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# VERTICAL EQUITY AND WELFARE: WHICH EFFECTIVE REDISTRIBUTION? AN APPLICATION TO ITALIAN DATA

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# Vertical Equity and Welfare: Which Effective Redistribution? An Application to Italian Data

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## Abstract

What matters for effective progression (or progressivity) is the tax income schedule and where the taxpayers are located. As a consequence a tax schedule with high marginal rates can have less progressivity than another with low marginal rates and the examination of schedular differences can be ambiguous when the examination itself is judged in isolation from the income distribution to which they apply. Unfortunately even recent empirical researches still continue to invoke the results of Jakobsson (1976) and Kakwani (1977a), which does not take into account the before-tax inequality and produce unequivocal evaluations of redistributive effects on the basis of a fixed and common distribution of before-tax income for all the schedules being compared. In this paper we present an application to Italian household micro-data and tax systems between 1995 and 2000, by considering distributional differences before undertaking comparisons between income tax schedules. We take critically advantage of a paper by Dardanoni-Lambert (2002). This involves a transplant-and-compare procedure that corrects post-tax incomes for inequality and size differences (if any) between the distributions to which they apply, by 'importing' the relevant income tax schedules from one regime into the other. The residual progression comparisons over transplanted schedules may lead, via the Jakobsson/Kakwani and Atkinson's (1970) seminal result, to mathematically valid and normatively significant redistributive judgement based on the actual distributions particularly when transplantation functions are isoelastic: in this case it is possible to show that it achieves an 'independence of baseline' property.

We find that before-tax log distributions differ essentially only by location and scale. By using the 1995 distribution as baseline, the other two are both its isoelastic transformations: local measures of residual progression of transplanted schedules and original ones by isoelasticity condition are the same.

In accordance with these results we apply the procedure by correcting log post-tax distributions by using the intercepts and slopes to take into account distributional differences.

We present and discuss empirical evidence and statistical questions arising from the application of this methodology and, of course, the main effectual redistributive results of Visco's personal income tax Reform. Some of the main findings are that the reform was able to defend the original degree of redistribution and, in the last period, also to decrease the total tax ratio.

**JEL Codes**: H23, H24, I30

**Key words**: personal income taxation, redistribution, welfare

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## Introduction

In the past twenty years many European and OECD countries have accomplished tax structure reforms. One of the policy goals was often to decrease the total tax burden in the economy, redistributing in some way the gains over tax units.

In Italy a restructuring of personal income tax (IRPEF) schedule was implementing during the mid 90's .

The direct taxation in Italy provides around 15 percent of the total GDP and IRPEF total revenue accounts for around ¾ of total revenues from direct taxation. Italian Constitutional Law states that IRPEF had to be progressive; as a matter of fact, it is the principal Italian device to catch up with vertical equity objective.

This paper focus on some aspects of the 1998 and 2000 IRPEF reform, that is, on the effects induced on the Italian distribution of income and inequality by personal income tax reforms[1], identifying the pure progressivity (effective progression) of regimes, and the related well-being in the economy. We shall do it using a static framework, that is (slightly changing the definition of Onrubia et al. (2004)), working with the *final* pre-tax income distribution - the pre-tax income distribution once individual behaviour variations (e.g. in labour supply) have been taken into account as a result of tax change - and the post-tax income distribution – the distribution of incomes resulting after the taxation process.

Redistributive effect and, in some cases, normative significance topics have been already investigated by other Italian researchers. Among others, Birindelli et al. (1998), CER (1998a, 1998b), Bosi et al. (1999) examined 1998 IRPEF reform *versus* 1997 IRPEF; Giannini and Guerra (1999) compared 1999 IRPEF *versus* 1990 IRPEF; Fiorio (2002) illustrated the post-tax inequality variation by 1998 IRPEF *versus* 1991 IRPEF, while Gastaldi and Liberati (2004) presented post-tax Lorenz curves comparisons by using 1995, 1998, 2000, 2002, 2003, IRE[2] pre- and post-tax data.

All these papers use the Bank of Italy *Survey of Households' Income and Wealth*, operating with a microsimulation model (*MSm*) to recover the pre-tax income since taxation information is not available for this survey.

[1] IRPEF is applied on a individual basis. Table 1 in the Appendix shows the reduction of the number of fiscal brackets - from seven to five - from 1995 onwards, and the variation of the nominal tax rates (the highest was reduced and the lowest was increased). From 1995 to 1998, tax allowances structure changed, increasing the amount and the number of tax allowances both for self employment and dependent work, and increasing tax allowance and tax credits (not refundable) for 'family burdens', and other minor attributes. From 1998 to 2000, the main aim of the second step of IRPEF reform was, together with the reinforcement of the redistribution between the tax units, to decrease the total IRPEF burden, diminishing the tax rate for the second bracket (- 1%) and acting on the tax credit structure,.

[2] This is the name of the proposal of personal income tax reform presented in Italy by the current Executive on 21 December 2001.

We are not entirely satisfied with procedures used to determine the results provided by the authors of all these contributions.[3] Some practitioners do not use any equivalence scale to compare incomes of individuals belonging households with different dimensions or compositions (CER, 1998a, 1998b); others provide index numbers, measures of relative inequality or progressivity, to summarise inequality and redistributive effects (Birindelli et al. (1998), Bosi et al. (1999), and Giannini and Guerra (1999)), while it is well known that welfare-theoretic significance about comparative degrees of income inequality comes out - after an appropriate yields standardisation - only by looking at the entire Lorenz curves configuration (Lorenz dominance criterion).[4] Finally, and from our point of view the relevant point, these papers do not deal with the fact that the income tax redistributive effect is determined by the matching between the tax schedule *and* the income distribution to which the tax schedule applies. They fix a common distribution of pre-tax income for all schedules being compared and thus, with reference to results, they implicitly ignore the possibility to be dependent on the before-tax distribution chosen as the 'reference' distribution. Is this realistic for accurate intertemporal, and international, comparisons when, as a matter of fact, tax schedules operate on different income distributions? There is certainly a lot of evidence relating to pre-tax income distributive changes over time and across nations, then if the purpose is to assess the real impact rather than the potential effects of different systems, we feel that a correct redistribution analysis should require incorporation of pre-tax inequality differences.

More recently, Fiorio (2002) and Gastaldi – Liberati (2004) presented comparisons of redistributive effects based on criterion of Lorenz dominance, but their works still continue to not take into account pre-tax inequality; Fiorio, it seems, do not advise the readers of the crucial 'independence of baseline' point, while Gastaldi and Liberati make use of the Atkinson Theorem providing judgement about inequality with respect to the potential effects, rather than the actual ones.

As we shall see below, theoretical questions are the reasons of this empirical approach, but a recent paper (Dardanoni and Lambert, 2002) has suggested to the

[3] We refer to Fiorio (2002, pp. 2 - 3) for a deeper critical analysis of controversy about Birindelli et al. (1998), Cer (1998a, 1998b), Bosi et al. (1999), and Giannini and Guerra (1999). About Fiorio (2002), and Gastaldi and Liberati (2002) see on.

[4] We implicitly prefer a partial orderings among the set of income distributions by unanimous preference, rather than a complete ordering. Advocating the fundamental Atkinson theorem (1970), which will be stated later, Formby and Smith (1986, p. 562) comment, "If Lorenz curves intersect, a social welfare function can always be found which ranks income distribution differently than does the Gini coefficient or other summary measures of inequality.". As a consequence, if Lorenz curves do not intersect any inequality index that fulfil the Pigou-Dalton transfer principle and Symmetry will be robust. See Kondor (1975) about "value judgements implied by the use of various measures of income inequality" and, on this, the Dalton's pioneering article (1920).

practitioners a way to try to solve this practical issue. By taking critically advantage from this new procedure, this paper explores the effect of personal income taxes when pre-tax distributional differences are explicitly taken into consideration *before* undertaking local progression comparisons. In particular, it contains the application of the new method to Italian household micro-data and personal income tax systems throughout the last five years of twentieth century.

The rest of the paper is organized as follows: in section 2 we describe the original data set and some general methodological issues on the microsimulation model that provide the data we need. Section 3 describes the usual analytical framework, and tools that are now on hand to empirical researchers. In section 4 we shall go on presenting the methodology, problems and choices involved in implementing the innovative procedure, and first results. Section 5 concludes the paper; we discuss the pro and cons of the new procedure in light of our results and offer some conclusive remarks.

## 1. Data Description and the *MSm*

The data set we use is based on the *Survey of Households' Income and Wealth* (SHIW) published by the Bank of Italy. The SHIW is largely used in empirical analyses of income and wealth in Italy, and in general on saving behaviour and household spending.

SHIW collects detailed micro-data containing information on different sources of disposable incomes, consumption, saving, monetary and financial variables, labor market, social-demographic characteristics of each household member, and other kind of figures: SHIW 1995 covers 8,135 households composed of 23,924 individuals, the number of households interviewed in 1998 SHIW is 7,147 with a total of 20,901 individuals, and in the 2000 survey the corresponding values are 8.001 and 22,268.[5] Households are selected randomly.

The sampling design involves unequal stratum sampling fractions, thus, we need to use of sampling weights to obtain unbiased estimates: by SHIW, to each household has been attached a sample weight inversely related to the probability to be included into the sample.[6]

[5] For a critical discussion about the data provided by the SHIW see Brandolini (1999). Brandolini and Cannari (1994) analysed the quality of these data and advocated that it is similar to analogous surveys in other countries.

[6] All household members have the same sample weight. The sum of the survey weights is equal to the total number of sampled units. As there is no obligation to take part or answer, the SHIW suffers from a very high no-response rate, but ex-post reweigthing is computed in the Survey to account for it. However, this weighting procedure did not help to adjust for missing data or other nonsampling errors related to the income data.

We need to work with pre- and post-tax personal income distribution. So we need to recover the pre-tax incomes, since all data in the Survey are net of taxes. We take advantage of micro-data produced by the "Istituto per gli Studi e l'Analisi Economica" (ISAE) with ITAXMOD package, to have on hand tax liabilities according to 1995, 1998 and 2000 tax codes.

ITAXMOD is a static microsimulation model that allow the simulation of the immediate impact of a change in the rules of households taxes.[7] It was, in 1989, the first microsimulation model of personal income taxation in Italy.[8]

It acquires the post-tax income data revealed by the interviewed and embodies a procedure to reconstruct gross income, correcting for tax evasion under the assumption that surveyed net income is halfway between the (minimum) after-tax declared income and 'true' net income.[9] ITAXMOD developers postulate that the tax evasion is substantially concentrated on self-employment income, while wage and salary earners declared incomes assumed to be near the 'true' values, thus, with an evasion parameter equal to zero. Then, after the application of the procedure, essentially based on the inverse of the algorithm that determines the individual tax burden, ITAXMOD provides gross income micro-data[10] that are validated by exogenous information on fiscal (the Finance Ministry's fiscal data stored by SOGEI) and national aggregates. Finally, ITAXMOD obtains post-tax income by using now directly the algorithm of IRPEF, including all the available information about the attributes of the household and its members. Pre- and post-tax incomes so computed are thus the start line for the application of the methodology: in this paper we necessitate to have on hand both the pre- and post-tax distribution of income, the two vectors that, for each period, allow to investigate the real redistributive effect of taxation process.

## 2. The Analytical Framework: Definitions, Tools

Here we first recall the formal framework and the core results of the established literature that, up to Dardanoni and Lambert findings, have been the usual reference in the practical work of assessing alternative tax systems.

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7 Di Biase et al. (1995).

8 Lugaresi (1989, 1990).

9 About the methodology, see Di Biase et al. (1995, pp. 22-23) and Marenzi (1996). Among others, Cannari et al. (1997) find that the surveyed net income is higher than the IRPEF one, in particular for self-employment source of income

10 Earned and self-employment incomes, pensions, income from immovable properties, entrepreneurial incomes (in the IRPEF tax basis), and other minor incomes sources are included. Capital incomes (not included in the IRPEF tax basis, but subject to separated taxation regime) and fringe benefits are excluded.

Let income $x$ be continually distributed over some support $[0, z]$ and represented by the function $F:[0,z]\to[0,1]$; the income pre-tax distribution function is denoted by the range $F(x)$; $f(x)$ is the associated density function defined on the same interval and assumed strictly positive throughout the lowest income $x_1 \geq 0$ and the highest income level $x_N \leq z$ ($z$ could be described as 'any income level in excess of the highest one actually occurred'), and $n$ is the number of observations. For each $p\in[0,1]$ there is just one income level $y$, which satisfies $p = F(y)$.[11] This means that the first $100p\%$ of income units are those with pre-tax income less than or equal to $y$. If mean pre-tax income is

$\mu_X = \int_0^z x\, f(x)\, dx$, and mean tax liability[12] is $\mu_T = \int_0^z t(x)\, f(x)\, dx$,

the *Total tax ratio* is $T/X = \dfrac{\mu_T}{\mu_X} = r$

To illustrate the Lorenz order consider the Lorenz function $L:[0,z]\to[0,1]$ for, respectively, the pre-tax income, post–tax income, and tax liability, $L_X$, $L_N$, and $L_T$:[13]

$$p = F(y) \longrightarrow L_X(p) = [1/\mu_X]\int_0^y x f(x)\mathrm{d}x$$

$$p = F(y) \longrightarrow L_N(p) = 1/[\mu_X(1-r)]\int_0^y N(x)f(x)\mathrm{d}x$$

$$p = F(y) \longrightarrow L_T(p) = 1/[\mu_X r]\int_0^y t(x)f(x)\mathrm{d}x$$

The graph of a Lorenz function is the Lorenz curve, which indicates the share of total income enjoyed by the bottom $p$ proportion of the population. For the sake of income distribution comparisons, the Lorenz curve always closer to the uniform one is said to represent less inequality. On these grounds, the seminal papers in static literature - Jakobsson (1976), Fellman (1976), and Kakwani (1977a) (JFK) - based their results.[14] Jakobsson (1976) and Fellman (1976) point out that:

[11] For minor complication which may arise when extremes (p=0 and p=1) are included, we refer to Lambert, 2001, p. 20 and 32.

[12] The $t(x)$ is the tax liability of an income unit having pre-tax monetary income $x$ and, for analytical convenience, will be assumed differentiable. We denoted $t'(x)$ as the first derivative that determines the marginal tax rate, and assume that $0 \leq t'(x) < 1\ \forall x$, thus $0 \leq t(x) < x\ \forall x$, and net income $N(x) = x - t(x)$ is a monotone increasing function of pre-tax income $x$. The individual tax burden is function only of the monetary income while the typical income tax structure usually is also function of other features: we shall see on how to take into account these non-income characteristics.

[13] Here, the $t(x)$ function characterisation allow to consider $L_N$ and $L_T$ concentration curves as Lorenz curves.

[14] Extensions to personal income taxes with positive thresholds can be found in Keen et al. (2000). Indeed, IRPEF tax code embodies a succession of upward and fixed marginal tax rates on bands of taxable income with different specified threshold values.

$$d\,[t(x)/x]\,/\,\mathrm{d}x \geq 0 \quad \forall x \quad \text{iff} \quad L_{\mathrm{N}}(p) \geq L_{\mathrm{X}}(p) \geq L_{\mathrm{T}}(p) \quad \forall\, p, \ \text{with} > \text{for some } p$$

Thus, with a tax code designed for any homogenous sub-population where the only difference among people are the income levels, a progressive income tax is within group inequality reducing according to the dominance of post-tax income Lorenz curve over the pre-tax income Lorenz curve, where the latter is nowhere above the former and at least somewhere strictly below.

Now, let denote residual progression at income $x$, $RP(x)$, as the elasticity of post-tax schedule $N(x)$ with respect to the income $x$; in regard to this well-known local measure, a necessary and sufficient condition for the existence of non-negative redistribution is $0 \leq RP(x) \leq 1$, $\forall x$.

According to Jakobsson (1976) and Kakwani (1977a) (JK), given any particular distribution of pre-tax income, let $N_1(x)$ and $N_2(x)$ be two post-tax income schedules induced by their respective tax liabilities, $t_1(x)$ and $t_2(x)$:

$$RP_1(x) \leq RP_2(x) \ \forall x \quad \text{iff} \quad L^1_{\mathrm{N}}(p) \geq L^2_{\mathrm{N}}(p) \quad \forall\, p, \text{with} > \text{for some } p$$

If the inequality on the left-hand side of the assertion holds for all the income parade, lower residual progression implies higher progressivity. The local ordering of schedules is equivalent to the Lorenz partial ordering (a local-to-global comparison), whenever the pre-tax distribution remains the same for all schedules being compared.

Within this framework, this is the key point that gives relevance to Dardanoni and Lambert (DL) findings. They were able to formulate what should be the plan that a practitioner needs to perform if he/she wish to take pre-tax inequality diversity into account, *still* continuing to exploit the standard result on redistribution of JK.

Two stages are required. First, he/she needs to avoid the problem to be dependent, about conclusions, on the selected reference distribution. The authors show that "the (residual) progression comparisons can be guaranteed invariant to the choice of baseline if and only if the candidate reference distributions are isoelastic transformations of one another" (DL, 2002, p. 105). Then, standard JK results are preserved under specific conditions on the structure of the income distributions.

Second, on these grounds, the authors demonstrate that a *transplant-and-compare* procedure is pertinent to draw out correct distributional implications.

To summarise, the procedure acts on the pre- and post-tax distributions under analysis; it looks for an isoelastic transformation between the former and, if this is the case, correct for the pre-tax distributional differences between the latter.

Following Dardanoni and Lambert (2002), let $g$: $\mathrm{R}_+ \longrightarrow \mathrm{R}_+$ be any monotone increasing function. By $g$, let define the *deformation* $N^{\mathrm{g}}$ of a post-tax income schedule $N$:

$$N^{\mathrm{g}} = g \circ N \circ g^{-1}$$

and the deformation $\langle N,F\rangle^g$ of a generic regime $\langle N,F\rangle$ consisting of actual post tax schedule and pre-tax income distribution pair:

$$\langle N,F\rangle^g = \langle N^g, F \circ g^{-1}\rangle$$

Thus,

- if $F$ is the pre-tax income distribution for $x$, $F \circ g^{-1}$ is the pre-tax income distribution when incomes are defined by $g(x)$;
- if $N$ maps an original income $x$ into a final income $y$, $N^g$ maps $g(x)$ into $g(y)$;
- $\langle N,F\rangle^g$ is the regime induced by $\langle N,F\rangle$ on the distribution of deformed incomes $g(x)$.

The function $g$ effects a variable shrink (or stretch) of pre-tax relative income differentials. It follows obviously that, with two regimes $\langle N_1,F_1\rangle$, $\langle N_2,F_2\rangle$, and a 'reference' distribution, call it $F_0$, if:

$$g_i = F_0^{-1} \circ F_i \quad \Rightarrow \quad \langle N_i,F_i\rangle^{g^i} = \langle N_i^{g^i}, F_0\rangle \qquad i = 1, 2$$

The authors argue that, to transplant two pre-tax income distributions under analysis into a reference distribution, some (presumably different) appropriate transformation functions $g_i$'s do exist; then, the respective transformation functions $g_i$'s themselves should be used to correct post-tax relative income distributions. Thus, we should proceed to compare the transplanted regimes $\langle N_i^{g^i}, F_0\rangle$'s and achieve unambiguous local progression comparison between $N_1^{g^1}$ and $N_2^{g^2}$, if any, that can be represented as a partial progressivity ordering over regimes conditioned by $F_0$.

Dardanoni and Lambert show that the isoelasticity linking-condition regarding to any possible reference distribution is the crucial point. In fact, a natural question to ask is if the same result obtained by using $F_0$ may be found selecting another baseline, say $G_0$.

Let $F_0$ and $G_0$ be two alternative reference distributions for the comparison of regimes, $\langle N_1,F_1\rangle$ and $\langle N_2,F_2\rangle$. The authors state (Theorem 1, p. 105) that:

*the partial orderings over $\langle N_1,F_1\rangle$ and $\langle N_2,F_2\rangle$ conditioned on $F_0$ and $G_0$ are the same $\Leftrightarrow$ $G_0^{-1} \circ F_0 = g$ is isoelastic ($\Leftrightarrow \exists\, A, b > 0 : g(x) = A x^b$).*

If the analyst were interested to transplant one distribution, $F_1$, directly into one another, say $F_2$, as a consequence - to avoid the risk to be dependent on the elected baseline about findings - he/she should verify if they are isoelastically linked; we shall do in the next section with respect to the Italian case.

What is on hand to practitioners is formally stated by Theorem 2 (DL, 2002, pp. 105-106).

Let $\langle N_1,F_1\rangle$ and $\langle N_2,F_2\rangle$ be two regimes. The partial orderings over regimes conditioned by a generic reference distribution $F$ is denoted by $\succ_{P|F}$:

a) *Let be* $F_0$ *any income distribution such that* $g_1 = F_0^{-1} \circ F_1$ *and* $g_2 = F_0^{-1} \circ F_2$ *are both isoelastic. If* $RP_1(g_1^{-1}(x)) \leq RP_2(g_2^{-1}(x))\ \forall x$ *then* $\langle N_1, F_1 \rangle \succ_{P|F_0} \langle N_2, F_2 \rangle$

b) *Assume that* $g = F_1^{-1} \circ F_2$ *is isoelastic. If* $RP_1(g(x)) \leq RP_2(x)\ \forall x$ *then* $\langle N_1, F_1 \rangle \succ_{P|F_1} \langle N_2, F_2 \rangle$ *and* $\langle N_1, F_1 \rangle \succ_{P|F_2} \langle N_2, F_2 \rangle$.

c) *If* $g = F_1^{-1} \circ F_2$ *is not isoelastic, the partial orderings over regimes by* $\succ_{P/F_1}$ *and* $\succ_{P/F_2}$ *are different.*

The part *a)* and *b)* of this theorem lead to give relevance to the isoelasticity conditions issue: if they hold, the potential for dependency of end results on the baseline is avoided. If this is not the case, the part *c)* affirms that conclusions may be uncertain, reflecting the distribution, $F_1$ or $F_2$, selected as baseline. The practitioner should verify by making successive pairwise tests by using all the potential different reference distributions under analysis whether outcomes are free-dependent, or not. Of course, we are more interested to the part *a)* and *b)* - less laborious to handle - of this theorem; if they are verified we can make use of JK results to infer the occurrence of Lorenz curves intersections. If net income schedules $N_i^{g^i}$ yields, (i = 1, 2,…), are the same for all comparisons and - by JK theorem - Lorenz curves do not cross, the Atkinson theorem is helpful to derive normative significance.[15]

In order to obtain a ranking of income distributions with respect to income inequality, Atkinson assumes that the social welfare function is an additively separable and symmetric function of individual incomes.[16]

$$W = \int_0^z U(x) f(x)\, dx$$

$\forall$ strictly increasing and concave utility function $U(x)$[17]

Let $H(x)$ e $G(x)$ be two income distributions with equal mean $\mu_H = \mu_G$, for a given population size:

$$L_H(p) \geq L_G(p) \quad \forall p \qquad \text{iff} \qquad W_H \geq W_G$$

[15] Non-equal yield taxes are usually a result of a personal income tax reform. In such a case, according to an appropriate *Residual Progression neutral tax* device we should standardise the different total tax burdens (Pfähler 1984, Lambert 2001). With a *RP* neutral tax cut/hike, the gain/the loss is equal for every sample household in percentage terms; for every *p*, $RP(x)$ remains constant; the Lorenz curves, with respect to the post-tax income distributions under analysis - before and after the *RP* neutral tax cut/hike - are exactly superimposes. The size of the cake changes, not how the shares are divided. See Foster (1985), Fields and Fei (1978), and Chakravarty and Muliere (2003), about correct procedures to rank inequality.

[16] Dasgupta, Sen and Starrett (1973), and Rotschild and Stiglitz (1973) prove that the Atkinson's results are more general: strict Schur-concavity of a social welfare function is sufficient to incorporate egalitarian bias into distributional judgements.

[17] $U'(x) > 0$, $U''(x) < 0$, $\forall x > 0$.

Endowed with these tools, we turn to the empirical analysis.[18]

## 3. Implementation

In this section we present the application of this new methodology to the Italian distribution of income. As we wrote in section 2 the micro-data used are the output of ITAXMOD package adapted from 1995, 1998 and 2000 SHIW original micro-data of the Bank of Italy. For each year we have two vectors, pre- and post-tax income distributions, but prior to apply the DL procedure some phases are required.[19]

First, household survey micro-data need to be adjusted to make them tell about well-being. We adopt an equivalence scale for the distribution of household income, both before and after tax. By using a double-parametric function suggested by Cutler and Katz (1992)[20] we deflate each household money income into units of equivalent income. The equivalence scale deflator provides what is named the "number of adult equivalents" and takes the form:

$$\mathrm{m}_h = (\mathrm{N}_a + \varphi\,\mathrm{N}_c)^{\theta} \qquad h = (1, 2,\ldots, n)$$

where, $\mathrm{N}_a$ and $\mathrm{N}_c$ are the numbers of adults and children in the household $h$; $\varphi$ is the parameter value which represents the weight of a children with respect the weight of an adult (=1); $\phi$ is the parameter value for economies of scale within the household $h$ and $(\phi, \theta) \in [0, 1]$. We present two cases, [1] $\theta = \varphi = 0.5$; [2] $\theta = 0$.

In both cases there are no conversion coefficient differences between adults (e.g., head *versus* spouse, or other adults). According to the OECD scale, the value 0.5 is assigned to children younger than 14 in [1]. We have no explanation for the value selected in regard to $\theta$ (= 0.5), nevertheless, even if we let vary this value it is possible to show that results about effective progression are qualitatively the same (they can be

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[18] Note that in the case of an equal-yield flat tax:

$$p = F(y) \;\rightarrow\; L_{\mathrm{N}}(p) = 1/[\mu_{\mathrm{X}}(1 - r)]\int_0^y (x - r\,x) f(x)\mathrm{d}x \;=\; L_{\mathrm{X}}(p) = (1/\mu_{\mathrm{X}})\int_0^y x\, f(x)\mathrm{d}x$$

then, even if positive taxation *per se* - proportional or progressive - is only social welfare reducing, nevertheless, a progressive income tax is social welfare reducing by less than a proportional tax raising the same revenue from the same before-tax income distribution.

[19] The starting point of the analysis is usually to deflate incomes time series to avoid the effect of inflation: we skip this phase. By assuming isoelasticity there is no need to convert nominal values into real values before applying the transplant-and-compare procedure (Dardanoni and Lambert, 2002, p. 111, footnote 19).

[20] It is well known that levels in measured income inequality can vary depending on the choice of equivalence scale, although none of them has been proved to be superior. Thus, there is a wide agreement about the lack of a unique equivalence scale. Other rules suggested come from Buhmann et al. (1988), Atkinson et al. (1995); they could be derived also from the Cutler and Katz deflator by the selection of particular parameter values. Marenzi (1995) adopts their double-parametric function.

provided by the author upon request).[21] The second case is an extreme one, it is appropriate if the analyst judges that households equivalent income coincides to households money income.

Second, even if we use an equivalence scale to determine living standards, the chance to have a horizontally inequitable income tax is very high. When the population is socially homogenous and the only source of difference among people is money income $x$, this turns out when the assumption $0 \le t'(x) < 1$, $\forall\ x$, is violated. When the population is not socially homogenous and it assumes that the only relevant differences between households are their money income, family sizes and composition, there is horizontally inequity (HI) when the income tax function, $t\,(\cdot)$, is not (Ebert 1997, 1999; Lambert 2001):

a) $t\,(x, h) = \mathrm{m}_h\ [\,\tau_h\ (\dfrac{x}{\mathrm{m}_h})]$

where $\tau_h$ is a tax function of the household equivalized income, $(\dfrac{x}{\mathrm{m}_h})$, which embodies the degree of vertical equity prescribed by the decision maker, and $\mathrm{m}_h$ is the equivalence scale deflator in accordance with the number of equivalent adults for the household $h$;

b) such that $\tau_h$ is not the same for all $h$ ($\tau_h = \tau$, $\forall\ h$);

c) such that $0 \le \tau'(\dfrac{x}{\mathrm{m}_h}) < 1$, $\forall\ (\dfrac{x}{\mathrm{m}_h})$.

The Italian personal income tax does not act like the income tax function just described. The Italian different tax treatment of urban and rural incomes could be seen as discriminatory; deductions for items of expenditure and, as a matter of fact, tax evasion concentrated in particular on self-employment income, can both easily cause HI.

The aim of the paper is to capture the pure IRPEF redistributive effect, then we must isolate and exclude the new inequality - e.g. within pre-tax income equal group - introducing by HI.[22] The literature provides two prevailing views on how to do this.

The starting point of the Classical HI approach highlights the fact that the before-tax equals have been unequally treated by the taxation: as a consequence, the dispersion of taxes at fixed income levels $x$ comes out. The *no-reranking* equity criterion refers to HI as a feature of the taxation process, rather than of its outcome. Both approaches lead to

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[21] In this context, Atkinson theorem and the strictly concavity of the individual utility function imply that we approve transfers of *living standard* from the better-off to the worse-off.

[22] See Marenzi (1995) for a Reynolds-Smolensky index decomposition, showing how much HI – separated into two parts, Classical Horizontal Inequity and Reranking - is delivered by IRPEF. About the decomposition analysis, see Lambert and Aronson (1993) and Aronson et al. (1994).

different ways to observe the pure vertical stance of a tax system. Without going deeper into the procedures for the Classical HI approach[23], we choose to adopt the no-reranking point of view, basically in accordance with the fact that no pre-tax equals are present in our own micro-data samples. According to the no-reranking approach, vertical equity is about the choice of post-tax equivalent income distribution *given* the pre-tax distribution[24]; there should be perfect association between households pre-and post-tax living standards. On the contrary, usually, households get reranked by actual tax systems: we must isolate the vertical equity effect from the reranking effect. We would like to have all $N_{\mathrm{i}}(x)$, (i = 1995, 1998, 2000) sample post-tax distributions generating *from* the sample pre-tax distributions. $N_{\mathrm{i}}(x)$ should be the post-tax equivalent income whose rank is the same as the pre-tax rank of $x$. The only way to construct such a function is to sort separately the pre- and post-tax equivalent income distribution in each sample: we have to break the disassociation, if any, which is present. Each $N_{\mathrm{i}}(x)$ still maps *existing* pre-tax living standards to *existing* post-tax living standards, but in a different order: they enjoy now perfect and positive association. We have no effect on post-tax inequality because the only variation is the rank of each household. Thus, the sorting procedure is inequality neutral.[25]

We have now on hand, for each period, two vectors that provide micro-data for HI-free pre- and equal yield post-tax equivalent incomes. The last stage before going on is to assure that no selectivity bias is present into the sample.

Survey weights have been assigned to each sample case by the SHIW of the Bank of Italy. To each household is attached a sample weight in inverse relation to his probability to be included into the sample. Thus, the procedure adopted here takes into account the weight structure in the sample design replicating the number of each household according to the difference of the respective sample weight with respect to the smallest one.

Let be $\omega_{\mathrm{h}} = 1 / P_{\mathrm{h}}$ the sample weight of a generic household $h$, where $P_{\mathrm{h}}$ indicates its probability to be included into the sample. Let the smallest sample weight be $\omega_{\mathrm{sm}}$ and $\{\omega\}_{\mathrm{h}}$ the sample weight set. For $\{\omega\}_{\mathrm{h}}$ the 'replication' factor is provided by:

$$(\omega_{\mathrm{h}} / \omega_{\mathrm{sm}}) \quad \forall h$$

The ratio $(\omega_{\mathrm{h}} / \omega_{\mathrm{sm}})$ is in general non-integer, then each $h$ should be replicated a number of terms equal to:

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[23] The interested reader may helpful look at Lambert and Ramos (1997), and Duclos and Lambert (2000).

[24] See Pechman – Okner (1974, pp. 55-57) and Blackorby and Donaldson (1984, p. 686).

[25] See King (1983), Dardanoni and Lambert (2002). We have to advert the reader that, however, the progressive stance obtained by sorting pre- and post-tax distributions reveals slight differences with respect to the alternative procedure characterizing the classical HI approach; see Dardanoni and Lambert (2001) on this.

$$[(\omega_h / \omega_{sm})_{integer} - 1]$$

We approve that the closer integer to ($\omega_h$ / $\omega_{sm}$) is a good proxy, thus we make use of this simple proceeding to take into account the sample design.

Finally, zero incomes are eliminated from the data since logarithmic transformations will be useful for the procedure below outlined, and the top 0.5% from each sample are removed to eliminate dependency of results on outliers.

### *3.1 Progressivity, Pre-Tax Distributional Differences, and Empirical Analysis*

The part a) of Dardanoni and Lambert (2002) Theorem 2 applies in particular whether the pre-tax income distribution functions under analysis can be fitted as members of some parametric family distribution. Transforming the data into logarithms, if the pre-tax income distribution function belongs to the location-and-scale invariant lognormal family of income distribution, any member of this family can be $F_0$, the host distribution for the procedure.[26] Thus, we have to test, with obvious notation, if:

$$\ln x_{1995} \sim N(\mu_{x1995}, \sigma^2_{x1995})$$

$$\ln x_{1998} = (a_1 + b_1 \ln x_{1995}) \sim N(a_1 + b_1 \mu_{x1995}, b_1^2 \sigma^2_{x1995})$$

$$\ln x_{2000} = (a_2 + b_2 \ln x_{1995}) \sim N(a_2 + b_2 \mu_{x1995}, b_2^2 \sigma^2_{x1995})$$

Besides the $\chi^2$ and the unilateral statistic-tests, others, named omnibus test (Omnibus test for departure from normality), jointly reflect the skewness and kurtosis, if any, present (for example, the Lilliefors' test, 1967, and the D'Agostino and Pearson test, 1973).

We wish to highlight that a parameter model requires the specification of a particular functional form, without any possibility to start with an explorative phase to identify the essential structure characteristics - for example the shape of the distribution and the number of modes. The parametric approach, after the estimation of parameters for each set of observations, requires only to perform statistical inference, the evaluating phase for the approximation degree by which the specific model estimates the empirical distribution. We would like to not exclude any a priori detection of irregular pattern of income distribution or multimodality; thus, we prefer pay attention to the non-parametric techniques, by using kernel density estimation approach. Given the exploratory character of this estimation, it provides relevant information about the underlying distribution without relying on arbitrary assumptions.

[26] Following Gibrat (1931) and Aitchinson and Brown (1969), the idea is that the income growth is governed by a proportional growth process.

The analyses based on kernel estimates confide heavily on the graphical presentation of the shape of the distribution. If the visual impression from the density estimates reveals, for example, the occurrence of two, or more, modes, this is adequate to imply rejection of lognormality. If those do not crop up, we should turn back to the goodness-of-fit test. The estimation method we use here is derived from a generalization of the kernel density estimator to account for the sample weights attached to each observation, namely, from the adaptive or variable kernel: we make use of an adaptive bandwith to handle data sparseness and a weighting variable to take into account the sample design.[27] To calculate the adaptive kernel, a two-stage procedure has been followed.

A density is settled in the first step in order to obtain the optimal bandwidth parameter; in the second step, a local bandwidth factor is then used for the construction of the adaptive kernel itself and the final density is computed. In detail:

**1(*a*)** – pilot estimate $\hat{f}(y)$: $\hat{f}(Y_j) > 0$ (j = 1, 2, …, $n$);

$$\hat{f}(y) = \sum_{j=1}^{n} \left(\frac{\omega_j}{h}\right) K\left(\frac{y - Y_j}{h}\right)$$

$K$ is the kernel function; $\omega_j$ are the sample weights, and $\sum_{j=1}^{n} \omega_j = 1$; $h$ is the fixed bandwith parameter;[28]

**1(*b*)** – definition of a local bandwith factor $\lambda_j$,

$$\lambda_j = \left\{ \frac{\hat{f}(Y_j)}{g} \right\}^{-\alpha}$$

where the normalization factor $g$ is the geometric mean of $\hat{f}(Y_j)$ and $\alpha$ is the sensitivity parameter;[29]

**2** – final estimate

$$\tilde{f}(y) = \sum_{j=1}^{n} \left(\frac{\omega_j}{h\lambda_j}\right) K\left(\frac{y - Y_j}{h\lambda_j}\right)$$

[27] Using the non-parametric density estimation and UK data, Cowell et al (1996) showed the emergence of bimodality from 1979 to 1988-89. Pittau and Zelli (2004) did the same using Italian data from SHIW between 1987-1998; see also D'Ambrosio (2001).

[28] For this *smoothing* parameter we adopt the statistical rule proposed by Silverman (1986), $h$=1.06*std($y$)*$n^{(-1/5)}$ ; $n$ is the number of observations; $K$ is the Gaussian. Note that for large samples, it is well known that the nonparametric estimation is not sensitive to the different choices of kernel functions (Silverman 1986).

[29] Here, $\alpha = ½$.

As the figure 1 reproduces (see the Appendix), the stylized facts represented by estimates for all the pre-tax equivalent income distributions under analysis (with $\phi = \theta = 0.5$) offer strong support for, at least, bimodality and irregularity for their shapes.[30]

If the lognormal fit is unacceptable, the part b) of the Theorem 2 calls for the identification of the coefficients $A$ and $b$ connecting isoelastically the pre-tax distribution functions.

After the logarithmic transformation, the OLS estimator seems to be the simplest way to derive the constant and slope in a regression of, say, $\ln(x_j)_{1995}$ on $\ln(x_j)_{1998}$. Note that in this paper OLS estimator is not used for statistical inference purposes. We adopt it as geometric method to obtain those parameters which minimize the Euclidean distance between the vectors, say, $\ln(x_j)_{1995}$ *versus* $\mathrm{a} + \mathrm{b}\ln(x_j)_{1998}$. The observation unit is always the same, the rank; we look for the *a* and *b* measuring the change for the size and inequality between, say, 1995 and 1998, from the poorer to the richer. The $R^2$ statistic is the goodness-of-fit measure in this procedure. If the $R^2$ statistic is extremely high, indeed close to one, it may have a good degree of confidence about the *a* and *b* parameters potential to transplant one pre-tax income distribution into the other one.

Finally, we should control for the graphical superimposition between the distribution functions, between the transplanted distribution function and the baseline.

Let be $A = e^{\mathrm{a}}$; for the general case, if $g$ is isoelastic, of course:

$$\ln g(x_j) = \ln e^{\mathrm{a}} + \mathrm{b} \ln x_j = \mathrm{a} + \mathrm{b} \ln x_j \qquad (\mathrm{j} = 1, 2, \ldots, n)$$

In this paper, we examine if :

$$(x_j)_{1998} = e^{\mathrm{a}_1} (x_j^{\mathrm{b}_1})_{1995} \qquad [= g^{-1}(x_j)_{1995}]_{1998} \quad \forall \mathrm{j}$$

and

$$(x_j)_{2000} = e^{\mathrm{a}_2} (x_j^{\mathrm{b}_2})_{1995} \qquad [= g^{-1}(x_j)_{1995}]_{2000} \quad \forall \mathrm{j}$$

then:

$$(x_j)_{1995} = e^{-\frac{\mathrm{a}_1}{\mathrm{b}_1}} ((x_j)_{1998})^{\frac{1}{\mathrm{b}_1}} \qquad [= g(x_j)_{1998}] \quad \forall \mathrm{j}$$

$$(x_j)_{1995} = e^{-\frac{\mathrm{a}_2}{\mathrm{b}_2}} ((x_j)_{2000})^{\frac{1}{\mathrm{b}_2}} \qquad [= g(x_j)_{2000}] \quad \forall \mathrm{j}$$

After a logarithmic transformation, the distributions should differ essentially only by location and scale:

$$\ln(x_j)_{1998} = \mathrm{a}_1 + \mathrm{b}_1 \ln(x_j)_{1995} \quad \text{and} \quad \ln(x_j)_{2000} = \mathrm{a}_2 + \mathrm{b}_2 \ln(x_j)_{1995} \quad \forall \mathrm{j}$$

---

[30] In this paper, the mode is considered a local maximum, a point where the estimated density function gradient changes sign, from a positive value to a negative one. On the multimodality statistical significance for 1995 and 1998 SHIW estimated kernel density function, we refer to Pittau and Zelli (2004).

Thus, if the equivalence scale parameters are $\phi = \theta = 0.5$, these are the results of OLS method:

*OLS*(1995 log pre-tax incomes = $x$, 1998 log pre-tax incomes = $y$)
R-squared = 0.9987
sigma^2 = 0.0006
Nobs, Nvars = 61311, 2
*********************************************

| Variable | Coefficient |
|---|---|
| $a_1$ | 0.148736 |
| $b_1$ | 0.995889 |

and

*OLS*(1995 log pre-tax incomes = $x$, 2000 log pre-tax incomes = $y$)
R-squared = 0.9985
sigma^2 = 0.0007
Nobs, Nvars = 61311, 2
*********************************************

| Variable | Coefficient |
|---|---|
| $a_2$ | 0.295818 |
| $b_2$ | 0.988186 |

Then:

$$\ln(x_j)_{1998} = 0.148736 + 0.995889 \ln(x_j)_{1995} \qquad \forall j$$
$$\ln(x_j)_{2000} = 0.295818 + 0.988186 \ln(x_j)_{1995} \qquad \forall j$$

According to OLS results, figures 2(*a*) and 2(*b*) show, respectively, the 1998 log pre-tax distribution fit to that of 1995, and the 2000 log pre-tax distribution fit to that of 1995 (Appendix). Considering the correspondent $R^2$ values, it seems that these distributions differ basically only by location and scale.[31] Then, with a good degree of confidence, we assert that:

$$\ln(x_j)_{1995} = \left(-\frac{a_1}{b_1}\right) + \left(\frac{1}{b_1}\right)\ln(x_j)_{1998} = \left(-\frac{a_2}{b_2}\right) + \left(\frac{1}{b_2}\right)\ln(x_j)_{2000} \qquad \forall j$$

---

[31] For the distribution of pre – tax equivalent incomes, observing that $e^{a_1} = A_1 = 1.1598$ and $b_1 = 0.995889$, the sample data display a nominal *living standard* growth rate equal to 0.1598 and a slight higher equality in 1998 with respect to 1995.

According to $e^{a_2} = A_2 = 1.3430$ and $b_2 = 0.988186$, the sample data show a slight higher equality in 2000 with respect to 1995, and in regard to 1998 ($b_2 < b_1$).

On these grounds, we proceed with the second phase of the procedure, that is, we correct for the pre-tax distributional differences acting on log post-tax equivalent income distributions. Given that the general deformation function that allows to implement the *transplant-and-compare* procedure is $N^{g} = g \circ N \circ g^{-1}$, with isoelasticity it is possible to show that:

$$\ln N^{g}_{1998}(x_j) = (-\tfrac{a_1}{b_1}) + (\tfrac{1}{b_1}) \ln N_{1998}(x_j) \qquad \forall j$$

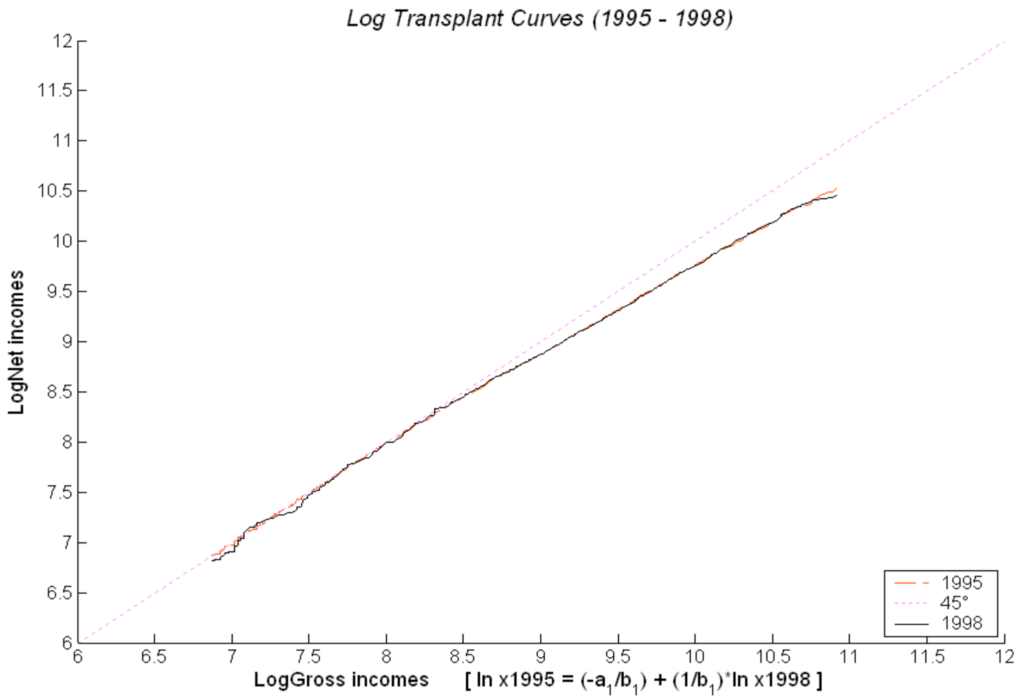
and

$$\ln N^{g}_{2000}(x_j) = (-\tfrac{a_2}{b_2}) + (\tfrac{1}{b_2}) \ln N_{2000}(x_j) \qquad \forall j$$

In accordance with Dardanoni and Lambert Theorem 2 (part *b)*), we need now to compare residual progression measures; the more suitable way to verify the residual progression elasticities is to plot in logs and then examine the *log transplant curve* slopes. Figure 3 and figure 4 plot, ∀j:

| | | | | |
|---|---|---|---|---|
| [fig. 3] | (*a*) | $\ln(x_j)_{1995}$ | *vs.* | $\ln N(x_j)_{1995}$ |
| | (*b*) | $\ln(x_j)_{1995}$ | *vs.* | $(-\frac{a_1}{b_1}) + (\frac{1}{b_1}) \ln N(x_j)_{1998}$ |
| [fig. 4] | (*a*) | $\ln(x_j)_{1995}$ | *vs* | $\ln N(x_j)_{1995}$ |
| | (*b*) | $\ln(x_j)_{1995}$ | *vs.* | $(-\frac{a_2}{b_2}) + (\frac{1}{b_2}) \ln N(x_j)_{2000}$ |

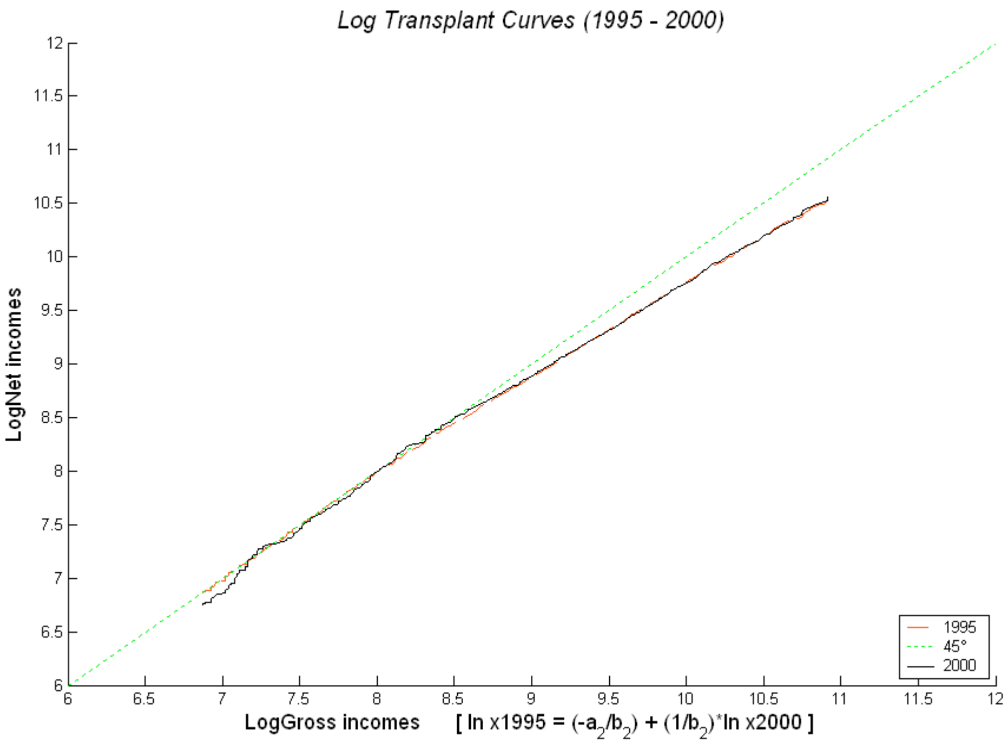
**Figure 3**



The isoelasticity transformation allows to consider, in regard to the *x*-axis of the graph, the distribution of 1995 log pre-tax equivalent income alone - that is, if we are ready to accept that it is a good proxy for the distribution of 1998 log pre-tax equivalent income corrected for distributional differences. The residual progression elasticities for the corrected 1998 log post-tax equivalent income distribution – 3(*b*) – are, of course, the actual ones. The figure supports the assertion that only for, around, the top 0.5 percent there is evidence for a 1998 higher degree of effective progression. Only there the 1998 transplant is clearly flatter than the 1995 curve. Out of this range, there are too many crossing between the estimated log transplant curves or, almost, perfect superimposition. It seems that 1998 IRPEF tax reform does not change so much the redistribution between income units with respect to 1995 IRPEF tax schedule.

What about the second log transplant curves comparison? The next chart illustrates it.

**Figure 4**




In this case too, the distribution of 1995 log pre-tax equivalent income is the variable on the horizontal axis - that is, by accepting that it is a good proxy for the distribution of 2000 log pre-tax equivalent income, corrected for specific distributional differences. The figure supports the assertion that only for, around, the lower 1 percent there is evidence concerning the lower degree of effective progression over the time path between 1995 and 2000. According to the figure, only in that point of the log income parade, the residual progression measure seems higher by the 2000 IRPEF tax schedule (the second step of IRPEF tax reform) than by 1995 IRPEF tax schedule. Furthermore, it appears that for a relevant range around the 'middle' of the distributions, the slope of 2000 log transplant curve (the solid line) is slight flatter than the 1995 one. However, there are many crossing between the estimated log transplant curves and, often, they look like almost perfectly superimposed.

In short, the redistributive differences driven by 2000 IRPEF tax schedule in regard to the 1995 IRPEF tax schedule are not so impressive.

We turn now to a less normatively significant case, that is, by using the equivalence scale parameter, $\theta = 0$.

The results of OLS procedures are:

*OLS*(1995 log pre-tax incomes = $x$, 1998 log pre-tax incomes = $y$)

R-squared = 0.9991

sigma^2 = 0.0005

Nobs, Nvars = 61311, 2

***************************************************

| Variable | Coefficient |
|---|---|
| $a_1$ | 0.183490 |
| $b_1$ | 0.990573 |

and

*OLS*(1995 log pre-tax incomes = $x$, 2000 log pre-tax incomes = $y$)

R-squared = 0.9980

sigma^2 = 0.0011

Nobs, Nvars = 61311, 2

***************************************************

| Variable | Coefficient |
|---|---|
| $a_2$ | 0.391951 |
| $b_2$ | 0.976069 |

Then, with a slight change in the notation:

$$\ln (x_j)_{1998m} = 0.183490 + 0.990573 \ln (x_j)_{1995m} \qquad \forall j$$

$$\ln (x_j)_{2000m} = 0.391951 + 0.976069 \ln (x_j)_{1995m} \qquad \forall j$$

According to OLS results, figures 5($a$) and 5($b$) show, respectively, the 1998 log pre-tax distribution fit to that of 1995 and the 2000 log pre-tax distribution fit to that of 1995 (see the Appendix). The $R^2$ values still continue to be extremely high. It seems that household money distributions differ essentially only by location and scale. Then, with a good degree of confidence, we assert that:

$$\ln (x_{\mathrm{j}})_{1995m} = (-\tfrac{\mathrm{a}_1}{\mathrm{b}_1}) + (\tfrac{1}{\mathrm{b}_1}) \ln (x_{\mathrm{j}})_{1998m} = (-\tfrac{\mathrm{a}_2}{\mathrm{b}_2}) + (\tfrac{1}{\mathrm{b}_2}) \ln (x_{\mathrm{j}})_{2000m} \qquad \forall \mathrm{j}$$

By correcting for the pre-tax distributional differences, i.e. transplanting $N(x_{\mathrm{j}})_{1998m}$ and $N(x_{\mathrm{j}})_{2000m}$ in logs into the distribution of 1995 log pre-tax equivalent income, figures 6 and 7 (see the Appendix) plot, ∀j:

| | | | | |
|---|---|---|---|---|
| [fig. 6] | (*a*) | $\ln (x_{\mathrm{j}})_{1995m}$ | *vs.* | $\ln N(x_{\mathrm{j}})_{1995m}$ |
| | (*b*) | $\ln (x_{\mathrm{j}})_{1995m}$ | *vs.* | $(-\tfrac{\mathrm{a}_1}{\mathrm{b}_1}) + (\tfrac{1}{\mathrm{b}_1}) \ln N(x_{\mathrm{j}})_{1998m}$ |
| [fig. 7] | (*a*) | $\ln (x_{\mathrm{j}})_{1995m}$ | *vs* | $\ln N(x_{\mathrm{j}})_{1995m}$ |
| | (*b*) | $\ln (x_{\mathrm{j}})_{1995m}$ | *vs.* | $(-\tfrac{\mathrm{a}_2}{\mathrm{b}_2}) + (\tfrac{1}{\mathrm{b}_2}) \ln N(x_{\mathrm{j}})_{2000m}$ |

Observing log transplant curves in figure 6, it notes that only for, around, the top 2 percent there is evidence concerning a higher degree of progressivity over the time path between 1995 and 1998: only there the 1998 transplant is clearly flatter than the 1995 curve. Along the income parade, also in this case, there are many crossing between the estimated log transplant curves and, often, they appears almost exactly superimposed.

Figure 7 describes the comparison about 1995 and 2000 IRPEF tax schedules. It comes out that the number of intersections is very frequent over all the income parade. 1995 tax system strongly mimicks the pattern of 2000 and, on this basis, to qualify the trend of redistributive differentials between 1995 and 2000 it is certainly complicated.

In short - according to both equivalence scale cases and by using actual tax laws and actual pre- and post-tax distributions - from 1998 and 2000 IRPEF tax reform we can not draw out global (and normative) properties, or, excluding some small subsets of the income parade reported above, find substantial different impact levels with respect to 1995 IRPEF tax schedule.

To obtain more definite conclusive assessments about the first (1998) and second (2000) step of Italian personal income tax reform, we provide effective progression comparisons directly based on the criterion of Lorenz dominance.[32]

We do only respect to the postulated more normatively significant case ([1] $\phi = \theta = 0.5$); we present both the usual Lorenz curve graphs (figures 8, and 9, appendix) and a figure concerning the difference between the gaps among post- and pre-tax household equivalent income cumulated shares, for each pair of years (figure 10).

The former provides the same scarce information of the log transplant curves, the latter allows to be a little more precise about the actual redistributive findings. For

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[32] Let denote the Lorenz partial ordering of regimes by $\succ_L$. The Lorenz dominance criterion states that:

$$\langle N_1, F_1 \rangle \succ_{\mathrm{L}} \langle N_2, F_2 \rangle \quad \text{iff} \quad L^1_{\mathrm{N}}(p) - L^1_{\mathrm{X}}(p) \geq L^2_{\mathrm{N}}(p) - L^2_{\mathrm{X}}(p) \quad \forall \text{ p, with} > \text{for some p}$$

example, 1995 vs. 1998 denotes the empirical evidence about the difference between 1995 post- and pre-tax Lorenz curve gap and the corresponding gap for 1998. A positive value means Lorenz dominance of 1995; while negative value means 1998 dominance.

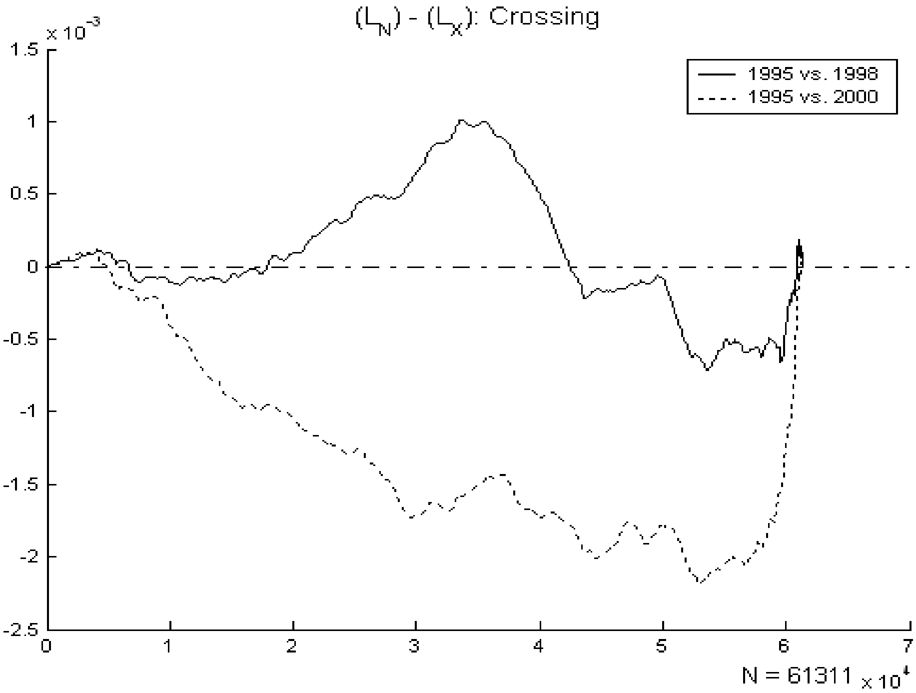
In figure 10 we plot:

[10*a*] $[L_{\mathrm{N}}^{1995}(p) - L_{\mathrm{X}}^{1995}(p)] - [L_{\mathrm{N}}^{1998}(p) - L_{\mathrm{X}}^{1998}(p)] \quad \forall p$

and

[10*b*] $[L_{\mathrm{N}}^{1995}(p) - L_{\mathrm{X}}^{1995}(p)] - [L_{\mathrm{N}}^{2000}(p) - L_{\mathrm{X}}^{2000}(p)] \quad \forall p$

**Figure 10**




What about the comparison labelled [10*a*]? We observe at least four intersections, then Atkinson theorem is not helpful. As easily observed, in accordance with [10*b*], there would be greater evidence for 2000 redistribution dominance on 1995, but given that at bottom range of the income parade the values are positive, the Atkinson theorem is always not satisfied.[33]

Taking into account the well-known Reynolds-Smolenky redistributive effect index (1977) - a measure that, given the estimated Lorenz curves cross condition, can now be considered a descriptive index - the redistributive story is more comprehensive.[34] Over

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[33] Adopting Dardanoni-Lambert (1988), the last comparison could imply welfare preference for 2000 redistribution.

[34] It can be shown that $\Pi^{RS} = [\, r \,/\, (1 - r)\,]\, \Pi^{K}$ , where $\Pi^{K}$ is the Kakwani index (1977b). If the $r$ term decreases, a significant counterfactual $\Pi^{K}$ index increase is essential to obtain a raising $\Pi^{RS}$. Here the $\Pi^{RS}$ index for each period is obtained by using pertinent pre- and post tax income distributions.

time its value increased, as expected in particular for 2000 (table 2, Appendix; equivalence scale: [1]$\phi = \theta = 0.5$).[35]

## 4. Conclusions

This paper proposes the application of a new procedure by Dardanoni and Lambert (2002) to the pure redistributive effect implied by the Italian personal income tax (IRPEF). We identified the effective progression produced by different regimes (1995, 1998, and 2000 IRPEF). Depending on the practitioner purpose, the distribution of pre-tax incomes matters, or does not. In order to make meaningful comparisons, if the aim is to reveal actual progressivity effects by tax reforms, pre-tax distributional differences matter; as a consequence, one of the goals of the researcher should be to avoid any baseline dependence controversy. This paper takes into account it.

It has not been possible to qualify a welfare ranking according to the Atkinson theorem. However, it seems that some of IRPEF reform objectives were reached. In the second step of the reform (2000), in accordance with the policy maker goals, the total tax ratio slightly fell (table 3, appendix; equivalence scale: [1]$\phi = \theta = 0.5$); the Reynolds-Smolensky index reduction demonstrates that the degree of progressivity for the whole economy is maintained or slightly increased . The alternative way to present the comparison of Lorenz curves supports this empirical evidence; however, sizes of the variation were small (e.g., for the Lorenz curve comparison between 1995 and 2000, figure 10, the highest redistributive effect gain is around 0.002) and their economical significance low. What about the log transplant curve device? By using the new tool we have been not able to reveal who are the gainers and the losers by IRPEF reforms (excluding some very small range of the income parade), but the same trouble goes up with comparisons based on the classical Lorenz curve graphical approach.

It seems that if variation sizes are small, log transplant curves are not able to capture them (indeed, a log transformation compresses the variable trend and then crushes differences); then, it is clear that only if a relevant change of the pre-tax distribution and tax system were at work, this tool would become helpful.

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[35] In regard to the issue of performing statistical inference for the transplant-and-compare procedure, and log transplant curves, to our knowledge testing procedures to infer statistical significance do not exist yet. For Lorenz curve orderings we refer to Dardanoni and Forcina,1999; Davidson and Duclos, 1997 and 2000. In this paper we do not present tests for equality of two empirical Lorenz curves, or tests for Lorenz dominance among two curves. We guess that statistical significance is relevant in particular when there is dominance empirical evidence significant from an economic point of view (the same holds for $\Pi^{RS}$ indexes). The stylized facts above displayed do not allow to state this assertion in the case of IRPEF reforms.

Nevertheless, figure 10 confirms the log transplant curve analyses regarding to the normative issue; for each pair of years, figure 10 shows at least one intersection, thus, it does not contradict log transplant curves.

Finally, let discuss about an interesting insight of this innovative procedure. If isoelasticity does hold between pre-tax income distributions in a nation in a succession of years, it seems reasonable enough to conclude that this will occur again, excluding the impact of exogenous structural crisis factors (however, these factors could affect all the population with, roughly, the same proportion). As a consequence, with some parameters *a* and *b* reflecting inequality and size differences, there should exist some isoelastic transformation able to transplant an unknown and coming pre-tax income distribution into an already existing distribution.

According to this hypothesis, an empirical researcher wishing to assess, *now*, which may be the outcomes of a *future* tax law, could proceed in two stages. First, as usual, he/she can simulate those outcomes by using the more recent available distribution of pre-tax income. Second, in addiction, he/she could take into account distributional differences by assuming different values for parameters *a* and *b*. In this way, it may control for a range of potential distributional variations. Of course, the choice of parameter values is the 'variable' that could influence, together with the new tax structure, the redistributive effect. If the length of time series is large enough, a good approximation could be an average of preceding estimates. In Italy, the IRE reform could be a good candidate for this more comprehensive approach.

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## APPENDIX

**Table 1**

*IRPEF national nominal tax rates* (*)

| ***brackets of taxable income*** | ***tax rates*** |
|---|---|
| **1995 IRPEF **** | **%** |
| **0 – 7,200** | **10** |
| **7,200 – 14,400** | **22** |
| **14,400 – 30,000** | **27** |
| **30,000 – 60,000** | **34** |
| **60,000 – 150,000** | **41** |
| **150,000 – 300,000** | **46** |
| **over 300,000** | **51** |

| **1998 IRPEF **** | **%** |
|---|---|
| **0 – 15,000** | **18.5** |
| **15,000 – 30,000** | **26.5** |
| **30,000 – 60,000** | **33.5** |
| **60,000 – 135,000** | **39.5** |
| **over 135,000** | **45.5** |

| **2000 IRPEF **** | **%** |
|---|---|
| **0 – 20,000** | **18.5** |
| **20,000 – 30,000** | **25.5** |
| **30,000 – 60,000** | **33.5** |
| **60,000 – 135,000** | **39.5** |
| **over 135,000** | **45.5** |

* no regional ‘addizionali IRPEF’, nor ‘comunali’ IRPEF 2000
** values in thousands of lire

**Table 2**

*Reynolds-Smolensky Index*

| | |
|---|---|
| $\Pi^{RS}_{1995}$ | 0.038825 |
| $\Pi^{RS}_{1998}$ | 0.039184 |
| $\Pi^{RS}_{2000}$ | 0.04165 |

**Table 3**

*Total Tax Ratio*

| | |
|---|---|
| $r_{1995}$ | 0.17205 |
| $r_{1998}$ | 0.17308 |
| $r_{2000}$ | 0.16627 |

Pre-Tax Household Equivalent Incomes *($\phi$ = $\theta$ = 0.5)*

**Figure 1(*a*)**

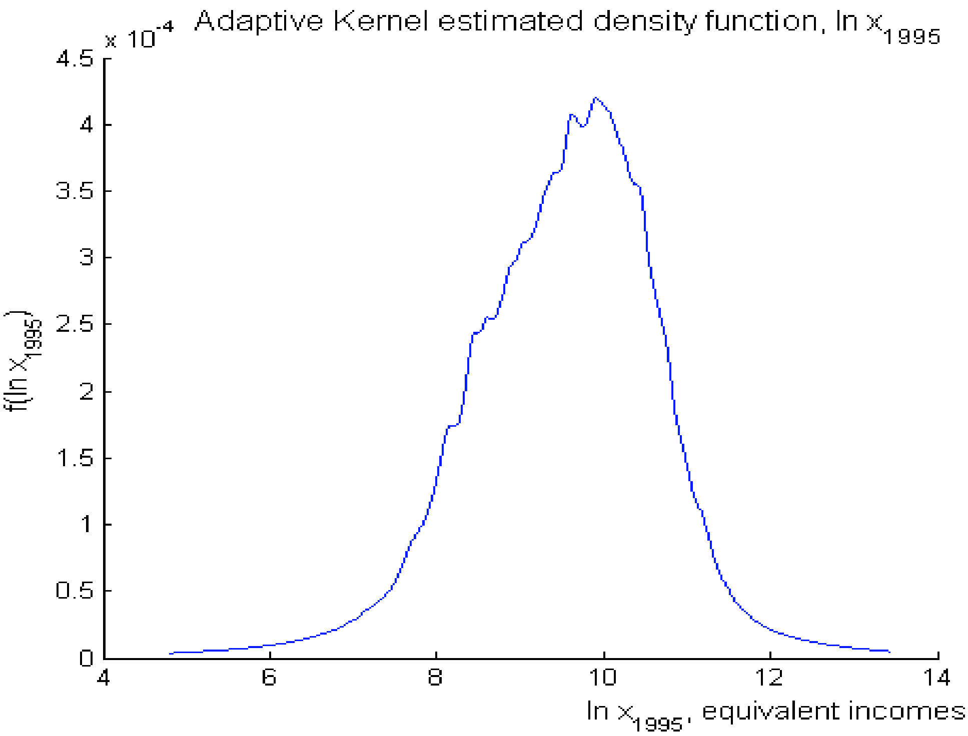


**Figure 1(*b*)**

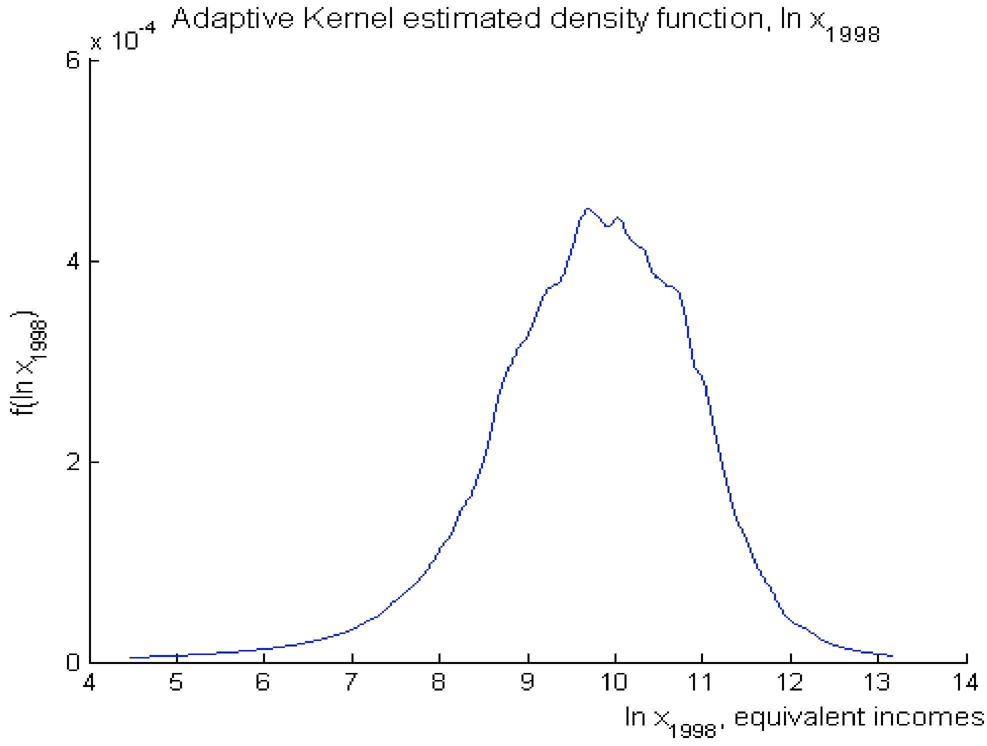
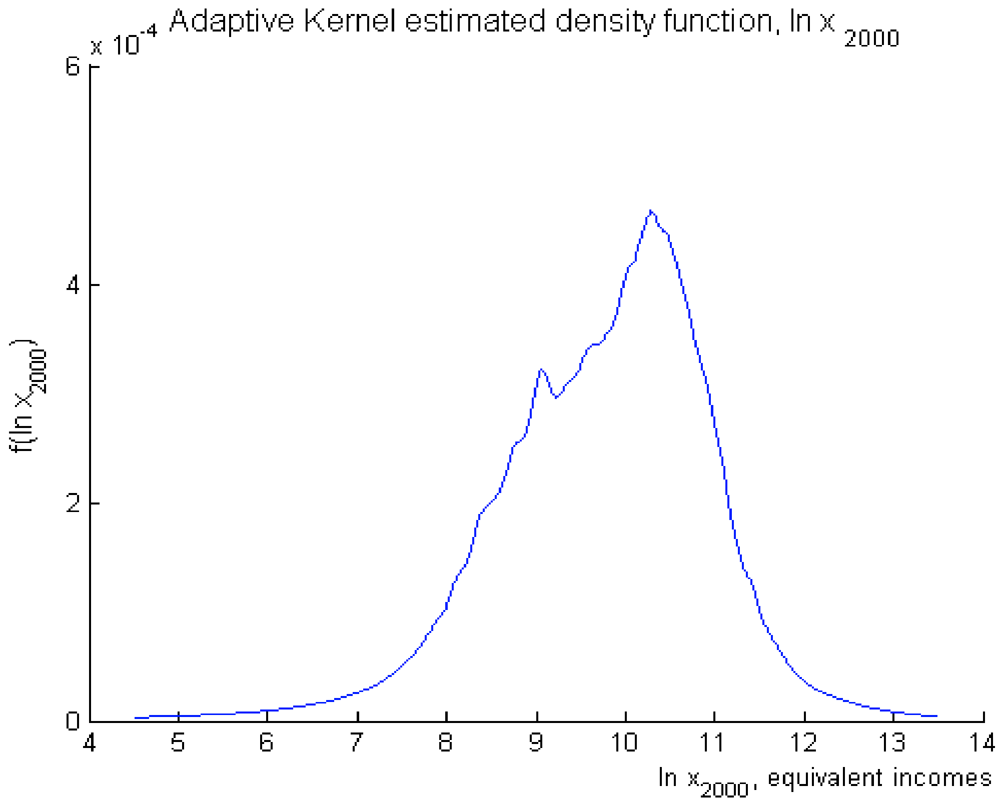



**Figure 1(*c*)**



Pre-tax household equivalent income distributions ($\phi = \theta = 0.5$)

**Figure 2(*a*)**

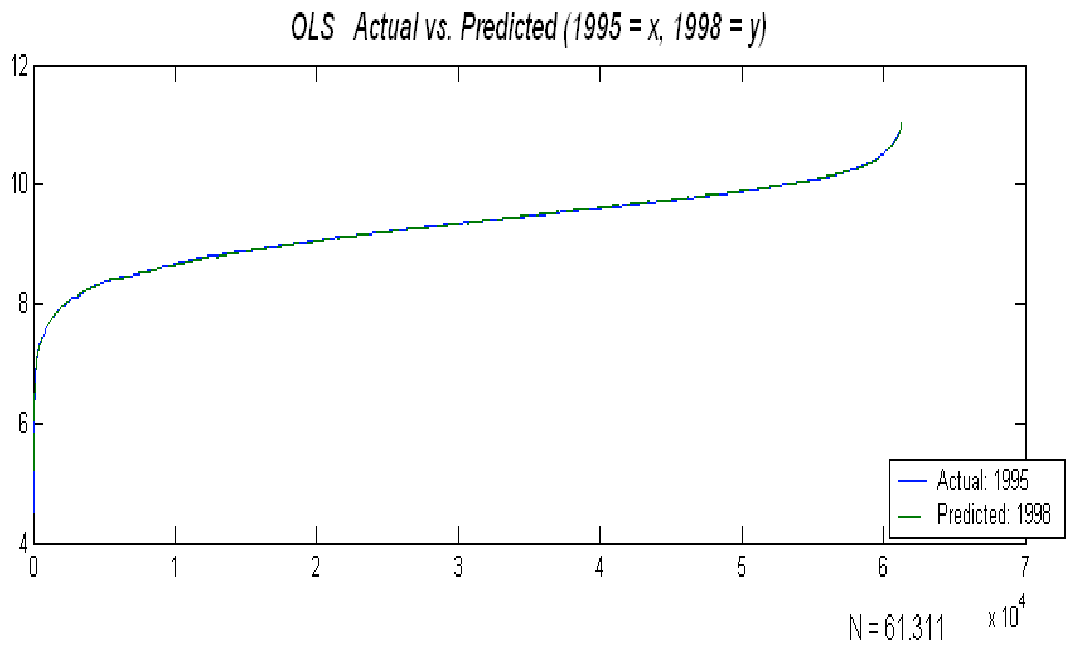
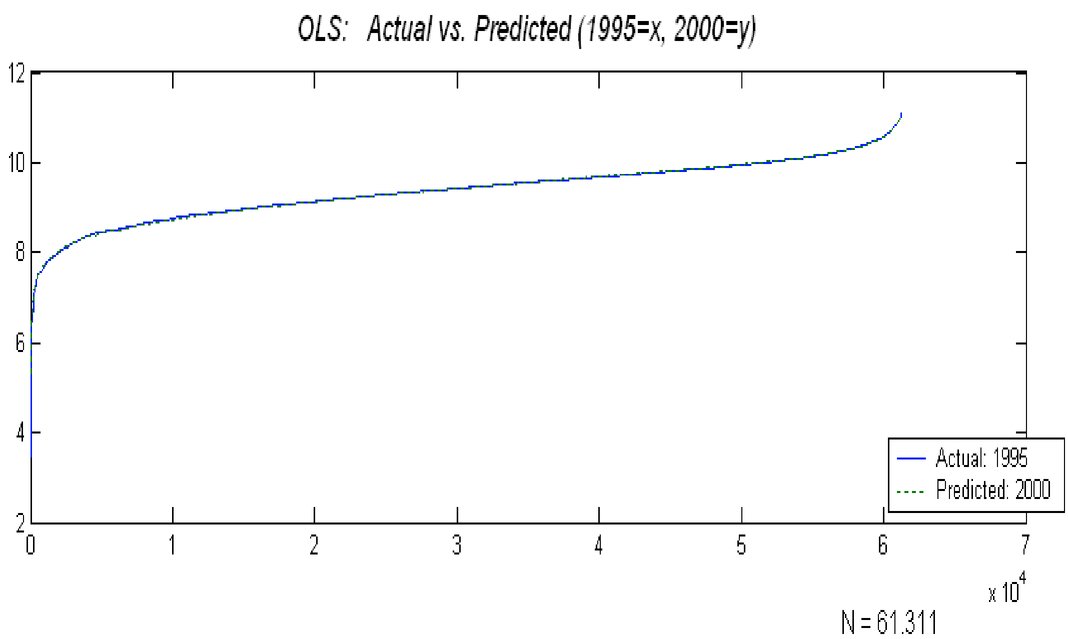



**Figure 2(*b*)**



Pre-tax household money income distributions ($\theta$ = 0)

**Figure 5(*a*)**

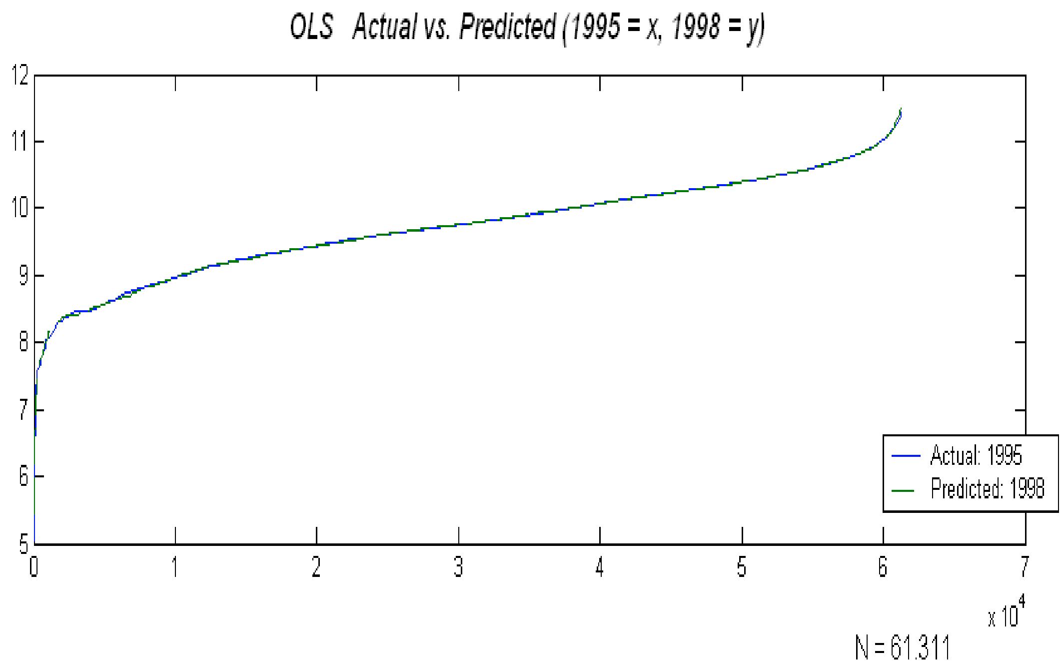
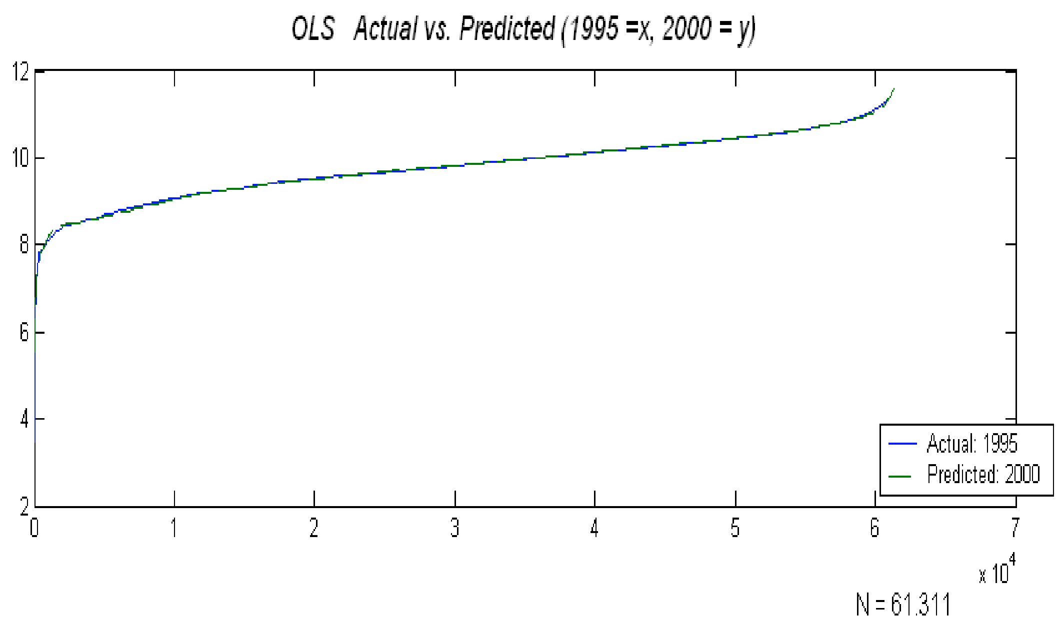



**Figure 5(*b*)**



Household money income Log transplant curves ($\theta$ = 0)

**Figure 6**

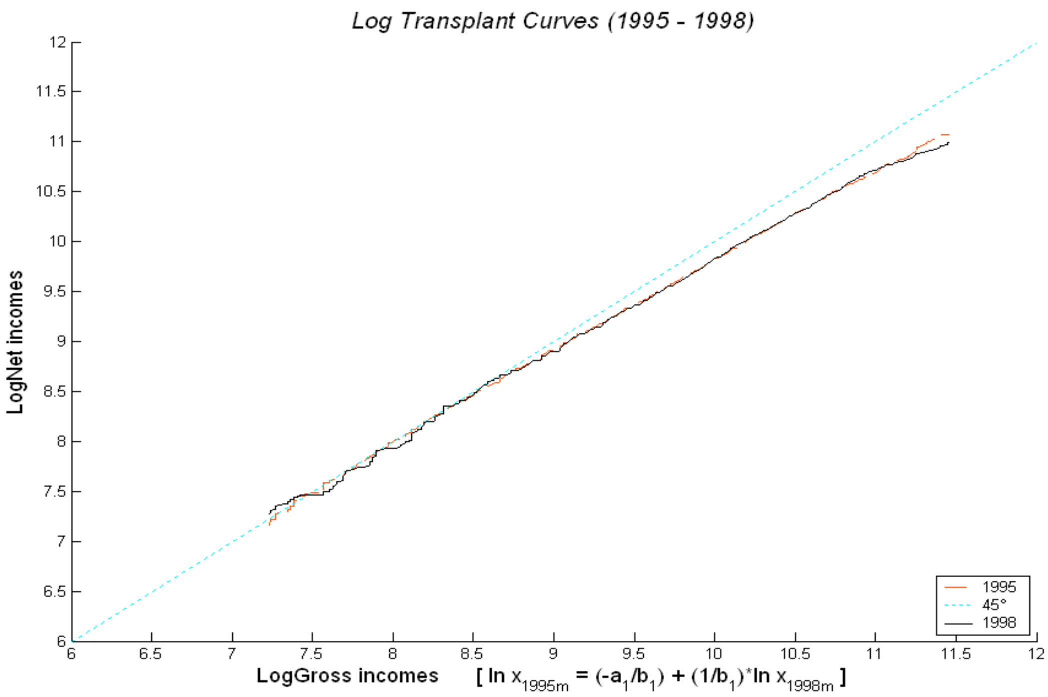
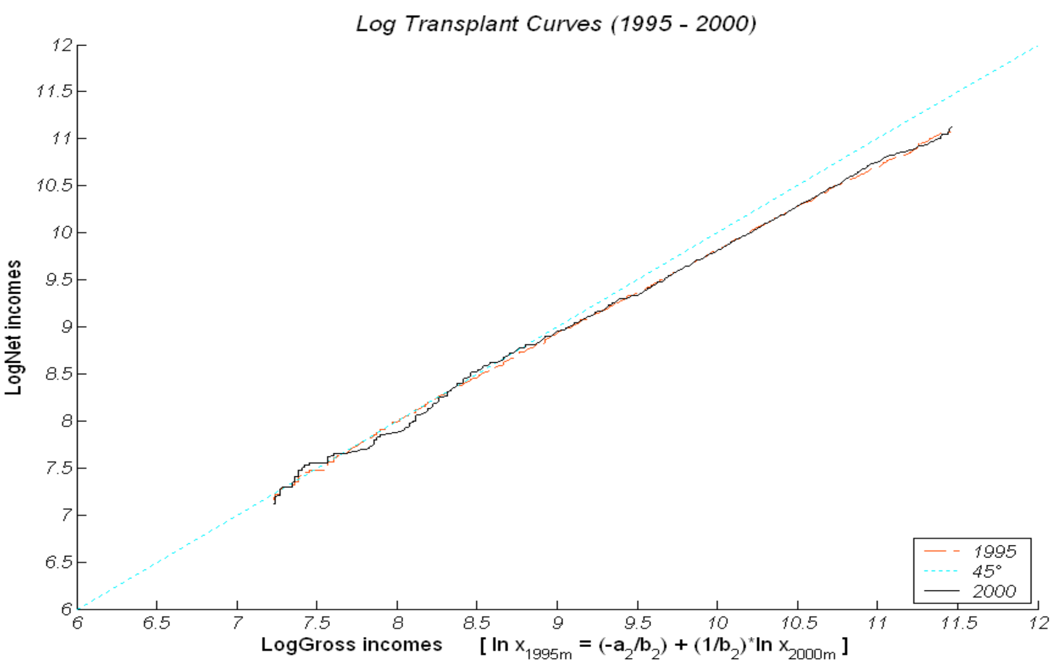



**Figure 7**



Household equivalent income comparisons ($\phi = \theta = 0.5$)

**Figure 8**

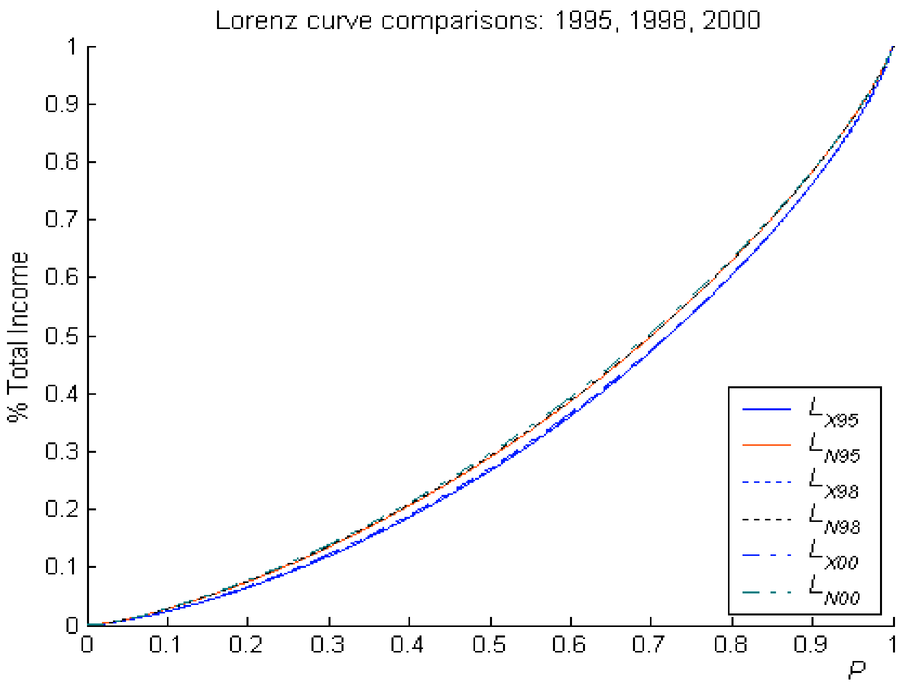



**Figure 9**