{R}_{++}$ for all $\tau \in T$, $\varphi\colon \mathbb{Q}_+ \to \mathbb{R}_+$ and $\psi\colon \mathbb{Q}_+ \to \mathbb{R}_+$ such that:*

(a) for all $i \in N$, for all $\tau \in T$ and for all $e^\tau \in \mathbb{Q}_+^n$,

$$E_i^\tau(e^\tau) = \begin{cases} 0 & \text{if } \mathcal{J}_i(e^\tau) = \emptyset \\ F^\tau(|\mathcal{J}_i(e^\tau)|) \sum_{j\in\mathcal{J}_i(e^\tau)} (e_i^\tau - e_j^\tau) & \text{if } \mathcal{J}_i(e^\tau) \neq \emptyset; \end{cases} \tag{5}$$

(b) for all $i \in N$ and for all $\mathbf{e} \in \mathbb{Q}_+^{tn}$,

$$\mathbf{E}_i(\mathbf{e}) = \varphi\left(\sum_{\tau\in T} E_i^\tau(e^\tau)\right);$$

(c) for all $\mathbf{e} \in \mathbb{Q}_+^{tn}$,

$$\mathbf{E}(\mathbf{e}) = \psi\left(\sum_{i\in N} \mathbf{E}_i(\mathbf{e})\right) = \psi\left(\sum_{i\in N} \varphi\left(\sum_{\tau\in T} E_i^\tau(e^\tau)\right)\right). \tag{6}$$

3 An application to EU countries

Social exclusion has recently become one of the main concepts in social policy debates in EU countries. With the Treaty of Amsterdam, which came into force in 1999, the EU has, indeed, enlarged its objectives to include the combating of social exclusion among its members. The purpose of this section is to illustrate the social exclusion measure, $\mathbf{E}$

in (6), and the deprivation measure, **D** in (4), proposed in the paper using the European Community Household Panel (ECHP). In the application, we let the functions φ, ψ, ρ and F^{τ} for all $\tau \in T$ be the identity mapping. We base our analysis on all the waves that are currently available of ECHP, which cover the period from 1994 to 2000. The surveys are conducted at a European national level. The ECHP is an ambitious effort at collecting information on the living standards of the households of the EU member states using common definitions, information collection methods and editing procedures. It contains detailed information on incomes, socio-economic characteristics, housing amenities, consumer durables, social relations, employment conditions, health status, subjective evaluation of well-being, etc.. Of the 15 EU member states, we could not consider Austria, Finland, Luxembourg and Sweden since the data for these countries were not available for all the waves. For similar reasons we had to exclude Germany and the UK. In particular, the ECHP surveys of these countries were substituted by national surveys, SOEP and BHPS respectively, that did not collect information on all the variables considered in our application. Information has been collected at the individual or the household level depending on the variable, but the unit of our analysis is the individual. The calculation uses required sample weights and, since we are interested in analyzing the persistence of deprivation, we considered only individuals that were interviewed in all the seven waves. In ECHP a person's life has been measured along the following domains: financial difficulties, basic needs and consumption, housing conditions, durables, health, social contacts and participation, and life satisfaction.

For the choice of the non-monetary indicators to be considered for measuring social exclusion and deprivation with ECHP, we follow the suggestions of Eurostat (2000) and analyze the well-being of EU societies focussing on the 14 non-monetary variables[1] proposed there. These are the following:

- Financial difficulties: 1. Proportion of persons living in households that have great difficulties in making ends meet; 2. Proportion of persons living in households that are in arrears with (re)payment of housing and/or utility bills;
- Basic necessities: 3. Proportion of persons living in households which cannot afford meat, fish or chicken every second day; 4. Proportion of persons living in households

[1] In fact, the non-monetary indicators recommended in Eurostat (2000) are 15. We decided to drop the one belonging to the health domain, namely the proportion of people that were severely hampered in their daily activity by long-lasting health problems, since there was a considerable discontinuity between the ECHP waves for this indicator.

which cannot afford to buy new clothes; 5. Proportion of persons living in households which cannot afford a week's holiday away from home;

- Housing conditions: 6. Proportion of persons living in the accommodation without a bath or shower; 7. Proportion of persons living in the dwelling with damp walls, floors, foundations, etc.; 8. Proportion of persons living in households which have a shortage of space;

- Durables: 9. Proportion of persons not having access to a car due to a lack of financial resources in the household; 10. Proportion of persons not having access to a telephone due to a lack of financial resources in the household; 11. Proportion of persons not having access to a color TV due to a lack of financial resources in the household;

- Health: 12. Proportion of persons (over 16) reporting bad or very bad health;

- Social contact: 13. Proportion of persons (over 16) who meet their friends or relatives less often than once a month (or never);

- Dissatisfaction: 14. Proportion of persons (over 16) being dissatisfied with their work or main activity.

We calculate **E** and **D** separately for two sets of indicators V_1 and V_2, where V_1 includes the indicators in the domains of financial difficulties, basic necessities, housing conditions, and durables, and V_2 includes the remaining indicators. The reason for separate calculations is that for indicators covered under V_1 we have household level information, whereas for the indicators in V_2 the available information is at the individual level, with the additional constraint that the minimum age of the reportee is 16.

Numerical estimates of social exclusion as measured by **E** for the EU member states are reported in Table 1, the values being plotted in Figures 1 and 2. The first column of the table gives the names of the countries for whom the required information was available. In column 2 we present, for each country, the estimates for V_1 (values plotted in Figure 1) while column 3 gives the analogous values for V_2 (values plotted in Figure 2). Several interesting features emerge from Table 1. Portugal is the most excluding country followed by Greece. At a distance we observe the other two Southern European countries, namely Spain and Italy. The value of **E** for Ireland is slightly higher than the one for Italy. If we consider the ranking of countries from high to low exclusion, then an unambiguous sequence is Portugal, Greece, Spain, Ireland, Italy, France, Belgium, the Netherlands and

Denmark. In V_2 as well, Portugal is the member state with maximum exclusion, Italy has the second worst off position and Ireland performs the best by showing the lowest values. Denmark and the Netherlands show values higher than Ireland but lower than all other member states. The other countries, namely Greece, Spain, France and Belgium are divided into two groups with Belgium belonging to one separate group with relatively lower values of exclusion. Finally, except for Portugal, the ranking of countries by any measure in V_2 is different from that in V_1.

Estimates of **D** are reported in Tables 2 (variables included in V_1) and 3 (variables included in V_2), and plotted in Figures 3 and 4 respectively. In the first column of the tables the names of the countries are indicated, while in all the following columns the values that the deprivation index assumes over the years are reported. In the deprivation measure, as opposed to social exclusion, we do not consider persistence in the deprivation state. In other words, the deprivation index is the same as the social exclusion one, the only difference being that the individual deprivation variable in the social exclusion measure is the sum (in this application, without discounting) of the individual deprivation variables of the seven waves. Persistence in the deprivation state is a key variable in understanding the different performance of EU member states in the two measures suggested in this paper. We focus first on the estimates of deprivation for V_1 (Figure 3). The countries appear to be grouped into three classes according to the level of deprivation reached: Portugal and Greece into the first; Ireland, Spain and Italy into the second; all the remaining countries into the remaining class. In all the years we observe a descending trend and convergence over time in particular of the second and third group that present values of deprivation much more similar in the last wave than they were in the first. Based on these observations on deprivation, we can re-read the values of the measures of social exclusion. We notice the three classes, definitely Portugal and Greece, perform very differently than all other countries, but the position of France is now more ambiguous being in between the second and third class. Portugal and Greece present a greater dissimilarity in social exclusion than in deprivation in all the years considered. This fact is caused by the higher persistence in the deprivation state that individuals face in Portugal than in Greece. In other words, in each period the percentage of the population that is deprived is slightly higher in Portugal than in Greece, but in the latter it is easier for individuals to escape from the deprivation state than it is in the former. Hence, the individuals deprived that we observe in each period vary more over time in Greece than in Portugal.

In deprivation for V_2 (Figure 4), we do not observe nor convergence over time, neither a common descending trend. In the first two waves Italy was the most deprived country, but a drop in the value observed starting from the third wave associated with lower persistence led Portugal to be more socially excluded than Italy.

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Table 1: Social Exclusion in EU Member-States (1994-2000).

	V1	V2
Belgium	276.424	78.182
Denmark	211.608	53.043
Greece	560.302	96.653
Spain	398.508	97.028
France	308.262	98.924
Ireland	373.358	43.474
Italy	372.036	137.782
Netherlands	240.198	51.418
Portugal	663.323	153.406

V1 considers jointly the variables included in the domains of financial difficulties, basic necessities, housing conditions, durables.
V2 considers jointly the variables included in the domains of health, social contact and dissatisfaction. The values reported are per persons, with the additional constraint of age being at least 16 for V2.

Table 2: Deprivation in V1 in EU Member-States (1994-2000).

V1	wave 1	wave 2	wave 3	wave 4	wave 5	wave 6	wave 7
Belgium	0.456	0.451	0.408	0.373	0.369	0.373	0.363
Denmark	0.408	0.306	0.316	0.264	0.288	0.293	0.278
Greece	0.986	0.908	0.882	0.863	0.781	0.787	0.761
Spain	0.656	0.630	0.630	0.614	0.545	0.491	0.464
France	0.507	0.468	0.473	0.459	0.437	0.427	0.405
Ireland	0.739	0.605	0.608	0.579	0.504	0.509	0.333
Italy	0.620	0.650	0.609	0.542	0.553	0.548	0.522
Netherlands	0.363	0.342	0.342	0.333	0.345	0.304	0.300
Portugal	0.993	0.966	0.915	0.911	0.890	0.873	0.788

V1 considers jointly the variables included in the domains of financial difficulties, basic necessities, housing conditions, durables.

Table 3: Deprivation in V2 in EU Member-States (1994-2000).

V2	wave 1	wave 2	wave 3	wave 4	wave 5	wave 6	wave 7
Belgium	0.133	0.125	0.130	0.136	0.128	0.102	0.101
Denmark	0.074	0.069	0.085	0.083	0.079	0.074	0.085
Greece	0.188	0.147	0.131	0.134	0.137	0.127	0.119
Spain	0.154	0.152	0.148	0.151	0.142	0.134	0.135
France	0.146	0.150	0.152	0.155	0.159	0.157	0.159
Ireland	0.092	0.066	0.058	0.066	0.053	0.058	0.064
Italy	0.243	0.246	0.189	0.185	0.192	0.191	0.192
Netherlands	0.073	0.067	0.069	0.069	0.075	0.072	0.083
Portugal	0.225	0.219	0.214	0.227	0.225	0.220	0.211

V2 considers jointly the variables included in the domains of health, social contact and dissatisfaction. The values reported are per persons, with the additional constraint of age being at least 16 for V2.

Figure 1: Social Exclusion in EU Member-States (1994-2000), V_1.

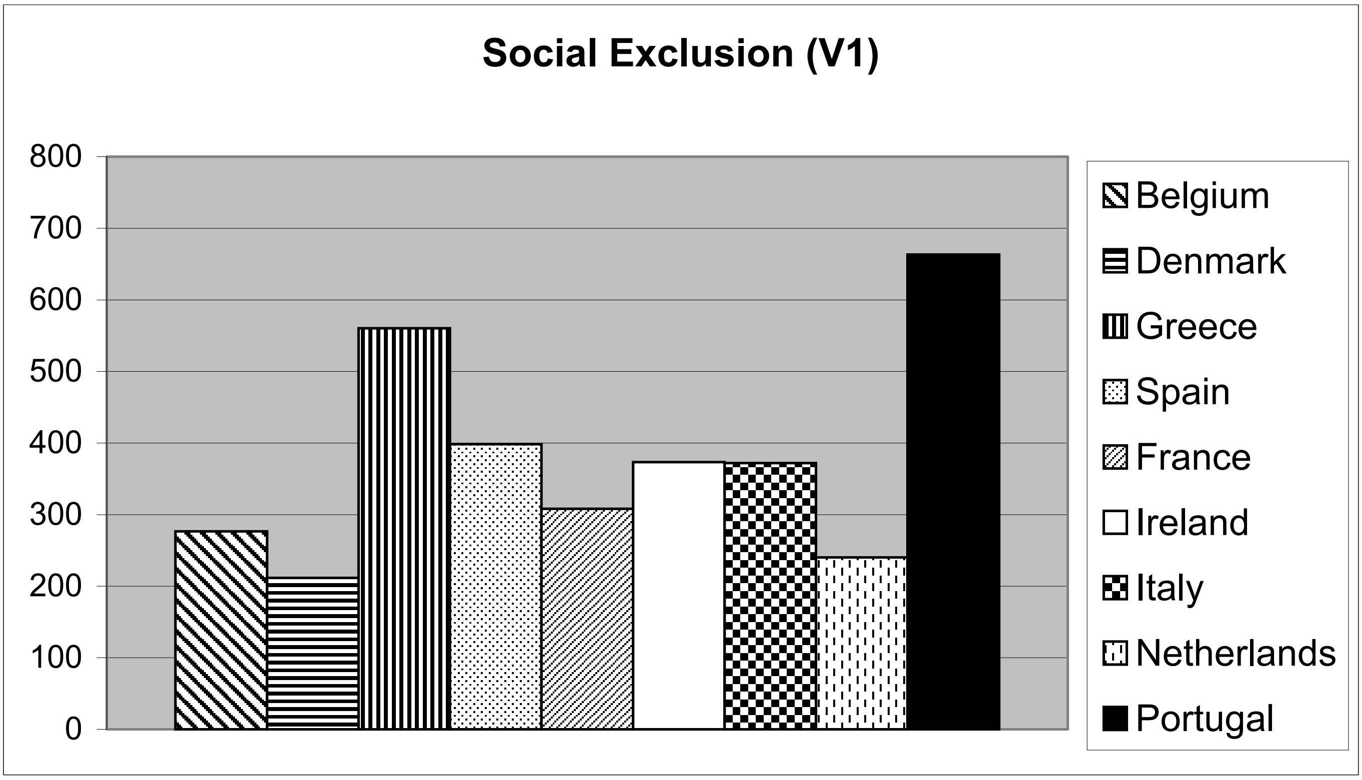

Figure 2: Social Exclusion in EU Member-States (1994-2000), V_2.

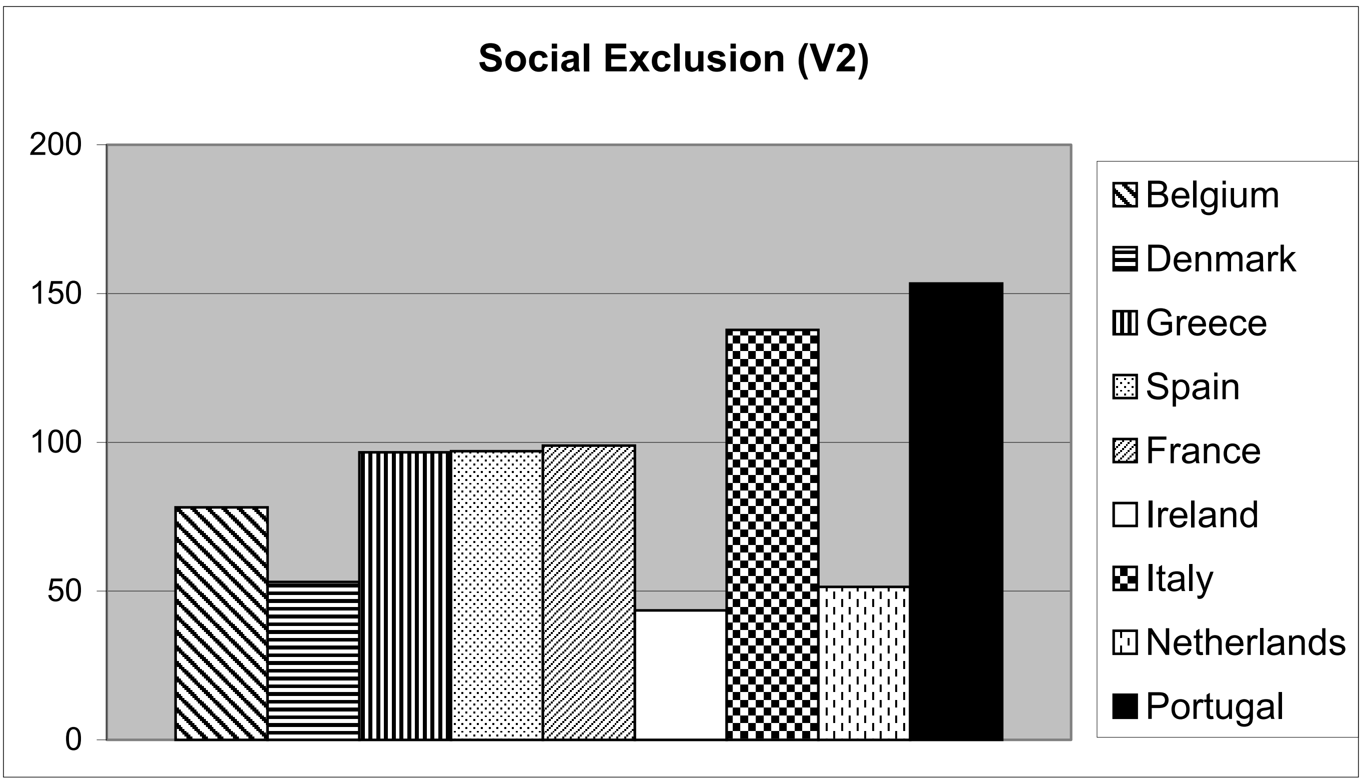

Figure 3: Deprivation in EU Member States (1994-2000), V_1.

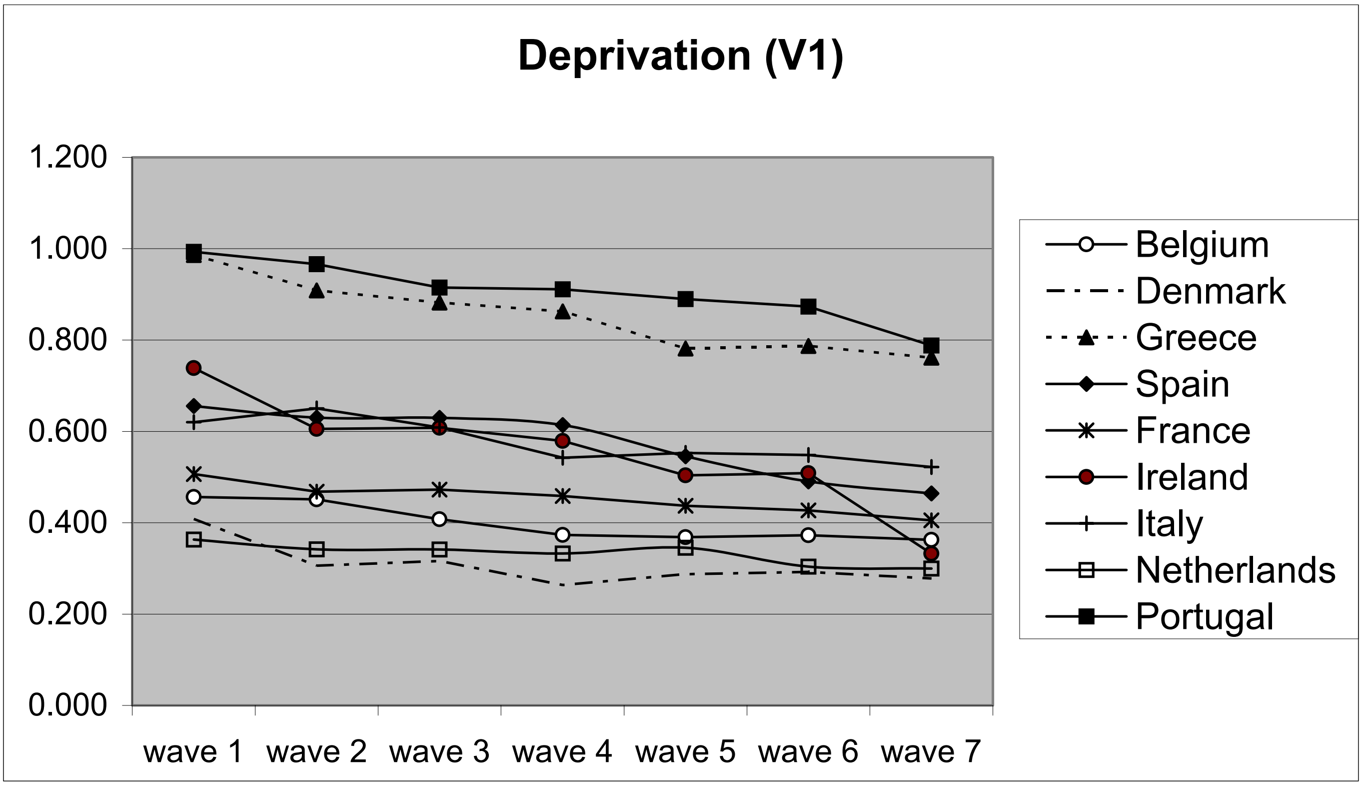

Figure 4: Deprivation in EU Member States (1994-2000), V_2.