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THE DISTRIBUTIONAL INCIDENCE OF GROWTH: A SOCIAL WELFARE APPROACH

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The distributional incidence of growth: a social welfare approach *

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Abstract

This paper provides a normative framework for the assessment of the distributional incidence of growth. By removing the anonymity axiom, such framework is able to evaluate the individual income changes over time and the reshuffling of individuals along the income distribution that are determined by the pattern of income growth. We adopt a rank dependent social welfare function expressed in terms of initial rank and individual income change and we obtain partial and complete dominance conditions over different growth paths. These dominance conditions account for the different components determining the overall impact of growth, that is the size of growth and its vertical and horizontal incidence. We then provide an empirical application for Italy: this analysis shows the distributional impact of the recent economic crisis suffered by the Italian populaltion.

Keywords: Growth; Pro-poor; Inequality; Income mobility; Social welfare.

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1 Introduction

In recent years a new literature has emerged, both theoretical and empirical, on the measurement of the distributive impact of growth (see Bourguignon, 2003, 2004; Ferreira, 2010; Son, 2004; Ravallion and Chen, 2003). Different tools have been proposed for the evaluation of the pro-poorness of growth (see Duclos, 2009; Kakwani and Son, 2008; Kraay, 2006; Kakwani and Pernia, 2000; Essama-Nssah, 2005; Essama-Nssah and Lambert, 2009; Zheng, 2010) and different decompositions have been obtained that relate the concepts of growth to poverty and inequality (see Datt and Ravallion; 1992; Jenkins and Van Kerm, 2006). One basic tool used in this literature is the growth incidence curve (GIC), measuring the quantile-specific rate of economic growth between two points in time as a function of each percentile (Ravallion and Chen, 2003).

In this literature the growth process is basically analyzed by comparing the pre-growth and post-growth distribution. There is a clear analogy here between the transformation of an income distribution throughout the effect of growth and the transformation obtained as the effect of an income tax. In the same way as progressive taxation reduces inequality, a "progressive" growth

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reduces the inequality in a distribution. In fact, such analogy has been deeply explored, and different results in the progressivity literature have been applied to this context (see Bénabou and Ok, 2001).

Now, with very few recent exceptions that will be discussed below, all this literature, when analyzing the distributional impact of growth, basically compares the phenomena under scrutiny before and after the growth process has taken place. Hence, for instance, to measure the pro-poorness of growth, the poverty levels (measured, say, according to the headcount or the poverty gap indices) are computed in the two periods of time and then compared. The same for different distributional indices. If this procedure is all right when one is interested in measuring the pure distributional change that takes place, this is unsatisfactory if one is interested in evaluating growth in terms of social welfare: from this viewpoint, it can make a big difference if the poor people in the first period are still the same poor people in the following period, thereby experiencing chronic poverty, or if there has been a substantial reshuffling of the individual positions in the population. To capture this aspect one needs to remove a basic assumption used in the literature: the axiom of anonymity. According to this axiom, distributional measures are required to be invariant to permutations of income vectors. As a consequence, the individual income dynamics along the distribution are ignored.

We believe, on the contrary, that, for the welfare evaluation of growth, the identities of individuals do matter. This information allows to find out who are the winners and the losers from growth and to focus on the individual income transformations that take place during the growth process, a useful information, for example, in the evaluation of the efficacy of policy reforms. This information is usually hidden by the anonymity assumption in the standard approach. We propose, instead, an analytical framework for the normative assessment and comparison of growth processes, within which the evaluation of a growth process depends on the evaluation of the income change experienced by each individual in that society.

Our approach is close in spirit to Grimm (2007), who formally develops the concept of non-anonymous GIC (na-GIC). The na-GIC measures the individual-specific rate of economic growth between two points in time, thus comparing the income of individuals which were in the same initial position, independently of the position they acquire in the final distribution of income. Bourguignon (2011) and Jenkins and Van Kerm (2011) also contribute to this strand of the literature by proposing a normative justification for the use of na-GIC in the evaluation of growth. Bourguignon (2011) develops complete and partial dominance conditions to rank growth processes taking place on the same base-initial distribution of income. For, he adopts a utilitarian social welfare function (SWF), sensitive to the horizontal and vertical inequality of growth. Jenkins and Van Kerm (2011) develop complete and partial dominance conditions to rank growth processes, taking place on different initial distributions of income. They adopt a rank dependent SWF, sensitive to the vertical impact of growth.

In this paper we consider bivariate distributions of initial incomes and income changes: we first identify individuals according to their rank in the initial distribution of income; then, we evaluate the income evolution of each of these individuals. We provide new partial and complete dominance conditions for ranking growth processes, taking place on different initial distributions, and show that these conditions can be interpreted in terms of non-anonymous Growth Incidence Curve. However, differently from Bourguignon (2011), we adopt a rank dependent approach to social welfare, and differently from Jenkins and Van Kerm (2011), we also focus on the horizontal inequality of growth. Finally we show the empirical relevance of our framework, by implementing it in order to analyze the distributional impact of growth in Italy in the last decade. A very limited growth has been characterizing this country since the beginning of the new millennium. In fact, the average household income increased by 2% for the period 2002-2004 and by 2.6% for the 2004-2006. These periods of limited growth were finally followed by a long spell of negative growth during the recent economic crisis. In particular, the equivalent disposable income of the Italian households decreased by 2.6% during the 2006-2008 period and by 0.6% during the 2008-2010 (Banca d'Italia, 2008, 2012).

It is therefore interesting to understand wether the non-anonymous perspective to the evaluation of growth can provide useful information to complement the actual knowledge on the economic crisis in Italy. For, we use the Bank of Italy "Survey on Household Income and Wealth" (SHIW) in order to assess the distributional impact of growth in Italy. In particular, we consider four of the most recent available waves to compare the 2002-2006 growth episode against the 2006-2010. The overall result of our analysis suggests that the impact of the crisis has been severe according to different features of growth.

The work is structured as follows. In section 2 we outline the framework. In section 3 we present the theoretical results. In section 4 we provide the empirical illustration. In section 5 we conclude.

2 The framework

2.1 Notation

Let $\mathbf{y}_t$ be the distribution of income at time t, the initial period, with cumulative distribution function (cdf) $F(y_t)$ and let $\delta = y_{t+1} - y_t$ be the income growth experienced by each individual in moving from date t to $t+1$. Our aim is to evaluate the distributional incidence of growth taking place over $\mathbf{y}_t$, according to the non-anonymous perspective. Therefore we need to keep track of the identity of individuals. We assume that such identity is represented by their membership to a specific quantile group at time t. We partition $F(y_t)$ into quantile groups and we identify each individual according to the initial quantile group he belongs to. We index each subgroup by $i = 1, ..., n$ in increasing order, starting from the poorest to the richest, and we denote by q_i the share of individuals in quantile group i, therefore i - the information about individuals' identity - corresponds to their rank in $F(y_t)$.[1] Hence, for each initial quantile group i it is possible to observe a distribution of income change with cdf $F_i(\delta)$ and $\delta_i^{(t,t+1)}(p)$ representing the income change of those individuals ranked p in $F_i(\delta)$.[2] Thus, $\Delta_i^{(t,t+1)}(p) = \int_0^p \delta_i^{(t,t+1)}(q)dq/p$ is the average income growth for those at the bottom p quantiles within that initial quantile group and $\Delta_i^{(t,t+1)}(1)$ the average income growth of that initial quantile group.[3] We define by $\Delta^{(t,t+1)} \mid y_t$ the income transformation process of all individuals conditional on their identity and by D the set of admissible growth paths.

We are interested in ranking members of D from a normative perspective and we assume that such ranking can be represented by a social evaluation function, $W : D \longrightarrow \mathbb{R}$. Given the assumption

[1] Obviously, $q_i = q_j \ \forall i, j = 1, ...n$.

[2] Letting $F_i(\delta)$ be the cdf of the individual income change within each initial quantile group, $p = F_i(\delta) \Longleftrightarrow \delta_i(p) = F_i^{-1}(p)$, $\delta_i(p) := \inf\{y : F_i(\delta) \geq p\}$. $\delta_i(p)$ is the left inverse function, denoting the level of income change of individuals ranked p in $F_i(\delta)$, $i = 1, ..., n$.

[3] Our framework can be extended to consider proportionate growth, $\delta = \frac{y_{t+1} - y_t}{y_t}$. All the dominance results will hold for distributions of proportionate income change. But the relationship with the na-GIC will be different.

Note that individual growth measures are not normalized by the average income growth across the population. Hence, according to our approach, what matters is the extent of growth of each individual, independently of the level of growth experienced by the other individuals. Hence a positive increase in individual income will increase aggregate growth wathever the growth (higher or lower) of anyone who is richer.

of non-anonymity, W will depend not only on the extent of the individual growth but also on the individual identity. According to this idea, a social preference over growth processes can be defined by:

$$W = \sum_{i=1}^{n} q_i \int_0^1 v_i(p)\, \delta_i^{(t,t+1)}(p)\, dp. \quad (1)$$

Eq. (1) aggregates the income change experienced by each individual, giving different weight to different individuals.[4] In particular, $\int_0^1 v_i(p)\, \delta_i^{(t,t+1)}(p)\, dp$ is the social evaluation of the growth experienced by the individuals belonging to a given initial quantile group $i = 1, ..., n$.[5] It is expressed by a weighted average of the income changes of its members, where each income change is weighted according to the position of each member in $F_i(\delta)$. The function $v_i(p) : [0,1] \longrightarrow \Re_+$ expresses the weight attached by the society to the change in income experienced by the individuals ranked p in $F_i(\delta)$ and $v_i(p)\, \delta_i^{(t,t+1)}(p)$ is the contribution of these individuals to the social evaluation of growth. The overall social evaluation of growth is then obtained by applying an additional weighted aggregation. That is, we aggregate the social evaluation of growth corresponding to each initial quantile group, weighted by the relevant population share, using initial quantile dependent weights. In sum, W expresses concerns for both the initial and final economic situation of the individuals it represents. The set of weight functions specifying the social evaluation of growth, $< v_1(p), ..., v_n(p) >$, will be called "weight profile". Different preferences over growth processes are encompassed in this framework by selecting different classes of "social weight" profiles. It is clear from eq. (1) that the differences in the initial distribution of income are neutralized. This enables to evaluate growth processes that take place over different initial distributions of income.[6]

The model provided in eq. (1) assumes that each initial quantile encompasses more than one individual. However, the higher the number on initial quantile groups used to partition the population, the higher the probability that each group will only count one individual. In the limiting case, it is possible to partition the population into as many groups as the number of individuals in the population. As it will be discussed later, our model also accommodates for this limiting situation. In this special case, i will denote the individual, q_i will collapse to $\frac{1}{n}$ and $\delta_i^{(t,t+1)}(p)$ to $\Delta_i^{(t,t+1)}$. Eq. (1) can, then, be rewritten as $W = \frac{1}{n}\sum_{i=1}^{n} v_i \Delta_i^{(t,t+1)}$.

2.2 Properties

In this section we discuss the properties that W must satisfy. Consequently, different families of W are derived, depending of the restrictions imposed on the weight function.

The first property is a standard monotonicity assumption.

Axiom 0 (Pro-Growth) *For all $i = 1, ..., n$, for all $p \in [0,1]$*

$$v_i(p) \geq 0.$$

[4]This approach is based on the idea that a social planner cares about the initial economic condition of individuals. This impact is taken into account, in our framework, through relaxing anonymity and ordering individuals by partitioning the population according to their belonging in the initial distribution of income.

[5]See Yaari (1988), Donaldson and Weymark (1980), Aaberge (2001). See also Zoli (2000) and Peragine (2002) for alternative applications.

[6]Note the main difference with Bourguignon (2011): while he conditions income changes to initial income levels and requires information about the density distribution of the base-year initial income distribution, we do not require such information, since we condition income changes to initial ranks. This implies that we can use our framework to compare growth processes that take place over different initial distributions of income.

According to this axiom a positive income change generates an increment of W, or at most leaves the social evaluation of growth unaffected; a negative income growth causes a reduction, or at most does not affect it. Let $\mathbf{V} = \{< v_1(p), ..., v_n(p) >$: Axiom 0 holds$\}$ and let $\mathbf{W}$ be the class of W based on the weight profile $\mathbf{V}$.

We proceed by imposing aversion to vertical inequality.

Axiom 1 (Pro-poor Growth) *For all* $p \in [0, 1]$, for all $i = 1, ..., n-1$

$$v_i(p) \geq v_{i+1}(p).$$

This property is expression of the transfer-sensitivity principle in the context of income growth among individuals having different ranks in $F(y_t)$. That is, a transfer of a small amount of income change from an individual ranked p in a richer initial quantile group $i+1$ to an individual ranked p in a poorer initial quantile group i does not decrease W.[7] According to Axiom 1, the social evaluation of growth would increase more if the income change concerns the initially poorest individuals. Let $\mathbf{V_1} = \{< v_1(p), ..., v_n(p) >$: Axioms 0 and 1 hold$\}$ and let $\mathbf{W}_1$ be the class of W based on the weight profile $\mathbf{V}_1$.

Axiom 2 (Diminishing Pro-poor Growth) For all $p \in [0, 1]$, for all $i = 1, ..., n-2$

$$v_i(p) - v_{i+1}(p) \geq v_{i+1}(p) - v_{i+2}(p).$$

This axiom states that the weight given to the income change of individuals, ranked the same in different initial quantile group distributions, decreases more the lower is the initial rank of those individuals. That is, the poorer is the initial quantile group to which an individual belongs, the more sensitive is W to the progressivity of the growth process under scrutiny. Let $V_{\mathbf{12}} = \{< v_1(p), ..., v_n(p) >$: Axioms 0, 1 and 2 hold$\}$ and let W_{12} be the class of W constructed in (1) and based on the weights profile V_{12}.

Note that in all these axioms, no concern is expressed with respect to horizontal equity, that is how individuals with same initial economic conditions - belonging to the same initial quantile group - are affected by growth. The following axiom, instead, goes in this direction.

Axiom 3 (Horizontally Equal Gains) For all $p \in [0, 1]$, for all $i = 1, ..., n$

$$v_i'(p) \leq 0.$$

This axiom states that a social planner evaluates more a growth process under which individuals with same starting economic conditions get equal gains. This implies that W will be higher the lower is the inequality in the incidence of growth for individuals belonging to the same initial quantile group. This kind of social evaluation function captures the inequality in gains conditionally on the individuals' initial rank. Let $V_{\mathbf{3}} = \{< v_1(p), ..., v_n(p) >$: Axioms 0 and 3 hold$\}$ and let W_3 be the class of W based on the weight profile in V_3.

Axiom 3* (Pro-poor Horizontally Equal Gains) *For all* $p \in [0, 1]$, *for all* $i = 1, ..., n-1$

$$v_i^{'}(p) \leq v_{i+1}^{'}(p) \leq 0.$$

This axiom encompasses the joint effect of being ranked differently in the initial distribution of income and experiencing a different incidence of growth in each i. The same progressive transfer of income change is evaluated differently if it takes place in different initial quantile groups. W

[7] This axiom reflects the preferences of a sicial planner that on a Rawlsian ground cares more about someone the poorer she is in the initial distribution of income.

increases more, the lower is the initial quantile group, within which the progressive transfer of income change takes place. Thus, the lower is the level of inequality of the individual gains within the poorest initial quantile groups, the higher is W. Let $\mathbf{V}_{13^*} = \{< v_1(p), ..., v_n(p) >$: Axioms 0, 1 and 3* hold$\}$ and let $\mathbf{W}_{13^*}$ be the class of W based on the weight profile $\mathbf{V}_{13^*}$.

Axiom 4 (Horizontal Inequality Neutrality in Gains) For all $p \in [0,1]$, for all $i = 1, ..., n$, $\exists \beta_i \in \Re$ such that

$$v_i\left(p\right) = \beta_i.$$

This axiom states that W is neutral to the inequality of growth in each i. Therefore, a social planner adopting this preference would give the same weight to the income change of individuals ranked the same in the initial distribution of income. Let $\mathbf{V_4} = \{< v_1(p), ..., v_n(p) >$: Axioms 0 and 4 hold$\}$ and let $\mathbf{W}_4$ be the class of W based on the weight profile $\mathbf{V}_4$. Furthermore, let $\mathbf{V_{14}} = \{< v_1(p), ..., v_n(p) >$: Axioms 0, 1 and 4 hold$\}$ and let $\mathbf{W}_{14}$ be the class of W based on the weight profile $\mathbf{V}_{14}$. Last, let $\mathbf{V_{124}} = \{< v_1(p), ..., v_n(p) >$: Axioms 0, 1, 2 and 4$\}$ and let $\mathbf{W}_{124}$ be the class of W based on the weight profile $\mathbf{V}_{124}$.

It is worth noticing that the model proposed in eq. (1) and the properties discussed above may have different justifications, each of them generating a different economic interpretation of the dominance results. Eq. (1) could also be used in other frameworks when income changes are substituted with other variables of interest. For example, Peragine et al. (2013) extend this model to the case of growth in the context of equality of opportunity. In their paper the groups are identified on the basis of a set of exogenous individual characteristics, such as the socioeconomic background.

3 Results

In this section we discuss the dominance conditions corresponding to different classes of social welfare evaluation functions W, all the proofs are gathered in the theoretical appendix.

We start considering the class of W satisfying the pro-growth axiom.[8]

Proposition 1 *For all* $\Delta_A^{(t,t+1)} \mid y_t$ and $\Delta_B^{(t,t+1)} \mid y_t \in D$, $W_A \geq W_B, \forall W \in \mathbf{W}$ *if and only if*

$$\delta_{Ai}^{(t,t+1)}\left(p\right) \geq \delta_{Bi}^{(t,t+1)}\left(p\right) \forall i = 1, ..., n, \forall p \in [0,1]\,. \tag{2}$$

The condition expressed in eq. (2) is a first order dominance. Given two distributions of income change $F_{Ai}\left(\delta\right)$ and $F_{Bi}\left(\delta\right)$, which are specific for each initial quantile group, the inverse of $F_{Ai}\left(\delta\right)$, that is $\delta_{Ai}^{(t,t+1)}\left(p\right)$, must lie nowhere above the inverse of $F_{Bi}\left(\delta\right)$, that is $\delta_{Bi}^{(t,t+1)}\left(p\right)$, for each $i = 1, ..., n$. When we only impose pro-growth, to determine which growth process is socially preferable one needs to check, for every initial quantile group, that each individual in that group shows higher income growth in the dominating process than in the dominated one. This class of W is expression of a simple efficiency-based criterion, no concern is expressed in terms of redistributional effect of growth. Therefore, the comparisons of growth processes according to this condition only rest on the extent of growth and it is independent from any redistributional features of growth.

[8] Since we partition the initial distribution of income into quantile groups, by definition $q_i = q_j$ for all $i, j = 1, ...n$ and they also will be the same across different distributions; therefore, the ordering provided by the dominance conditions we characterize here does not depend on q_i. This observation allows to leave out them in the derivation of our results.

We now turn to the family of $W \in W_1$.

Proposition 2 *For all* $\Delta_A^{(t,t+1)} \mid y_t$ and $\Delta_B^{(t,t+1)} \mid y_t \in D$, $W_A \geq W_B, \forall W \in \mathbf{W}_1$ *if and only if*

$$\sum_{i=1}^{k} \delta_{Ai}^{(t,t+1)}(p) \geq \sum_{i=1}^{k} \delta_{Bi}^{(t,t+1)}(p), \forall k = 1, ..., n, \forall p \in [0,1]. \tag{3}$$

The condition expressed in eq. (3) is a first order sequential distributional test, to be checked on the initial quantile group specific distribution of income change, starting from the lowest initial quantile group, then adding the second, then the third, and so on. The condition to be satisfied at each stage is a standard first order dominance of $\delta_{Ai}^{(t,t+1)}(p)$ over $\delta_{Bi}^{(t,t+1)}(p)$. That is, the quantile function of $F_{Ai}(\delta)$, $\delta_{Ai}(p)$, must be higher than the corresponding one in $F_{Bi}(\delta)$, and this dominance must hold for every p, by sequential aggregation of the initial quantile groups. Hence, first we have to check that, for the poorest initial quantile group, $i = 1$, at every p, the dominant distribution shows higher income changes than the dominated one. Then, we have to add the second lowest initial quantile group, and so on, and perform the same check at every step.[9] In order to compare two processes according to this proposition, it is not enough to focus on the extent of growth; the final judgement will also depend on its vertical distributional impact.

The next family of W we consider satisfies also axiom 2.

Proposition 3 *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_B^{(t,t+1)} \mid y_t \in D$, $W_A \geq W_B, \forall W \in \mathbf{W}_{12}$ *and* $\forall p \in [0,1]$ such that $v_n(p) = 0$ *if and only if*

$$\sum_{i=1}^{j} \sum_{k=1}^{i} \delta_{Ak}^{(t,t+1)}(p) \geq \sum_{i=1}^{j} \sum_{k=1}^{i} \delta_{Bk}^{(t,t+1)}(p), \ \forall j = 1, ..., n-1, \ \forall p \in [0,1]. \tag{4}$$

The condition expressed in eq. (4) is a first order "sequentially cumulated" distributional test. It provides a weaker dominance condition to be applied when it is not possible to rank distributions according to proposition 1 and 2. Note that the condition $v_n(p) = 0$ can be relaxed if the additional condition $\sum_{k=1}^{n} \delta_{Ak}^{(t,t+1)}(p) \geq \sum_{k=1}^{n} \delta_{Bk}^{(t,t+1)}(p) \ \forall p \in [0,1]$ is satisfied.

We now introduce a concern toward the horizontal incidence of growth, thus we turn to the family of $W \in W_3$.

Proposition 4 *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_B^{(t,t+1)} \mid y_t \in D$, $W_A \geq W_B, \forall W \in \mathbf{W}_3$ *if and only if*

$$\int_0^p \delta_{Ai}^{(t,t+1)}(q) - \delta_{Bi}^{(t,t+1)}(q)\, dq \geq 0, \ \forall p \in [0,1], \ \forall i = 1, ..., n. \tag{5}$$

The condition in eq. (5) is a second order distributional test to be applied at each initial quantile group. That is, for every $i = 1, ..., n$, we have to check that the the average income growth for those at the bottom p quantiles, within that initial quantile group, is higher in the dominating process than in the dominated one. If this condition is satisfied, it means that under the dominating process the gains (or losses) from growth are distributed more equally within each initial quantile groups than under the dominated one. This evaluation only focuses on the impact of growth among individuals with same initial economic status and it is independent from pro-poorness features.

In fact, for the condition characterized in proposition 4 what matters is only the distribution of growth among individuals belonging to each group, that is among individuals that can be considered

[9] Recall that the sequential aggregation is to be implemented on individuals ranked the same in the different quantile groups being aggregated.

as "equals". The different impact of growth between different groups, instead, does not represent a matter of concern. That is, the importance attributed to horizontal inequality aversion does not depend on the income level at which growth is experienced. We believe this is a desirable property as it reflects the logical independence between the horizontal and vertical equity principles.

Moving to the family of W satisfying axiom 3*, we get the next result.

Proposition 5 *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_A^{(t,t+1)} \mid y_t \in D$, $W_A \geq W_B, \forall W \in \mathbf{W}_{13^*}$ *if and only if*

$$\sum_{j=1}^{i} \int_0^p \delta_{Aj}^{(t,t+1)}(q)\, dq \geq \sum_{j=1}^{i} \int_0^p \delta_{Bj}^{(t,t+1)}(q)\, dq, \forall i = 1, ..., n, \forall p \in [0,1]. \tag{6}$$

The condition expressed in eq. (6) is a second order sequential distributional test, to be checked starting from the poorest initial quantile group, then adding the second, then the third, and so on. The condition to be satisfied at each stage is that the cumulated sum of the individual income growth, within each initial quantile group, be higher in $\Delta_A^{(t,t+1)} \mid y_t$ than in $\Delta_B^{(t,t+1)} \mid y_t$. Hence, first we have to check that, for the poorest initial quantile, the cumulated income change of the p least growing individuals in $F_{Ai}(\delta)$ is higher than the corresponding one in $F_{Bi}(\delta)$, and this dominance must hold for all p. Then, we have to add the second lowest initial quantile, and so on, and perform the same check at every step. When the condition in this proposition is used to compare two growth processes, the evaluation becomes finer as it builds upon all the possible features of growth, that is, the extent of growth and its vertical and horizontal incidence.[10]

The condition characterized in proposition 5 also incorporates aversion toward the horizontal inequality of growth; in fact, within each group, more importance is given to the individuals growing less than to the ones growing more. However, this condition is also sensitive to the progressivity of growth: subsequent comparisons successively include less and less initially poorer groups, and at the last stage all groups are considered.

The next family of W we consider satisfies horizontal inequality neutrality in gains.

Proposition 6 *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_B^{(t,t+1)} \mid y_t \in D$, $W_A \geq W_B, \forall W \in \mathbf{W}_4$ *if and only if*

$$\Delta_{Ai}^{(t,t+1)}(1) \geq \Delta_{Bi}^{t,t+1}(1), \forall i = 1, ..., n. \tag{7}$$

Eq. (7) is a first order direct dominance condition to be applied on distributions of income change means.[11] That is, we have to check that each initial quantile group shows higher mean income growth in $\Delta_A^{(t,t+1)} \mid y_t$ than in $\Delta_B^{(t,t+1)} \mid y_t$. Clearly this condition depends on the extent of growth of each initial quantile group and it is independent from distributional considerations.

We then consider the family of W satisfying also axiom 1.

Proposition 7 *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_B^{(t,t+1)} \mid y_t \in D$, $W_A \geq W_B, \forall W \in \mathbf{W}_{14}$ *if and only if*

$$\sum_{i=1}^{k} \Delta_{Ai}^{(t,t+1)}(1) \geq \sum_{i=1}^{k} \Delta_{Bi}^{(t,t+1)}(1), \forall k = 1, ..., n. \tag{8}$$

[10] See Atkinson and Bourguignon (1987), Bourguignon (1989) and Jenkins and Lambert (1993) for the implementation of the sequential dominance procedure to characterize social welfare dominances based on a utilitarian ground. See, also, Atkinson (1992) for the implementation of the sequential dominance in the context of poverty comparisons.

[11] Note that the dominance condition stated in this proposition is equivalent to the last stage of the dominance condition state in proposition 4.

Eq. (8) is a second order direct dominance to be applied on distributions of income change means.[12] That is, we have to check that the cumulated sum of the initial quantile group specific mean income growth in higher in $\Delta_A^{(t,t+1)} \mid y_t$ than in $\Delta_B^{(t,t+1)} \mid y_t$. In this case, the dominance condition depends, not only on the extent of growth, but also on the incidence of growth on each initial quantile group; it is, instead, neutral with respect to the incidence of growth within each group.

Finally, we consider the family of $W \in W_{124}$.

Proposition 8 *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_B^{(t,t+1)} \mid y_t \in D$, $W_A \geq W_B, \forall W \in \mathbf{W}_{124}$ *and* $\forall p \in [0,1]$ such that $v_n(p) = 0$ if and only if

$$\sum_{i=1}^{j} \sum_{k=1}^{i} \Delta_{Ak}^{(t,t+1)}(1) \geq \sum_{i=1}^{j} \sum_{k=1}^{i} \Delta_{Bk}^{(t,t+1)}(1), \forall j = 1, ..., n-1. \tag{9}$$

Eq. (9) is a third order direct dominance to be applied on distributions of mean income change.[13] It is a weaker condition, allowing to order distributions when it is not possible to rank them according to proposition 6 and 7. Note that the condition $v_n(p) = 0$ can be relaxed if the additional condition $\sum_{k=1}^{n} \Delta_{Ak}^{(t,t+1)}(1) \geq \sum_{k=1}^{n} \Delta_{Bk}^{(t,t+1)}(1)$ is satisfied.

In may be interesting to notice how these results can be used in the special case of initial quantile groups encompassing only one individual. In this case, the results in eq. (2), (5) and (7) are equivalent and they require the existence of a dominance between two curves, plotting against each individual her respective income change experienced in the periods compared, with individuals sorted according to their position in the initial distribution of income. In particular, for proposition 1, 4 and 6 this is equivalent to check that each individual experiences higher growth under the dominating process than under the dominated one, and this check must be done individual by individual. For proposition 2, 5 and 7 this is equivalent to check that the cumulated sum of the growth experienced by each individual is higher under the dominating process than under the dominated one, where individuals are ordered according to their level of initial income. Hence starting from the initially poorest individual, then adding the second poorest individual and so on, up to considering the whole distribution, and checking at every step that there is higher growth under the dominating process than under the dominated one. Equivalently, these conditions require that the cumulative curve of the dominating growth process must lie nowhere below than the cumulative curve of the dominated process. Last, for proposition 3 and 8 this is equivalent to check the difference between the area under the cumulative curve relative to each growth process.

In summary, our dominance results are different from those derived by Bourguingon (2011). First, our conditions rest on the (sequential) comparison of quantile functions, whereas Bourguingon's (2011) rest on the (sequential) comparison of cdfs. Second, while Bourguingon's model builds on the assumption of same initial distributions of income, our model can be extended to evaluate growth processes taking place over different initial distributions of income. Still, our dominance results are related to Jenkins and Van Kerm's (2011) contribution. The essential difference is that we introduce a concern toward the horizontal inequality of the incidence of growth, which allows to obtain a set of new different dominance conditions. The introduction of horizontal inequality

[12] Note that the dominance condition stated in this proposition is equivalent to the last stage of the dominance condition stated in proposition 5.

[13] The dominance conditions expressed in Proposition 6, 7 and 8 are equivalent respectively to "Type A", "Type B" and "Type C" dominance of absolute mobility profiles in Van Kerm (2006;2009).

concerns is relevant not only because it allows to draw a more complete picture of the overall redistributive effect of growth, but also because it could help to explain possible divergences between anonymous and non-anonymous evaluations of growth.

Furthermore, notice that the relevance of the horizontal inequality issue can also be evaluated by considering that the framework we propose could be extended to different definitions of "equals groups", alternative to those based on the rank in the initial distribution. It allows, for instance, to assess how growth affects a population adequately partitioned into groups which reflect different demographic characteristics or households' needs. The basic idea is the standard one, that is, attributing different social evaluation of income growth functions to households characterized by different needs. These social evaluation functions will differ each other in the way they will reflect the different needs of the population.[14] To be more specific, suppose that the aim is to investigate the distributional effects of growth on a population composed by n groups of households, differentiated by needs. Then, after partitioning the population into these n groups, one needs to rank them in a non-decreasing order, starting from the most needy to the less needy group. Thus, in this specific context $i = 1, ..., n$ will denote the ith needy group and $F_i(\delta)$ will refer to the cdf of the income changes experienced by the ith equally needy individuals and $\Delta_i^{(t,t+1)}(1)$ will denote the average growth the ith needy group.

3.1 Welfare dominance and na-GIC

In this section we study the relationship between our dominance conditions and the na-GIC (or Mobility Profile). The na-GIC, formally defined by: $\int_0^1 \delta_i^{(t,t+1)}(p)\,dp/y_t\left(\frac{i}{n}\right)$, plots against each quantile of the initial distribution of income, the rate of income growth of that quantile, where the income refers to the same individuals in t and $t+1$ (Grimm, 2007; Bourguignon, 2011; Jenkins and Van Kerm 2011).

Now, focussing on the family of $W \in \mathbf{W}$ and $W \in \mathbf{W}_3$, it is possible to note that if the dominance in eq. (2) or in eq. (5) holds for every p, then it must be the case that it holds if one integrates the quantile function of the income change from $p = 0$ up to $p = 1$, for all p belonging to the same initial quantile. Hence, we have:

$$W^A \geq W^B \Longrightarrow \int_0^1 \delta_{Ai}^{(t,t+1)}(p)\,dp \geq \int_0^1 \delta_{Bi}^{(t,t+1)}(p)\,dp \Longleftrightarrow \tilde{g}_{Ai} \geq \tilde{g}_{Bi}, \forall i = 1, ..., n.$$

where $\tilde{g}_{Ai}$ represents the absolute version of the na-GIC, which plots, against each quantile of the initial distribution, the level of income change of that quantile. We can summarize this discussion in the following remark.

Remark 1 a) *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_B^{(t,t+1)} \mid y_t \in D$, *if* $W_A \geq W_B$, $\forall W \in \mathbf{W}$ *or* $\forall W \in \mathbf{W}_3$ *then* $\tilde{g}_{Ai} \geq \tilde{g}_{Bi}, \forall i = 1, ..., n$; **b)** *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_B^{(t,t+1)} \mid y_t \in D$, $W_A \geq W_B$, $\forall W \in \mathbf{W}_4$ *if and only if* $\tilde{g}_{Ai} \geq \tilde{g}_{Bi}, \forall i = 1, ..., n$.

According to remark 1, first order dominance of na-GIC, weighted by the initial income of each quantile (or the absolute na-GIC), is a necessary condition, but not sufficient, for a normative dominance of two given growth processes, when the extent of growth or when both the extent of growth and its horizontal incidence matter. If we assume same initial distribution, this dominance reduces to first order na-GIC dominance. By contrast, when the dominance condition between two growth processes is based on social evaluation functions which are neutral to the horizontal

[14]See Lambert (2001) for more details on this issue.

incidence of growth, these dominance conditions arise to be equivalent to the absolute na-GIC dominance.

We now turn to consider the family of W satisfying axioms 0, 1 and 3*. Note that if the dominance in eq. (3) or in eq. (6) holds for every p, then it must be the case that it holds for $p = 1$. Integrating sequentially the inverse cumulative distribution function up to $p = 1$, the social evaluation dominance implies that:

$$\sum_{i=1}^{k} \Delta_{Ai}^{(t,t+1)}(1) \geq \sum_{i=1}^{k} \Delta_{Bi}^{(t,t+1)}(1), \forall k = 1, ..., n. \tag{10}$$

That is, at every stage we have to evaluate that the partial income growth mean is higher in one growth process than in the other. Thus

$$W^A \geq W^B \Longrightarrow \sum_{i=1}^{k} \tilde{g}_{Ai} \geq \sum_{i=1}^{k} \tilde{g}_{Bi}, \forall k = 1, ..., n. \tag{11}$$

We can summarize this discussion in the following remark.

Remark 2 a) *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_B^{(t,t+1)} \mid y_t \in D$, *if* $W_A \geq W_B$, $\forall W \in \mathbf{W}_1$ *or* $\forall W \in \mathbf{W}_{13^*}$ *then* $\sum_{i=1}^{k} \tilde{g}_{Ai} \geq \sum_{i=1}^{k} \tilde{g}_{Bi}, \forall k = 1, ..., n$; **b)** *For all* $\Delta_A^{(t,t+1)} \mid y_t, \Delta_B^{(t,t+1)} \mid y_t \in D, W_A \geq W_B$, $\forall W \in \mathbf{W}_{14}$ *if and only if* $\sum_{i=1}^{k} \tilde{g}_{Ai} \geq \sum_{i=1}^{k} \tilde{g}_{Bi}, \forall k = 1, ..., n$.

According to remark 2, second order dominance of absolute na-GICs is a necessary, but not sufficient, condition for the dominance between growth processes, evaluated on the base of preferences expressing a concern toward the extent of growth and its vertical impact or to the extent of growth and both its vertical and horizontal impact. If we assume same initial distributions, this dominance reduces to the second order na-GIC dominance. Further imposing axiom 4, in addition to axiom 0 and 1, we have an equivalence result.

3.2 Aggregate measures of non-anonymous growth

The dominance conditions established above provide robust but only partial rankings of growth processes. Complete rankings, instead, can be obtained adopting scalar measures. This is the aim of the following paragraphs.

3.2.1 Progressivity-adjusted growth

In this section we introduce a family of aggregate measures of growth which are sensitive to its progressivity. We do this by considering social evaluation functions endorsing a preference for the pro-poorness of growth and neutrality for the horizontal inequality of growth, hence satisfying axioms 0, 1 and 4. A general index of growth can, then, be obtained by normalizing eq. (1) as follows:

$$W^* = \frac{\sum_{i=1}^{n} q_i v_i \Delta_i^{(t,t+1)}(1)}{\sum_{i=1}^{n} q_i v_i}. \tag{12}$$

Eq. (12) represents a general family of aggregate measures of growth. Specific scalar measures can be obtained from eq. (12) by simply choosing the proper functional form for the weighting function v_i, consistent with axiom 0, 1 and 4. Eq. (12) evaluates growth accounting for both the size and the vertical redistribution impact of growth and can take both positive and negative values. Therefore, W^* can be interpreted as a progressivity-sensitive measure of growth.

It is also possible to capture only the pure redistributional effect of an income transformation, in order to evaluate growth processes on the base of their ability to favor the growth of the initially poorest individuals as compared to the initially richest. This can be done using the following general formulation:

$$G = \frac{\sum_{i=1}^{n} q_i v_i \Delta_i^{(t,t+1)}(1)}{\sum_{i=1}^{n} q_i v_i} - \bar{W}^*. \tag{13}$$

Where $\bar{W}^* = \sum_{i=1}^{n} q_i \Delta_i^{(t,t+1)}(1)$ is the average income growth and can be interpreted as the growth every initial quantile group would experience in case of equal growth. In eq. (13) we introduce a general family of aggregate measures of progressivity of growth. Specific scalar measures can be obtained by defining the functional form of the social weight v_i. G measures the relative distance between the effective process of growth and an hypothetical proportional process.[15] When $G > 0$ we are in presence of a progressive growth process, that is growth is concentrated more among individuals ranked lower in the initial distribution of income; $G < 0$ means that the growth process is regressive, that is income growth is concentrated more among the initially richer individuals; $G = 0$ means that the social evaluator is indifferent between the observed growth pattern and a neutral growth pattern.

Thus, this index can be interpreted as the gain (loss) in the social evaluation of growth due to its progressivity (regressivity).[16]

3.2.2 Horizontal inequality-adjusted growth

In this section we propose a family of scalar measures of growth consistent with the horizontal equality principle in axiom 3. We consider social evaluation functions endorsing aversion to the horizontal inequality of growth, hence satisfying axioms 0 and 3 ($v_i(p) \geq 0$ and $v_i'(p) \leq 0 \forall i$ and$\forall p$). Further by imposing pro-poorness neutrality (i.e. $v_i(p) = v_{i+1}(p) \forall i = 1, ...n$), a general index of horizontal inequality adjusted growth can be obtained by normalizing eq. (1) as follows:

$$W^{HE} = \sum_{i=1}^{n} q_i W_i^{HE}. \tag{14}$$

where $W_i^{HE} = \int_0^1 v_i(p)\, \delta_{Ai}^{(t,t+1)}(p)\, dp \ \forall i = 1, ..., n$ with $\int_0^1 v_i(p)\, dp = 1$ measure the weighted average of the growth experienced by the individuals belonging to the group i, where individuals experiencing lower growth get more weight than those experiencing higher growth. W_i^{HE} accounts for the cost of the inequality of growth among equal individuals. Aggregating this measure across initial groups we get W^{HE}: the social horizontal inequality adjusted measure of growth. By ranking growth processes using the index reported in eq. (14) one takes into account the trade-off between the extent of income change and the extent of the inequality of growth. Also in this case, specific measures of horizontal inequality adjusted growth can be obtained by specifying the functional form of the social weight $v_i(p)$.

[15] A special case of this measure is provided by Jenkins and Van Kerm (2011; 2006) and Van Kerm (2006).
[16] See Demuynck and Van de gaer (2012) for alternative scalar measures to evaluate growth.

4 An empirical illustration

4.1 Data

In this section we propose an empirical analysis of the distributional changes that took place in Italy during the last decade. In addition to show its empirical implementation, we use our framework to investigate the (very early) distributional impact of the economic crisis that took place in Italy in 2008 and is still going on.

We use the panel component of the "Survey on Household Income and Wealth" (SHIW). The SHIW is realized by the Bank of Italy every two years and is based on a representative sample of the Italian resident population. In particular, we consider eight years, from 2002 to 2010, and we divide the period in two time spells: 2002-2006 and 2006-2010. Hence, in order to grasp some information about the distributional impact of the crisis, we compare the growth process 2006-2010, encompassing the first phase of the economic crisis, to the immediately preceding 2002-2006 growth process. The unit of observation is the household, defined as all persons sharing the same dwelling. The variable of analysis is the household equivalent income, expressed in terms of 2010 euro.[17] Income includes all household earnings, transfers, pensions, and capital incomes, net of taxes and social security contributions. For each wave, we drop the richest and poorest 1% of the households in order to avoid outliers. We then obtain two samples of size: 2549 households for the period 2002/06 and 3384 for the period 2006/10.

We partition the initial distributions of income - 2002 for the first process and 2006 for the second - into 20 quantile groups, which we need in order to analyze the pro-poorness of growth.[18] Each initial quantile group is then partitioned into quintiles of income change, which we need in order to analyze the horizontal incidence of growth.

All our estimates are computed using sample weights. We give each household the sample weight corresponding to the sampling in the first wave of the survey in our analysis (2002). The standard errors of our estimates are obtained using the non-parametric block bootstrap procedure described by Cameron and Trivedi (2010), which accounts for the dependence structure of our observations.

The yearly overall mean income growth rate computed for the panel component in the first period is 1,78%; it is 0,034% in the second (see Table 1A in the appendix).

4.2 Results

We now turn to show the result of the tests presented in section 3.

The dominance conditions from proposition 1 to proposition 5 are represented by sequential tests. Hence, given that we have divided the initial distribution of income into 20 quantile groups, we need to perform 20 tests for each of the propositions to prove the existence of a possible dominance between the growth processes compared. For the sake of brevity, the detailed results for every step are gathered in the online empirical appendix. Here, instead, we report a summary table (Table 1) indicating whether the dominance stated in each proposition holds and its sign.

[17] We use as equivalence scale the square root of the household size.

[18] The choice of the number of quantile groups is dictated by the trade-off between the detail of information and the statistical reliability of the estimates. We consider only 20 quantile groups in order to have sufficient observations within each quantile group to obtain statisitcally reliable estimates of our measure. 20 is the maximum number of quantile groups that allow us to have groups whit a sample size sufficient for applying the bootstrap procedure. Our results are, in general, robust to the choice of a smaller number of quantile groups.

We start from proposition 1, where only the size of growth and its direction (positive vs. negative growth) matter. As reported in Table 1, although we compare a period of crisis with a period of growth, the two processes cannot be ordered. This happens because the condition imposed in this proposition is quite strong; it requires a first order dominance of the income change experienced by each individual within each initial quantile group. The impossibility of ordering the two processes derives from the fact that there exists first order dominance, statistically significant, of the 2002/06 on the 2006/10 for many but not all the steps, which is the condition required to validate the test (see Table 1B in the online empirical appendix). For example, the 2006/10 growth process dominates the 2002/06 for the first two quantiles within the initial group 19 and this dominance is statistically significant. We bump into the same inconclusiveness when we assume that, in this evaluation, priority is given to the growth experienced by those individuals initially ranked lowest, as in proposition 2 (see Table 2B in the online empirical appendix). According to this proposition, in fact, we are again unable to order the two growth processes.

A different conclusion, instead, can be obtained, by imposing the more stringent requirement of diminishing pro-poor growth and, which is the case of proposition 3. When we apply the condition characterized in this proposition, in fact, we obtain that the first process dominates the second with statistical significance. In fact, the existence of first order dominance of the 2002/06 over the 2006/10 growth process can be proved at every step of the sequential aggregation (see Table 3B in the online empirical appendix).[19] Hence, the 2002/06 episode can be considered as more socially desirable than the 2006/10 when, in addition to the extent of growth, its "non-anonymous" diminishing pro-poorness plays a role in the evaluation procedure.

We then move to compare the two processes on the base of a social preference encompassing only a concern toward the direction and the horizontal inequality of growth. This is the case of the test presented in proposition 4. This test deals with the comparison of cumulated distributions of income change between the 2002/06 and 2006/10, for each of the 20 initial quantile groups. However, these cumulated distributions overlap in some of the steps of the proposition, with conflicting divergences that are statistical significant.[20] This makes it impossible to establish a clear rank between the 2002/06 and the 2006/10 (see Table 4B in the online empirical appendix).

By contrast, a clear cut picture can be obtained looking at the results of proposition 5. In fact, the first growth process arises to be, with statistical significance, more socially preferable than the second, implying that the non-anonymous income transformation process is more desirable for the first four-year period, when concerns for the extent of growth and for both vertical and horizontal incidence are introduced.

The conditions applied till now are aimed at capturing and explaining both aspects of the redistributive effect of growth, that is its vertical and horizontal inequality. Although these conditions are quite restrictive, the 2002/06 growth process arises to dominate the 2006/10. It is possible to obtain further support to this result by applying the less restrictive dominances presented in proposition 6, 7 and 8. They are derived imposing horizontal inequality neutrality, thus the only concerns are the extent of growth and its vertical redistribution impact, according to the non-anonymous approach. The results are reported in Table 1 and in Figure 1 (a) and Figure 2 (a) (see also Table 6B, 7B, 8B in the online empirical appendix).

place table 1 here

[19] Although it is not significant for some of the steps, the test is still validated. In fact, recall that in order to prove the dominance we need to verify the existence of an inequality, but not a strict inequality, at every step.

[20] See for example the initial quintile group 18.

Figure 1 (a) contains the curves obtained applying proposition 6. They are constructed by plotting the level of income change experienced by the individuals belonging to each of the 20 initial quantile groups, therefore they can be considered as the absolute version of the non-anonymous GIC (or absolute "mobility profiles" according to Van Kerm, 2009 and Jenkins and Van Kerm, 2011). Although we cannot state that a clear dominance exists between the two processes since they overlap at the initial quintile group 19 and this divergence is statistically significant, some interesting features arise. First, looking at both the overall processes, it is straightforward to notice that the non-anonymous growth dynamics under scrutiny are pro-poor; in fact, the two absolute naGICs are characterized by the same decreasing pattern implying that growth favors more the initially poorest individuals in both periods considered. Second, the curve representing the 2002/06 lies almost always above the curve representing the 2006/10, however there is one initial quantile group in the upper part of the distribution, namely the 19, where the 2006/10 dominates the 2002/06. Hence, from a comparative point of view it is clear what part of the distribution has been affected less by the crisis: it is a rich quantile group (group 19) to lose less under the 2006/10 than the 2002/06.

Place figure 1 panel (a) and panel (b) here
Place figure 2 panel (a) and panel (b) here

In order to grasp the relevance of adopting our framework as a complementary tool to the standard analysis of growth, it is interesting to compare the curves in Proposition 6 which are absolute version of the non-anonymous growth incidence curve, with the absolute version of the GIC reported in Figure 1(b). The divergence between our approach and the standard approach is striking. First the absolute GIC for the first process (Figure 1 (b)) lies always above the second process implying that there is a clear dominance of the 2002/06 over the 2006/10 growth process. Second, the trend shown by each single curve is very different form that arising in Figure 1 (a), in particular for the first process. Notice, in fact, that when we adopt an anonymous approach, every percentile of the initial distribution experiences a positive change in the level of income, but this change appears strongly regressive. Whereas, under the second process, the trend is neither progressive nor regressive and the most part of the distribution, in particular the poorest and the richest quantiles, experiences negative growth. The implications in terms of the effect of the crisis seem also to be reverted by this graph. In fact, the divergence between the two processes is impressive for the richest individuals. Hence, the employment of the non-anonymous approach to evaluate growth may be a useful complement to the standard approach. As shown clearly for the first process, an absolute GIC with a regressive pattern may be associated to an absolute naGIC with a progressive one, which is realistic and witnesses how the focus of the two approaches differs: the GIC is concerned with the income growth of each part of the distribution, independently of the individuals sitting in that part, whereas the naGIC is concerned with the income dynamic of each individual in a distribution (or income mobility).

We now turn to apply the dominance condition of proposition 7, which is based on the size and on the pro-poorness features of growth. Figure 2 (a) reports the curves derived from proposition 7, they plot the cumulated sum of the income change experienced by each initial quantile (see also Table 7B in the online empirical appendix). The comparison between the two dynamics is now very clear. Both curves increase steeply up to the 15th quantile, becoming decreasing for the richest quantiles. However, the curve relative to the first process is always higher than the second implying that the 2002/06 dominates the 2006/10 and the dominance is statistically significant, when the social evaluation of growth is based on efficiency and pro-poorness features of growth. They also show a concave shape which appears to be more pronounced in the second process, implying that this process might have been more effective in terms of vertical impact.

Again the difference between our approach and the standard anonymous one appears from the comparison of the curves reported in Figure 2 (a) with the curves reported in Figure 2 (b). The sign of the dominance is the same (2002/06 dominates the 2006/10) however, the specific shape of each curve is quite different, in particular for the first growth episode.[21]

The results obtained implementing proposition 6 and 7 are confirmed by the employment of the dominance condition in proposition 8 (see Table 8B in the online empirical appendix) and by the adoption of the scalar measures defined in eq. (12) and (13). The two indices give us a clear information about the complete ranking and the features of each income dynamic.[22] In particular, while the index in eq. (12), the progressivity-sensitive measure of growth, reflects both size and vertical redistributive effects of growth, the index in eq. (13) is only concerned with the latter component, hence they may generate opposing ranking. Their results are reported in Table 2. As concerned the progressivity-sensitive index of growth, it arises to be higher for the first growth process than for the second. The complete ranking provided clearly indicates that the 2002/06 growth process dominates the 2006/10, when considering both the extent of growth and its vertical impact. By contrast, the complete ranking provided by eq. (13) indicates that the second process dominates the first one if only the vertical redistributive effect is taken into consideration. In summary, the larger value of W^* for the first period reflects the grater average growth in this period which compensates for the lower return to progressivity. Whereas, the higher value of G for the second period states that although a lower growth in level, the second process shows higher degree of return to progressivity (in absolute terms). However, note that the order of the dominance between the two processes, provided by G, is reverted when relative changes, instead of absolute changes, are considered (Table 2).

We complete our analysis on the evaluation of the Italian growth by computing the measure provided in eq. (14), which gives the extent of growth corrected for the cost of the horizontal inequality of growth. The value of this index for both processes and for both absolute and relative growth are reported in Table 2 and they confirm the dominance of the 2002/06 on the 2006/2010 growth process.[23] The scalar nature of the index allows to appreciate also the magnitude of the dominance.

place table 2 here

In sum, the results of the implementation of the partial and complete dominance conditions introduced in section 3 - with the only exception of the index of progressivity - witness the severe impact of the crisis according to different features of growth.[24] It can be argued that the increase in unemployment caused by the crisis is among the main determinants of the negative performance of the growth process in 2006/10 with respect to the growth process in 2002/06. In particular, the

[21] For an additional comparison see Figure 1A and 3A reporting the relative version of the curve in proposition 6 and 7, and Figure 2A and 4A reporting the relative version of the GIC and cumulative GIC.

[22] We adopt the following parametrization: $v_i = 2\left(1 - \frac{i}{n}\right)$.

[23] We adopt the following parametrization: $v\left(p\right) = 2\left(1 - p\right)$.

[24] These results are robust to the partition of the initial distributions into different and smaller numbers of initial quintiles. This robustness, however, tend to disappear with the partition of the population into a larger number of initial income quintiles. This is mostly due to the small size of the sample within each quintile.

flexibility introduced in the Italian labor market at the end of the 90s could have played a relevant role. In fact, job flexibility generates more opportunities during periods of positive growth, as also shown in our analysis for the growth episode 2002/06, increasing the income of most individuals in the population. This trend is then reverted with the advent of the crisis. In fact, the workers who benefit from flexibility, at the beginning of the decade, are the same to be exposed to the risk of being fired and were effectively fired because of the crisis, with the consequent huge fall in their disposable income.[25]

4.3 Robustness check to measurement error

In the last decades, the literature has shown that the reliability of mobility measurement analyses greatly depend on the extent of the measurement error, when the results of these analyses are based on incomes drawn from household surveys. In particular, it has been argued that a decreasing slope in a curve plotting income change against base year income may in part be due to a measurement error in base year income, generating a spurious negative relation between observed year income and measured income change (see Jarvis and Jenkins 1998; Fields et al. 2003). This is the case of classical measurement error, that is, errors uncorrelated with the true value and over time. Before checking the robustness of our results to measurement error, a few considerations are in order.

First, dropping the 1% top/bottom and using quintiles in part reduces the impact of this problem by smoothing out measurement error. Second, our main focus is the comparisons of non-anonymous growth processes over time. Hence, given that there have not been considerable changes in the design of the SHIW in the years considered, a bias in the extent of non-anonymous growth may be consistent with no bias in the estimated difference between two growth processes.[26]

In order to check the robustness of our results to measurement error we have applied the method proposed by Fields et al. (2003).[27] We have used a OLS regression to estimate the relationship between initial income and income changes for both periods considered. The estimated coefficient represents an approximation of the slope of the curves plotted in Figure 1(a). The results of the regressions for the two periods considered are reported in Table 2A in the appendix. As expected the coefficients are negative for both periods, witnessing that growth has been progressive. To verify that the negativity of the coefficient is not simply the result of a measurement error in the base year observed income, we have proceeded by computing the lower bound level of the measurement error for different values of the parameters involved on the estimation of this value, that is, the correlation with the true income and the serial correlation (see Fields et al. 2003 for the analytical details). The results are reported in Table 3A in the appendix. In 2002-2006 for the divergence in the value of the relationship between initial income and income change (a divergence in the slope of the curve in Figure 6) to have taken place, the variance of measurement error would need to be at least 27 to 36 percent of the variance of true incomes. In 2006-2010 it should be between 20 and 34 percent. In order to quantify the extent of measurement error in our data, we use as a basis for

[25]See on this Boeri and Garibaldi (2007).

[26]It is compelling to underline that non-classical measurement error may also occur. However, Gottschalk and Huynh (2010), Dragoset and Fields (2006), Fields et al. (2003) found that the bias in the estimations of mobility due to non-classical measurement error may be negligible. Clearly these results were obtained using surveys different from the one we are using here. However, they made a more general argument that can be extended to other surveys. In particular, they argued that the bias arising from the variance inflation aspect of measurement error and the bias arising from aspects such as mean reversion in errors cancel out each other. As a result, a bias in the measurement of mobility may be of little importance.

[27]We thank an anonymous referee for this suggestion.

comparison the same value considered by Fields et al. (2003), which derives from two validation studies of U.S. earnings data which compare the Current Population Survey and the Panel Study of Income Dynamics to Social Security or firm records. It amounts to about ten percent of true income. Thus, comparing this value to the value of the lower bound of the measurement error for the Italian case, we can state that the measurement error characterizing the data we employ is not so quantitatively important as to reverse the conclusion of our analysis.

5 Conclusions

Recent contributions in the economic literature show the need to modify the standard frameworks for measuring the distributional effect of growth, in order to take into account the identity of individuals and their reshuffling among income classes. In this work we have provided a normative approach to rank growth processes when these further aspects are a matter of concern. We have adopted a bi-dimensional framework, where the two dimensions are respectively the initial rank of individuals and their income transformation. We have proposed to aggregate these information according to a rank dependent approach, which makes it possible to account for the size of growth and its vertical and horizontal impacts. We have provided partial dominance conditions for ordering growth processes and we have shown how these conditions relate to na-GIC. Moreover, we have proposed three classes of indices: the first aimed at capturing at the same time the extent of growth and its vertical redistributive effect; the second aimed at isolating the latter component; the third aimed at evaluating the extent of growth corrected for the cost of the horizontal inequality in the distribution of growth. We have then applied our framework for the assessment of the distributional impact of growth in Italy in the last decade, with a particular focus on the effect of the recent economic crisis. Specifically, we have considered four of the most recent available waves to compare the 2002-2006 growth episode against the 2006-2010. The results obtained in our analysis allow us to argue that the economic crisis has been a negative event not only from a macro-perspective, but it has also let to a worse performance of the Italian income dynamic from a micro-perspective.

Our empirical illustration witnesses that the measurement framework we have introduced can usefully complement existing tools for the evaluation of growth. In particular, it stems out that further research for the understanding of the underlying relationship between anonymous and non-anonymous growth (or income mobility) seems to be a promising path.[28]

Another field of application of our framework is the analysis of tax-benefit reforms. Their distributional aspects are, in general, evaluated in terms of income inequality reduction. At the same time, we argue that the evaluation and comparison of these reforms in terms of their impact on the individual income trajectories, with the help of the tools developed in this paper, might be potentially relevant.

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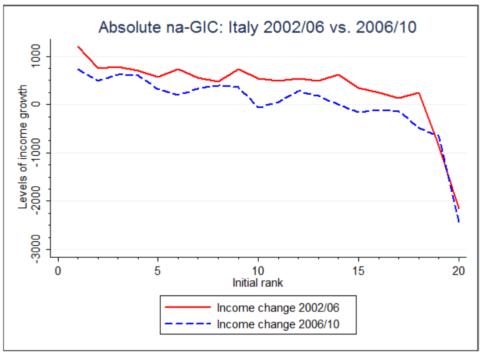
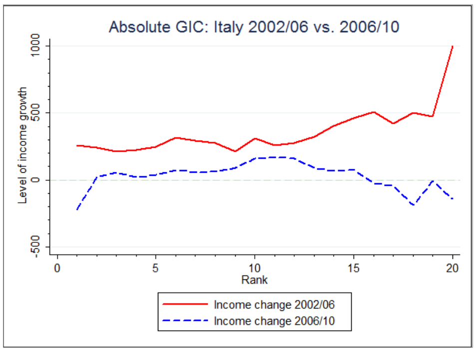
Figure 1. Panel (a) Proposition 6: Italy, 2002/06 vs. 2006/10 and panel (b) Absolute GIC: Italy, 2002/06 vs. 2006/10.

Panel (a)



Source: Authors'calculation from SHIW.

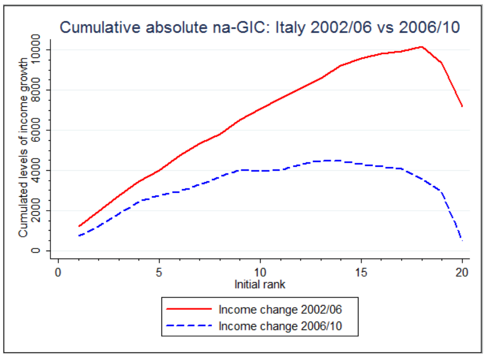
Panel (b)



Source: Authors'calculation from SHIW

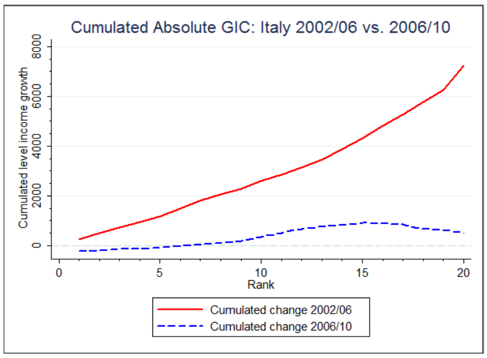
Figure 2. Panel (a) Proposition 7: Italy, 2002/06 vs. 2006/10 and panel (b) Cumulated Absolute GIC: Italy, 2002/06 vs. 2006/10.

Panel (a)



Source: Authors'calculation from SHIW.

Panel (b)



Source: Authors'calculation from SHIW.

Table 1. Results of the tests presented in Proposition 1, 2, 3, 4, 5, 6, 7 and 8: 2002/06 vs. 2006/10.

Proposition	2002/06 vs. 2006/10	Proposition	2002/06 vs. 2006/10
1	No	6	No
2	No	7	>
3	>	8	>
4	No		
5	>		

Source: Authors'calculation from SHIW.

Table 2. Indices of growth: Italy 2002/06 and 2006/10.

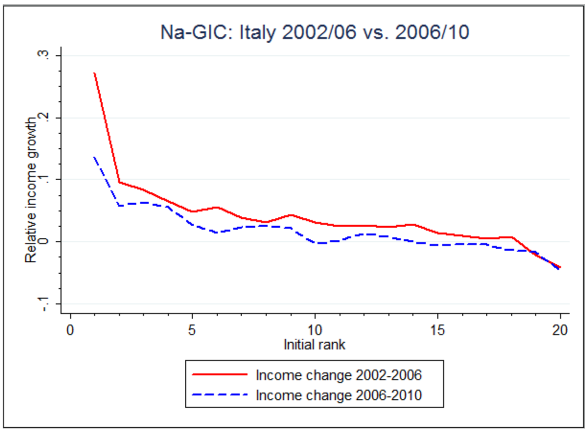
Period	Progressivity sensitive growth (W^*)	Return to progressivity (G)	Mean income growth (γ_a)	Inequality adjusted growth
Absolute incom'e growth				
2002/06	658.3697	299.6799	359.873	66.5636
2006/10	333.1501	308.5492	6.98	-229.9324
Relative income growth (%)				
Period	Progressivity sensitive growth (W^*)	Return to progressivity (G)	Mean income growth (γ_r)	Inequality adjusted growth
2002/06	7.052	2.86	1.78	2.60
2006/10	3.731	1.967	0.034	0.38

Source: Authors'calculation from SHIW.

Note: $\gamma_r = \frac{\mu(y_{t+1})-\mu(y_t)}{\mu(y_t)}$ and $\gamma_a = \mu(y_{t+1}) - \mu(y_t)$.

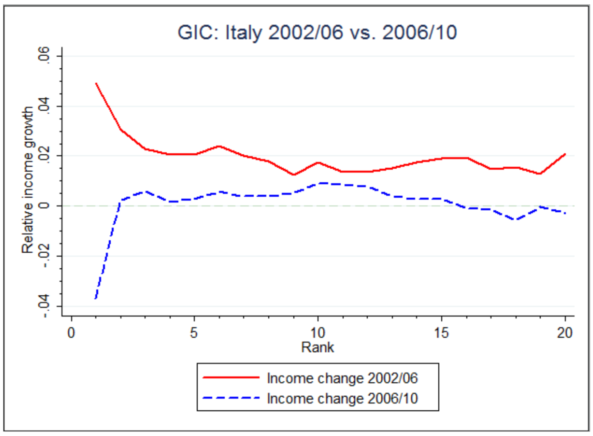
Empirical Appendix

Figure 1A. Non-anonymous GIC for Italy, 2002/06 vs. 2006/10.



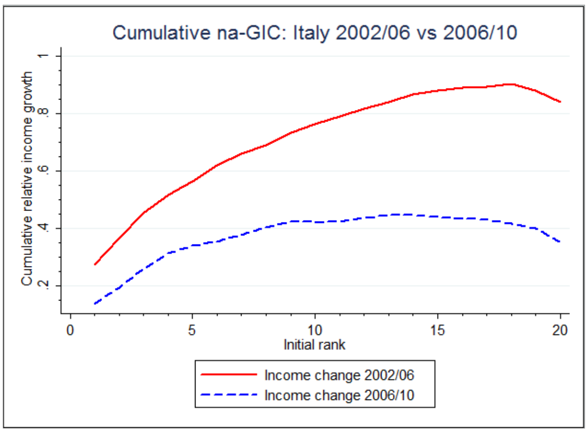
Source: Authors' calculation from SHIW

Figure 2A. GIC for Italy, 2002/06 vs. 2006/10.



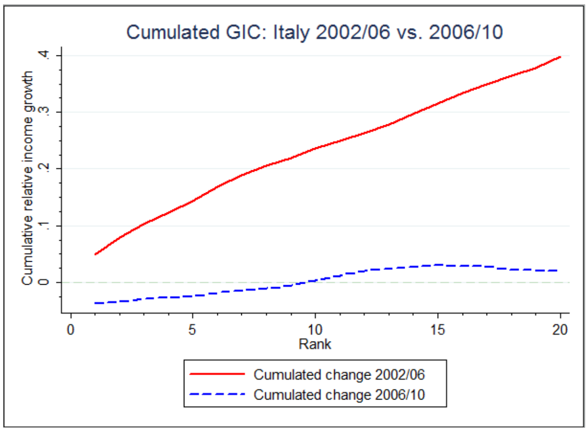
Source: Authors' calculation from SHIW

Figure 3A. Cumulative non- anonymous GIC for Italy, 2002/06 vs. 2006/10.



Source: Authors' calculation from SHIW

Figure 4A. Cumulative GIC for Italy, 2002/06 vs. 2006/10.



Source: Authors' calculation from SHIW

Table 1A. Descriptive Statistics: Italy 2002/06 and 2006/10.

Period	Sample size	Absolute income growth	Relative income growth (%)
2002/2006	2549	359.6225	1.78
2006/2010	3384	6.98	0.034

Source: Authors'calculation from SHIW.

Table 2A. Coefficients from a regression of income change on base year income

Dependent Variable	Income change	
2002-2006	-.2550***	Pro-poor
2006-2010	-.23086***	Pro-poor

Notes: *** means statistically significant at 99%;
Source: Authors' calculation from SHIW

Table 3A

Correlation with true income	Serial correlation	2002-2006	2006-20010
0	0	.3422916	.3001516
0	0.1	.3953605	.3450079
0	0.2	.4679044	.3450079
-0.1	0	.2772562	.2431228
-0.1	0.1	.320242	.2794564
-0.1	0.2	.3790026	.3285578
-0.2	0	.2190666	.192097
-0.2	0.1	.2530307	.220805
-0.2	0.2	.2994588	.2596012

Source: Authors' calculation from SHIW

Theoretical Appendix

Proof of Proposition 1

We want to find a necessary and sufficient condition for

$$\Delta W=\sum_{i=1}^{n}\int_{0}^{1}v_i\left(p\right)\left[\delta_{Ai}^{(t,t+1)}\left(p\right)-\delta_{Bi}^{(t,t+1)}\left(p\right)\right]dp\geq 0,\ \forall W\in\mathbf{W} \tag{1}$$

Sufficiency clearly derives from the fact that since $v_i\left(p\right)\geq 0$, $\forall p\in[0,1]$ and $\forall i=1,2,...,n$, $\delta_{Ai}^{(t,t+1)}\left(p\right)-\delta_{Ai}^{(t,t+1)}\left(p\right)\geq 0$, $\forall p\in[0,1]$ and $\forall i=1,2,...,n$, implies $\Delta W\geq 0$.

For the necessity, suppose for a contradiction that $\Delta W\geq 0$, $\forall W\in\mathbf{W}_1$, but there is a quantile group $h\in\{1,...,n\}$ and an interval $I\equiv[a,b]\subseteq[0,1]$ such that $\delta_{Ah}^{(t,t+1)}\left(p\right)-\delta_{Bh}^{(t,t+1)}\left(p\right)<0,\forall p\in I$. Now select a set of functions $\{v_i\left(p\right)\}_{i\in\{1,...,m\}}$ such that $v_i\left(p\right)\searrow 0$, $\forall i\neq h$ and $v_h\left(p\right)\searrow 0$, $\forall p\in[0,1]\setminus I$. in this case ΔW would reduce to $\int_{a}^{b}v_h\left(p\right)\left[\delta_{Ah}^{(t,t+1)}\left(p\right)-\delta_{Bh}^{(t,t+1)}\left(p\right)\right]dp<0$, a contradiction. **QED**

Proof of Proposition 2

Before proving this Proposition we state Abel's Lemma[1]

Abel's Lemma. If $v_1\geq...\geq v_i\geq...\geq v_n\geq 0$, a sufficient condition for $\sum_{i=1}^{n}v_iw_i\geq 0$ is $\sum_{i=1}^{j}w_i\geq 0\forall j=1,...,n$. If $v_1\leq...\leq v_i\leq...\leq v_n\leq 0$, the same condition is sufficient for $\sum_{i=1}^{n}v_iw_i\leq 0$.

We now turn to the proof of the proposition.

We now want to find a necessary and sufficient condition for

$$\Delta W=\sum_{i=1}^{n}\int_{0}^{1}v_i\left(p\right)\left[\delta_{Ai}^{(t,t+1)}\left(p\right)-\delta_{Bi}^{(t,t+1)}\left(p\right)\right]dp\geq 0,\ \forall W\in\mathbf{W}_1 \tag{2}$$

Sufficiency can be shown as follows. First, reverse the order of integration and summation, such that

$$\Delta W=\int_{0}^{1}\sum_{i=1}^{n}v_i\left(p\right)\left[\delta_{Ai}^{(t,t+1)}\left(p\right)-\delta_{Bi}^{(t,t+1)}\left(p\right)\right]dp\geq 0 \tag{3}$$

Letting $S_i\left(p\right)=\delta_{Ai}^{(t,t+1)}\left(p\right)-\delta_{Bi}^{(t,t+1)}\left(p\right)$ and rewriting (3):

$$\Delta W=\int_{0}^{1}\sum_{i=1}^{n}v_i\left(p\right)S_i\left(p\right)dp\geq 0 \tag{4}$$

[1] See Jenkins and Lambert (1993) for a formal proof.

Empirical Appendix

Table 1B. Results of each stage of the test in Proposition 1: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>**	>	>***	>***	>***
2	>***	>***	>***	>***	>
3	>**	>***	>**	>***	<
4	>*	>**	>	<	>
5	<	>	>***	>***	>
6	>**	>***	>***	>***	>**
7	>*	>***	>***	>***	<
8	>	>	>***	>	<
9	<	>	>	>***	<**
10	>***	>***	>***	>***	>*
11	>	>**	>	>	>*
12	>	>***	>***	>***	>
13	>	>**	>***	>	>
14	<	<	>***	>***	>***
15	>	>	>**	>***	>***
16	>	>***	>***	>***	>
17	>	<	>**	>**	>
18	<**	>***	>***	>***	>**
19	<***	<**	>***	<	>
20	<	>	>*	>***	>***

Note: authors' calculation, based on the Italian "Survey on Household Income and Wealth". > means the 2002/06 process dominates the 2006/10;c< the 2002/06 process is dominated by the 2006/10. *** means statistically significant at 99%; ** statistically significant at 95%; * statistically significant at 90%.
Source: Authors' calculation from SHIW

Table 2B. Results at each stage of the test in Proposition 2: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>***	>	>***	>***	>***
2	>***	>***	>***	>***	>
3	>***	>***	>***	>***	>
4	>***	>***	>***	>***	>***
5	>***	>***	>***	>***	>***
6	>***	>***	>***	>***	>***
7	>***	>***	>***	>***	<
8	>***	>***	>***	>	<
9	>***	>***	>***	>	<
10	>***	>***	>***	>	<
11	>***	>***	>***	>***	>
12	>***	>***	>***	>	>
13	>***	>	>***	>***	>
14	>***	>	>***	>	>***
15	>***	>	>***	>	>
16	>***	>*	>***	>	>**
17	>**	>	>***	>	>
18	>***	>	>***	>**	>
19	>	>	>***	>	>
20	>	>	>***	>	>

Notes: See Notes on Table 1B.

Table 3B. Results of each stage of the test in Proposition 3: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>***	>***	>	>***	>***
2	>*	>*	>**	>***	>**
3	>***	>***	>***	>***	>*
4	>***	>***	>***	>***	>
5	>***	>***	>***	>***	>*
6	>***	>***	>***	>***	>***
7	>***	>***	>***	>***	>
8	>***	>***	>***	>	>
9	>***	>***	>***	>	>
10	>***	>***	>***	>*	>
11	>***	>***	>***	>*	>
12	>***	>***	>***	>***	>*
13	>***	>***	>***	>***	>*
14	>***	>***	>***	>**	>**
15	>***	>***	>***	>*	>
16	>***	>***	>***	>**	>*
17	>***	>***	>***	>***	>
18	>***	>***	>***	>**	>
19	>***	>***	>***	>*	>**
20	>***	>***	>***	>	>

Notes: See Notes on Table 1B.

Table 4B. Results at each stage of the test in Proposition 4: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>**	>***	>***	>***	>***
2	>***	>***	>***	>***	>***
3	>**	>***	>***	>***	>**
4	>*	>***	>***	>**	>
5	<	>*	>***	>***	>***
6	>**	>	>	>*	>**
7	>*	>***	>***	>***	>***
8	>	>***	>***	>***	>***
9	<	>	>**	>***	>***
10	>***	>***	>***	>***	>***
11	>	>*	>***	>***	>***
12	>	>***	>***	>***	>***
13	>	>***	>***	>***	>***
14	<	>*	>***	>***	>***
15	>	>**	>***	>***	>***
16	>	>***	>***	>***	>***
17	>	>*	>**	>***	>***
18	<**	>***	>***	>***	>***
19	<***	<	<	<	<**
20	<	>*	>***	>***	>***

Notes: See Notes on Table 1B.

Table 5B. Results at each stage of the test in Proposition 5: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>**	>***	>***	>***	>***
2	>***	>***	>**	>**	>***
3	>***	>***	>***	>***	>***
4	>*	>***	>***	>***	>***
5	>***	>**	>***	>**	>***
6	>***	>***	>***	>***	>***
7	>***	>***	>***	>*	>***
8	>***	>*	>***	>***	>***
9	>***	>***	>**	>***	>***
10	>***	>***	>***	>***	>*
11	>***	>*	>***	>***	>***
12	>***	>***	>***	>***	>***
13	>***	>***	>**	>**	>***
14	>**	>*	>**	>	>*
15	>***	>***	>***	>***	>***
16	>***	>***	>***	>***	>*
17	>**	>***	>***	>***	>***
18	>***	>**	>***	>*	>*
19	>**	>***	>***	>***	>***
20	>***	>***	>***	>***	>***

Notes: See Notes on Table 1B.

Table 6B. Results at each stage of the test in Proposition 6: 2002/06 vs. 2006/10.

Initial quantile group	Absolute na-gic
1	>***
2	>***
3	>**
4	>***
5	>*
6	>
7	>***
8	>***
9	>***
10	<***
11	>***
12	>***
13	>***
14	>***
15	>***
16	>***
17	>***
18	>***
19	<**
20	>*

Notes: See Notes on Table 1B.

Table 7B. Results at each stage of the test in Proposition 7: 2002/06 vs. 2006/10.

Initial quantile group	Absolute na-gic
1	>***
2	>***
3	>***
4	>***
5	>***
6	>***
7	>***
8	>***
9	>***
10	>*
11	>***
12	>***
13	>***
14	>*
15	>***
16	>*
17	>***
18	>*
19	>***
20	>***

Notes: See Notes on Table 1B.

Table 8B. Results at each stage of the test in Proposition 8: 2002/06 vs. 2006/10.

Initial quantile group	Absolute na-gic
1	>***
2	>***
3	>***
4	>***
5	>***
6	>***
7	>***
8	>***
9	>***
10	>***
11	>***
12	>***
13	>***
14	>***
15	>***
16	>***
17	>***
18	>***
19	>***
20	>***

Notes: See Notes on Table 1B.

Since $v_i(p) \geq v_{i+1}(p) \geq 0$, $\forall i = 1, ..., n-1$ and $\forall p \in [0,1]$, we can apply the Abel's Lemma and obtain that $\sum_{i=1}^{n} v_i(p) S_i(p) \geq 0$ if $\sum_{i=1}^{k} S_i(p) \geq 0, \forall k = 1, ..., n$ and $\forall p \in [0,1]$. It follows that $\sum_{i=1}^{n} v_i(p) S_i(p) \geq 0$, $\forall p \in [0,1]$, implies that, integrating with respect to p, $\int_0^1 \sum_{i=1}^{n} v_i(p) S_i(p)\, dp \geq 0$.

For the necessity, suppose for a contradiction that $\Delta W \geq 0$, $\forall W \in \mathbf{W}_{12}$, but there is an initial quantile $h \in \{1, ..., n\}$ and an interval $I \equiv [a,b] \subseteq [0,1]$ such that $\sum_{i=1}^{h} S_i(p) < 0, \forall p \in I$. Now, applying Abel's Lemma, there exists a set of functions $\{v_i(p) \geq 0\} : [0,1] \longrightarrow \Re_+$, $i = 1, ..., n$, such that $\sum_{i=1}^{n} v_i(p) S_i(p) < 0$, $\forall p \in I$. Writing $\sum_{i=1}^{n} v_i(p) S_i(p) = T(p)$, $\Delta W \left(\Delta^{(t,t+1)} \mid y_t\right)$ reduces to $\int_0^1 T(p)\, dp$, where $T(p) < 0$, $\forall p \in I$. Selecting a set of function $T(p)$, such that $T(p) \longrightarrow 0$, $\forall p \in [0,1] \setminus I$, ΔW would reduce to $\int_a^b T(p)\, dp < 0$, a contradiction. **QED**

Proof of Proposition 3

We want to find a necessary and sufficient condition for

$$\Delta W = \sum_{i=1}^{n} \int_0^1 v_i(p) \left[\delta_{Ai}^{(t,t+1)}(p) - \delta_{Bi}^{(t,t+1)}(p)\right] dp \geq 0, \forall W \in \mathbf{W}_{12} \tag{5}$$

For the sufficiency, note that if $v_i(p)$ satisfies axiom 0, 1 and 2, we can revert the order of integration and summation and apply Abel's decomposition: $\sum_{i=1}^{n} v_i w_i = v_n \sum_{i=1}^{n} w_i + \sum_{i=1}^{n-1} (v_i - v_{i+1}) \sum_{j=1}^{i} w_j$ to obtain: $\Delta W = \int_0^1 \left[v_n(p) \sum_{i=1}^{n} S_i(p) + \sum_{i=1}^{n-1} (v_i(p) - v_{i+1}(p)) \sum_{j=1}^{i} S_j(p)\right] dp$, where $S_j(p) = \delta_{Aj}^{(t,t+1)}(p) - \delta_{Bj}^{(t,t+1)}(p)$. Let $v_i(p) - v_{i+1}(p) = \omega_i(p)$ and $\sum_{j=1}^{i} S_j(p) = \kappa_i(p)$, by axiom 2 $\omega_i(p) > \omega_{i+1}(p)$, $\forall i = 1, ..., n-1$, $\forall p \in [0,1]$. Now ΔW becomes $\int_0^1 \left[v_n(p) \kappa_n(p) + \sum_{i=1}^{n-1} \omega_i(p) \kappa_i(p)\right] dp$. Given the condition $v_n(p) = 0\ \forall p$, ΔW becomes $\int_0^1 \sum_{i=1}^{n-1} \omega_i(p) \kappa_i(p)\, dp$. We can apply Abel's Lemma to get that $\sum_{i=1}^{n-1} \omega_i(p) \kappa_i(p) \geq 0$ if $\sum_{i=1}^{k} \kappa_i(p) \geq 0$, $\forall k$. Thus $\sum_{i=1}^{k} \sum_{j=1}^{i} \delta_{A_j}^{(t,t+1)}(p) - \delta_{Bj}^{(t,t+1)}(p)$, $\forall k$, $\forall p \in [0,1]$ is sufficient for $\Delta W \geq 0$.

For the necessity, let $T(p) \equiv \sum_{i=1}^{n-1} \omega_i(p)\kappa_i(p)$, we can write the following $\Delta W = \int_0^1 T(p)\,dp$. Suppose that $\Delta W \geq 0$, $\forall W \in \mathbf{W}_{123}$, but $\exists h = 1, ..., n-1$ and $\exists I \equiv [a,b] \subseteq [0,1]$ such that $\sum_{i=1}^{h} \kappa_i(p) < 0$, $\forall p \in I$. Then by Abel's Lemma $\exists$ a set of functions $\omega_i(p) : [0,1] \longrightarrow \Re_+$, $i = 1, ..., n-1$, such that $T(p) < 0$, $\forall p \in I$. We can chose a function $T(p)$ such that $T(p) \longrightarrow 0$, $\forall p \in [0,1] \setminus I$, thus ΔW would reduce to $\int_a^b T(p)\,dp < 0$, a contradiction. **QED**

Proof of Proposition 4

We want to find a necessary and sufficient condition for

$$\Delta W = \sum_{i=1}^{n} \int_0^1 v_i(p) \left[\delta_{Ai}^{(t,t+1)}(p) - \delta_{Bi}^{(t,t+1)}(p)\right] dp \geq 0, \forall W \in \mathbf{W}_3 \tag{6}$$

For the sufficiency, let $S_i(p) = \delta_{Ai}^{(t,t+1)}(p) - \delta_{Bi}^{(t,t+1)}(p)$, we can integrate eq. (6) by parts to get:

$$\Delta W = \sum_{i=1}^{n} \left[v_i(1) \int_0^1 S_i(p)\,dp\right] - \sum_{i=1}^{n} \int_0^1 v_i'(p) \int_0^p S_i(q)\,dqdp \tag{7}$$

It follows that

$$\int_0^p \delta_{Ai}^{(t,t+1)}(q) - \delta_{Bi}^{(t,t+1)}(q)\,dq \geq 0, \forall p \in [0,1], \forall i = 1, ..., n \tag{8}$$

is sufficient for welfare dominance, since by axiom 0 $v_i(p) \geq 0$ eq. (8) implies the positivity of $v_i(1) \int_0^1 S_i(p)\,dp$ and by axiom 3 $v_i'(p) \leq 0$ it implies the negativity of the second term of eq. (7). It follows that $\Delta W \geq 0$.

For the necessity, suppose that $\Delta W \geq 0$, but $\exists h = 1, ..., n$ and $\exists I \equiv [a,b] \subseteq [0,1]$ such that $\int_0^p \delta_{Ah}^{(t,t+1)}(q) - \delta_{Bh}^{(t,t+1)}(q)\,dq < 0$, $\forall p \in I$. Let $T_i(p) = v_i'(p) \int_0^p \delta_{Ai}^{(t,t+1)}(q) - \delta_{Bi}^{(t,t+1)}(q)\,dq$, $\forall i = 1, ..., n$, $\forall p \in [0,1]$. We can chose a set of functions $T_i(p)$ such that $v_i'(p) \longrightarrow 0$, $\forall i \neq h$ and $T(p) \longrightarrow 0$, $\forall p \in [0,1] \setminus I$ and we can chose a combination of $v_i(1)$ and $S_i(p)$ such that $\sum_{i=1}^{n} v_i(1) \int_0^1 S_i(p)\,dp = 0$. Then, ΔWwould reduce to $-\int_a^b T_h(p)\,dp \leq 0$, a contradiction. **QED**

Proof of Proposition 5

We want to find a necessary and sufficient condition for

$$\Delta W = \sum_{i=1}^{n} \int_0^1 v_i(p) \left[\delta_{Ai}^{(t,t+1)}(p) - \delta_{Bi}^{(t,t+1)}(p)\right] dp \geq 0, \ \forall W \in \mathbf{W}_{13^*} \tag{9}$$

Sufficiency can be shown as follows. First, letting $S_i(p) = \delta_{Ai}^{(t,t+1)}(p) - \delta_{Bi}^{(t,t+1)}(p)$ and integrating by parts eq. (9)

$$\Delta W = \sum_{i=1}^{n}\left[v_i(1)\int_0^1 S_i(p)\,dp\right] - \sum_{i=1}^{n}\int_0^1 v_i'(p)\int_0^p S_i(q)\,dq dp \tag{10}$$

reversing the order of integration and summation in the second part of eq. (24):

$$\Delta W = \sum_{i=1}^{n}\left[v_i(1)\int_0^1 S_i(p)\,dp\right] - \int_0^1 \sum_{i=1}^{n} v_i'(p)\int_0^p S_i(q)\,dq dp \tag{11}$$

By axiom 3*, we can apply Abel's Lemma to get that $\sum_{i=1}^{j}\int_0^p S_i(q)\,dq \geq 0, \forall_j = 1,...,n$ and $\forall p \in [0,1]$ implies $-\int_0^1 \sum_{i=1}^{n} v_i'(p)\int_0^p S_i(q)\,dq dp \geq 0$, but given axiom 1, this also implies that $\sum_{i=1}^{n}\left[v_i(1)\int_0^1 S_i(p)\,dp\right] \geq 0$, hence it is sufficient for $\Delta W \geq 0$.

For the necessity, by axiom 0 and application of Abel's decomposition, write eq. (11) as follows:

$$\Delta W = \sum_{i=1}^{n}\left[v_i(1)\int_0^1 S_i(p)\,dp\right] + \int_0^1 \varepsilon_n(p)\sum_{i=1}^{n}\int_0^p S_i(q)\,dq dp + \int_0^1 \sum_{i=1}^{n-1}\omega_i(p)\sum_{j=1}^{i}\int_0^p S_j(q)\,dq dp \tag{12}$$

with $\varepsilon_n(p) = -v_n'(p)\,\forall p$ and $\omega_i(p) = -\left(v_i'(p) - v_{i+1}'(p)\right)\forall_i$ and $\forall p$. Now, suppose for a contradiction that $\Delta W \geq 0$, but $\exists h \in \{1,...,n-1\}$ and $\exists h = n$ and interval $I \equiv [a,b] \subseteq [0,1]$ such that $\sum_{j=1}^{h}\int_0^p S_j(q)\,dq < 0, \forall p \in I$. Now, given that $\{\omega_i(p) \geq 0\}_{i\in\{1,...,n-1\}}$, applying Lemma 1 in Chambaz and Maurin (1998) we will have $\sum_{i=1}^{n-1}\omega_i(p)\left(\sum_{j=1}^{i}\int_0^p S_i(q)\,dq\right) < 0\ \forall p \in I$ and given that $\varepsilon_n(p) \geq 0$, by Lemma 1 in Atkinson and Bourguignon (1987) we will have that $\int_0^1 \varepsilon_n(p)\sum_{j=1}^{n}\int_0^p S_j(q)\,dq <$ 0. Now, Let $R(p) = \sum_{i=1}^{n-1}\omega_i(p)\left(\sum_{j=1}^{i}\int_0^p S_j(q)\,dq\right)$ and $Q(p) = \varepsilon_n(p)\sum_{j=i}^{n}\int_0^p S_j(q)\,dq$, $\Delta W = \sum_{i=1}^{n}\left(v_i(1)\int_0^1 S_i(p)\,dp\right) + \int_0^1 Q(p)\,dp + \int_0^1 R(p)\,dp$. Now choosing $R(p)$ such that $R(p) \longrightarrow 0$ for some $p \in [0,1] \setminus I$, it follows

$$\Delta W = \sum_{i=1}^{n} \left(v_i(1) \int_0^1 S_j(p)\, dp \right) + \int_0^1 Q(p) + \int_a^b R(p)\, dp$$

Given that $\int_a^b R(p)\, dp < 0$ and $\int_0^1 Q(p) < 0$ we can choose a combination of $v_i(1)$ and $\int_0^1 S_j(p)\, dp$ such that $\Delta W < 0$, a desired contradiction. **QED**.

Proof of Proposition 6

We want to find a necessary and sufficient condition for

$$\Delta W = \sum_{i=1}^{n} \int_0^1 v_i(p) \left[\delta_{Ai}^{(t,t+1)}(p) - \delta_{Bi}^{(t,t+1)}(p) \right] dp \geq 0,\ \forall W \in \mathbf{W}_4 \tag{13}$$

For the sufficiency, by axiom 4 $v_i(p) = \beta_i$, $\forall p \in [0,1]$ and $\forall i = 1,2,...,n$, therefore we can write eq. (13) as follows:

$$\Delta W = \sum_{i=1}^{n} \beta_i \int_0^1 \left[\delta_{Ai}^{(t,t+1)}(p) - \delta_{Bi}^{(t,t+1)}(p) \right] dp =$$

$$= \sum_{i=1}^{n} \beta_i \left[\Delta_{Ai}^{(t,t+1)}(1) - \Delta_{Bi}^{(t,t+1)}(1) \right] \geq 0 \tag{14}$$

by axiom 0 $v_i(p) = \beta_i \geq 0$, $\forall p \in [0,1]$ and $\forall i = 1,2,...,n$, $\Delta W \geq 0$ if $\Delta_{Ai}^{(t,t+1)}(1) - \Delta_{Bi}^{(t,t+1)}(1) \geq 0$, $\forall i = 1,...,n$.

For the necessity, suppose that

$$\Delta W = \sum_{i=1}^{n} \beta_i \left[\Delta_{Ai}^{(t,t+1)}(1) - \Delta_{Bi}^{(t,t+1)}(1) \right] \geq 0$$

but $\exists k = 1,...,n$ such that $\Delta_{Ak}^{(t,t+1)}(1) < \Delta_{Bk}^{(t,t+1)}(1)$. We can choose a set of numbers $\{\beta_i\}_{i=1,...,n}$ such that $\beta_i \searrow 0$, $\forall i \neq k$. ΔW would reduce to $\beta_k \left(\Delta_{Ak}^{(t,t+1)}(1) - \Delta_{Bk}^{(t,t+1)}(1) \right) < 0$, a contradiction. **QED**

Proof of Proposition 7

We want to find a necessary and sufficient condition for

$$\Delta W = \sum_{i=1}^{n} \int_0^1 v_i(p) \left[\delta_{Ai}^{(t,t+1)}(p) - \delta_{Bi}^{(t,t+1)}(p) \right] dp \geq 0,\ \forall W \in \mathbf{W}_{14} \tag{15}$$

For the sufficiency, note that by axiom 4 we can write: $\Delta W = \sum_{i=1}^{n} \beta_i \left[\Delta_{Ai}^{(t,t+1)}(1) - \Delta_{Bi}^{(t,t+1)}(1) \right] \geq 0$. Let $S_i = \left[\Delta_{Ai}^{(t,t+1)}(1) - \Delta_{Bi}^{(t,t+1)}(1) \right]$, $\forall i = 1,...,n$. Since by axiom 1 $\beta_i \geq \beta_{i+1}$ and $\beta_i \geq 0$ by axiom 0, we can apply Abel's lemma. Therefore, $\Delta W = \sum_{i=1}^{n} \beta_i S_i \geq 0$ if $\sum_{i=1}^{k} S_i \geq 0$, $\forall k = 1, ..., n$. Hence, $\Delta W \geq 0$ if $\sum_{i=1}^{k} \Delta_{Ai}^{(t,t+1)}(1) - \Delta_{Bi}^{(t,t+1)}(1)$, $\forall k = 1, ..., n$.

For the necessity, assume that $\Delta W = \sum_{i=1}^{n} \beta_i \left[\Delta_{Ai}^{(t,t+1)}(1) - \Delta_{Bi}^{(t,t+1)}(1)\right] \geq 0$ but $\exists k \in \{1, ...n\}$: $\sum_{i=1}^{k} S_i < 0$. If $(v_j - v_{j+1}) \searrow 0 \forall j \neq k$, $\Delta W = \sum_{i=1}^{n} \beta_i S_i = \beta_n \sum_{i=1}^{n} S_i + (v_k - v_{k+1}) \sum_{i=1}^{k} S_i$. Given that $(v_k - v_{k+1}) \sum_{i=1}^{k} S_i < 0$, it is always possible to choose combinations of β_n and $\sum_{i=1}^{n} S_i$ to contradict eq. (15). **QED**

Proof of Proposition 8

We want to find a necessary and sufficient condition for

$$\Delta W = \sum_{i=1}^{n} \int_0^1 v_i(p) \left[\delta_{Ai}^{(t,t+1)}(p) - \delta_{Bi}^{(t,t+1)}(p)\right] dp \geq 0, \forall W \in \mathbf{W}_{124} \tag{16}$$

For the necessity note that we can apply axiom 4 to obtain that $\Delta W = \sum_{i=1}^{n} \beta_i S_i$, where $S_i = \Delta_{Ai}^{(t,t+1)}(1) - \Delta_{Bi}^{(t,t+1)}(1)$. Given axiom 0 and 1 we can apply Abel's decomposition to obtain $\Delta W = \beta_n \sum_{i=1}^{n} S_i + \sum_{i=1}^{n-1} \left(\beta_i - \beta_{i+1}\right) \sum_{j=1}^{i} S_j$. Let $\sum_{j=1}^{i} S_j = \kappa_i$ and $\beta_i - \beta_{i+1} = \omega_i$, by axiom 2 $\omega_i > \omega_{i+1}$, $\forall i = 1, ..., n-1$ and recall that $v_n(p) = 0\ \forall p$ implies $\beta_n = 0$. Applying Abel's Lemma, $\Delta W = \sum_{i=1}^{n-1} \omega_i \kappa_i \geq 0$ if $\sum_{i=1}^{k} \kappa_i \geq 0$, $\forall k = 1, ..., n-1$. Substituting in the above expression: $\Delta W \geq 0$ if $\sum_{i=1}^{k} \sum_{j=1}^{i} \Delta_{Ak}^{(t,t+1)}(1) \geq \sum_{i=1}^{k} \sum_{j=1}^{i} \Delta_{Bk}^{(t,t+1)}(1)$, $\forall k = 1, ..., n-1$.

For the necessity the proof follows exactly the same argument as for proposition 7. **QED**

Empirical Appendix

Table 1B. Results of each stage of the test in Proposition 1: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>**	>	>***	>***	>***
2	>***	>***	>***	>***	>
3	>**	>***	>**	>***	<
4	>*	>**	>	<	>
5	<	>	>***	>***	>
6	>**	>***	>***	>***	>**
7	>*	>***	>***	>***	<
8	>	>	>***	>	<
9	<	>	>	>***	<**
10	>***	>***	>***	>***	>*
11	>	>**	>	>	>*
12	>	>***	>***	>***	>
13	>	>**	>***	>	>
14	<	<	>***	>***	>***
15	>	>	>**	>***	>***
16	>	>***	>***	>***	>
17	>	<	>**	>**	>
18	<**	>***	>***	>***	>**
19	<***	<**	>***	<	>
20	<	>	>*	>***	>***

Note: authors' calculation, based on the Italian "Survey on Household Income and Wealth". > means the 2002/06 process dominates the 2006/10;c< the 2002/06 process is dominated by the 2006/10. *** means statistically significant at 99%; ** statistically significant at 95%; * statistically significant at 90%.
Source: Authors' calculation from SHIW

Table 2B. Results at each stage of the test in Proposition 2: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>***	>	>***	>***	>***
2	>***	>***	>***	>***	>
3	>***	>***	>***	>***	>
4	>***	>***	>***	>***	>***
5	>***	>***	>***	>***	>***
6	>***	>***	>***	>***	>***
7	>***	>***	>***	>***	<
8	>***	>***	>***	>	<
9	>***	>***	>***	>	<
10	>***	>***	>***	>	<
11	>***	>***	>***	>***	>
12	>***	>***	>***	>	>
13	>***	>	>***	>***	>
14	>***	>	>***	>	>***
15	>***	>	>***	>	>
16	>***	>*	>***	>	>**
17	>**	>	>***	>	>
18	>***	>	>***	>**	>
19	>	>	>***	>	>
20	>	>	>***	>	>

Notes: See Notes on Table 1B.

Table 3B. Results of each stage of the test in Proposition 3: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>***	>***	>	>***	>***
2	>*	>*	>**	>***	>**
3	>***	>***	>***	>***	>*
4	>***	>***	>***	>***	>
5	>***	>***	>***	>***	>*
6	>***	>***	>***	>***	>***
7	>***	>***	>***	>***	>
8	>***	>***	>***	>	>
9	>***	>***	>***	>	>
10	>***	>***	>***	>*	>
11	>***	>***	>***	>*	>
12	>***	>***	>***	>***	>*
13	>***	>***	>***	>***	>*
14	>***	>***	>***	>**	>**
15	>***	>***	>***	>*	>
16	>***	>***	>***	>**	>*
17	>***	>***	>***	>***	>
18	>***	>***	>***	>**	>
19	>***	>***	>***	>*	>**
20	>***	>***	>***	>	>

Notes: See Notes on Table 1B.

Table 4B. Results at each stage of the test in Proposition 4: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>**	>***	>***	>***	>***
2	>***	>***	>***	>***	>***
3	>**	>***	>***	>***	>**
4	>*	>***	>***	>**	>
5	<	>*	>***	>***	>***
6	>**	>	>	>*	>**
7	>*	>***	>***	>***	>***
8	>	>***	>***	>***	>***
9	<	>	>**	>***	>***
10	>***	>***	>***	>***	>***
11	>	>*	>***	>***	>***
12	>	>***	>***	>***	>***
13	>	>***	>***	>***	>***
14	<	>*	>***	>***	>***
15	>	>**	>***	>***	>***
16	>	>***	>***	>***	>***
17	>	>*	>**	>***	>***
18	<**	>***	>***	>***	>***
19	<***	<	<	<	<**
20	<	>*	>***	>***	>***

Notes: See Notes on Table 1B.

Table 5B. Results at each stage of the test in Proposition 5: 2002/06 vs. 2006/10.

Initial quantile group	Quintiles of income change within each initial quantile group				
	0.2	0.4	0.6	0.8	1
1	>**	>***	>***	>***	>***
2	>***	>***	>**	>**	>***
3	>***	>***	>***	>***	>***
4	>*	>***	>***	>***	>***
5	>***	>**	>***	>**	>***
6	>***	>***	>***	>***	>***
7	>***	>***	>***	>*	>***
8	>***	>*	>***	>***	>***
9	>***	>***	>**	>***	>***
10	>***	>***	>***	>***	>*
11	>***	>*	>***	>***	>***
12	>***	>***	>***	>***	>***
13	>***	>***	>**	>**	>***
14	>**	>*	>**	>	>*
15	>***	>***	>***	>***	>***
16	>***	>***	>***	>***	>*
17	>**	>***	>***	>***	>***
18	>***	>**	>***	>*	>*
19	>**	>***	>***	>***	>***
20	>***	>***	>***	>***	>***

Notes: See Notes on Table 1B.

Table 6B. Results at each stage of the test in Proposition 6: 2002/06 vs. 2006/10.

Initial quantile group	Absolute na-gic
1	>***
2	>***
3	>**
4	>***
5	>*
6	>
7	>***
8	>***
9	>***
10	<***
11	>***
12	>***
13	>***
14	>***
15	>***
16	>***
17	>***
18	>***
19	<**
20	>*

Notes: See Notes on Table 1B.

Table 7B. Results at each stage of the test in Proposition 7: 2002/06 vs. 2006/10.

Initial quantile group	Absolute na-gic
1	>***
2	>***
3	>***
4	>***
5	>***
6	>***
7	>***
8	>***
9	>***
10	>*
11	>***
12	>***
13	>***
14	>*
15	>***
16	>*
17	>***
18	>*
19	>***
20	>***

Notes: See Notes on Table 1B.

Table 8B. Results at each stage of the test in Proposition 8: 2002/06 vs. 2006/10.

Initial quantile group	Absolute na-gic
1	>***
2	>***
3	>***
4	>***
5	>***
6	>***
7	>***
8	>***
9	>***
10	>***
11	>***
12	>***
13	>***
14	>***
15	>***
16	>***
17	>***
18	>***
19	>***
20	>***

Notes: See Notes on Table 1B.